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Contents

Approximation of functions on the unit bicircle in generalized Hölder spaces

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Let $\omega(\delta_1, \delta_2)$ be a modulus of continuity; $\Omega_1(\delta)$, $\Omega_2(\delta)$ are the corresponding elementary moduli of continuity [1] satisfying Zygmund-Bari-Stechkin conditions; $\gamma = \gamma_1 \times \gamma_2$ is Cartesian product of unit bicircles. Let $H_\omega(\gamma)$ denotes the space of continuous functions $x(t, \tau)$ (on γ) whose moduli of continuity satisfy the next conditions:

$$\omega(x; \delta_1, \delta_2) \le c_1 \omega(\delta_1, \delta_2);$$

$$\omega_{1,1}(x; \delta_1, \delta_2) \le c_2 \Omega_1(\delta_1) \Omega_2(\delta_2)$$

(where $\omega_{1,1}(x; \delta_1, \delta_2)$ is mixed modulus of continuity of the second order). Introduce a norm in this space in the following way:

$$\|x\|_{H_\omega} = \max_{(t,\tau)\in\gamma} |x(t,\tau)| + \sup_{\delta_1^2+\delta_2^2\neq 0} \frac{\omega(x; \delta_1, \delta_2)}{\omega(\delta_1, \delta_2)} + \sup_{\delta_1^2+\delta_2^2\neq 0} \frac{\omega_{1,1}(x; \delta_1, \delta_2)}{\Omega_1(\delta_1)\Omega_2(\delta_2)} =$$

$$= \|x\|_C + H(x;\omega) + H^{t\tau}(x;\omega).$$

$H_\omega(\gamma)$ is Banach space for this norm.
Let $(L_{mn}x)(t;\tau)$ denotes an interpolational Lagrange polynomial of the function $x(t;\tau)$ with respect to the system of equidistant points. Let operator Φ_{mn} assignes to any function $x(t;\tau) \in H_\omega$ the mn-th partial sum ot its Fourier series.
The following results are established

Theorem 1. *Let $x(t;\tau) \in H_\omega$. Then*

$$\|x - L_{mn}x\|_C \leq c \ln m \ln n H(x;\omega)\, \omega\left(\frac{1}{m}, \frac{1}{n}\right),$$

$$\|x - \Phi_{mn}x\|_C \leq (1 + c\ln(2m+1)\ln(2n+1))\, H(x;\omega)\, \omega\left(\frac{1}{m}, \frac{1}{n}\right).$$

Theorem 2. *Let $x(t;\tau) \in H_{\omega^{(1)}}$; $\omega^{(2)}(\delta_1, \delta_2)$ is such that $H_{\omega^{(1)}} \subset H_{\omega^{(2)}}$ and $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ are increasing functions. Then*

$$\|x - L_{mn}x\|_{H_{\omega^{(2)}}} \leq c \ln m \ln n \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}}, \quad (1)$$

$$\|x - \Phi_{mn}x\|_{H_{\omega^{(2)}}} \leq c \ln m \ln n \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}}.$$

Let's now consider the space L_p, $p > 1$.

Theorem 3. *Let $x(t;\tau) \in H_\omega$. Then*

$$\|x - L_{mn}x\|_{L_p} \leq (1 + c(p))\, H(x;\omega)\, \omega\left(\frac{1}{m}, \frac{1}{n}\right).$$

Remark 1. *If under the conditions of Theorems 1–3 the partial derivatives of the function $x(t;\tau)$ of orders p and q with respect to t and τ belong to the space H_ω ($x(t;\tau) \in H_\omega^{(p,q)}$), that the estimations in the mentioned theorems can be considerably improved. For example, formula (1) will look like this:*

$$\|x - L_{mn}x\|_{H_{\omega^{(2)}}} \leq c \ln m \ln n \left(\frac{1}{m^p} + \frac{1}{n^q}\right) \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}}.$$

Bibliography

[1] Natanson I.P. Constructive Function Theory. – M.-L. : Gostechizdat, 1949. – 588 p. (in Russian).