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OPTIMAL STRATEGIES CONTINUUM FOR PROJECTING THE FOUR-MOUNT CONSTRUCTION UNDER INTERVAL UNCERTAINTIES WITH INCORRECTLY PRE-EVALUATED TWO LEFT AND ONE RIGHT ENDPOINTS

There is investigated a two-person game model of optimizing cross-section squares of the four-mount construction, where the model kernel is defined on the six-dimensional hyperparallelepiped as the product of three closed intervals of unit-normed loads and of three closed intervals of unit-normed cross-section squares. For the case of incorrectly pre-evaluated two left and one right endpoints of those interval uncertainties there has been proved that the projector may obtain an optimal strategies continuum. A criterion for singularizing that continuum has been proposed.

Key words: optimizing cross-section squares, two-person game model, four-mount construction, incorrect pre-evaluation.

INVESTIGATION AREA

There are many uncertain factors in building mount constructions, one of which is interval-valued potential load on the construction pivots, pillars, bars or other mount elements [1, 2]. If the potential load on the construction with four mounts is unit-normed, then the unit-normed load on the i-th mount x_i is enclosed within the closed interval $[a_i; b_i] \subset (0; 1)$ by $b_i > a_i$ for $i = \overline{1, 3}$ [3, 4]. The nonzero fourth unit-normed load is

$$x_4 = 1 - x_1 - x_2 - x_3 \tag{1}$$

due to the unit-normalization. The problem is to take some mounting square against that potential load, and this square may be unit-normed also. Thus the i-th mount cross-section square is y_i, and due to analogous unit-normalization $y_i \in [a_i;\, b_i] \subset (0;\, 1)$ for $i = \overline{1,\, 3}$ by

$$y_4 = 1 - y_1 - y_2 - y_3 . \tag{2}$$

Obviously, that taking the unit-normed values shouldn't be arbitrary, but be optimal in the sense of minimizing the potential load and mounting square relationship [3, 5, 6].

AVAILABLE REFERENCES ANALYSIS

There is a known model of optimizing the values $\{y_i\}_{i=1}^{3}$, stated as a convex game with the kernel [3, 7]

$$T(\mathbf{X},\, \mathbf{Y}) = T(x_1,\, x_2,\, x_3;\, y_1,\, y_2,\, y_3) =$$

$$= \max\left\{ x_1 y_1^{-2},\, x_2 y_2^{-2},\, x_3 y_3^{-2},\, \frac{1 - x_1 - x_2 - x_3}{(1 - y_1 - y_2 - y_3)^2} \right\} \tag{3}$$

on the hyperparallelepiped

$$\boldsymbol{X} \times \boldsymbol{Y} = \prod_{p=1}^{2} [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] =$$

$$= \prod_{p=1}^{2} \left(\prod_{i=1}^{3} [a_i;\, b_i] \right) \subset \prod_{d=1}^{6} (0;\, 1) \subset \prod_{d=1}^{6} [0;\, 1] \subset \mathbb{R}^6 \tag{4}$$

of pure strategies

$$\mathbf{X} = [x_1 \quad x_2 \quad x_3] \in$$

$$\in \boldsymbol{X} = [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] =$$

$$= \prod_{i=1}^{3} [a_i;\, b_i] \subset \prod_{i=1}^{3} (0;\, 1) \subset \prod_{i=1}^{3} [0;\, 1] \subset \mathbb{R}^3 \tag{5}$$

of the first player and of pure strategies

$$\mathbf{Y} = [y_1 \quad y_2 \quad y_3] \in$$

$$\in \boldsymbol{Y} = [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] =$$

$$= \prod_{i=1}^{3} [a_i;\, b_i] \subset \prod_{i=1}^{3} (0;\, 1) \subset \prod_{i=1}^{3} [0;\, 1] \subset \mathbb{R}^3 \tag{6}$$

of the second. The first player personifies the natural factors, which can't be foreseen, and the second player personifies the projector or the person responsible for projecting the four-mount construction, being investigated. The optimal strategy

$$\mathbf{Y}_* = [y_1^* \quad y_2^* \quad y_3^*] \in [a_1;\, b_1] \times [a_2;\, b_2] \times [a_3;\, b_3] = \boldsymbol{Y} \tag{7}$$

of the projector, existing by the theorem on the second player optimal strategies in the convex game [3, 8], is determined from the four-parted equality

$$\frac{b_i}{(y_i^*)^2} = \frac{1 - a_1 - a_2 - a_3}{(1 - y_1^* - y_2^* - y_3^*)^2} \text{ by } i = \overline{1,\, 3} . \tag{8}$$

However, the given bounded intervals $\{[a_i;\, b_i]\}_{i=1}^{3}$ uncertainties may occur such that the equality (8) is not true [9] within the parallelepiped (6). Then it is spoken about incorrectness of left or right endpoints of those intervals. If these endpoints were corrected (increased or decreased, as needed) then the equality (8) would have been turned true within the parallelepiped (6). But this is impossible, so pre-evaluations $\{a_i\}_{i=1}^{3}$ and $\{b_i\}_{i=1}^{3}$ are held still.

WORK GOAL

Will find the projector optimal strategy (7) in supposition that here are interval uncertainties $\{[a_i;\, b_i]\}_{i=1}^{3}$ with incorrectly pre-evaluated two left and one right endpoints. If they all were correct the components of the projector optimal strategy (7) would have been [7]

$$y_i^* = \frac{\sqrt{b_i}}{\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3}} \quad \forall\, i = \overline{1,\, 3} . \tag{9}$$

But the endpoints a_p, a_q and b_k by $\{p,\, q,\, k\} = \{1,\, 2,\, 3\}$ have occurred such that

$$\frac{\sqrt{b_p}}{\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3}} < a_p , \tag{10}$$

$$\frac{\sqrt{b_q}}{\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3}} < a_q , \tag{11}$$

$$\frac{\sqrt{b_k}}{\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3}} > b_k . \tag{12}$$

It means that

$$y_p^* > \frac{\sqrt{b_p}}{\sqrt{b_1} + \sqrt{b_2} + \sqrt{b_3} + \sqrt{1 - a_1 - a_2 - a_3}} , \tag{13}$$

$$y_q^* > \frac{\sqrt{b_q}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}}, \qquad (14)$$

$$y_k^* < \frac{\sqrt{b_k}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}}, \qquad (15)$$

and for finding the projector optimal strategy (7) there has to be used the impossibility of the equality (8) within the parallelepiped (6) up with the conditions (10)–(12). Clearly that the conditions (10)–(12), meaning (13)–(15), also give one of the four following inequalities, which breaks the equality (8):

$$\frac{1}{b_k} > \frac{b_r}{a_r^2} > \frac{1-a_1-a_2-a_3}{\left(1-a_p-a_q-b_k\right)^2} \text{ by } r \in \{p, q\}, \qquad (16)$$

$$\frac{1}{b_k} > \frac{b_p}{a_p^2} \geqslant \frac{1-a_1-a_2-a_3}{\left(1-a_p-a_q-b_k\right)^2} \geqslant \frac{b_q}{a_q^2}, \qquad (17)$$

$$\frac{1}{b_k} \geqslant \frac{1-a_1-a_2-a_3}{\left(1-a_p-a_q-b_k\right)^2} > \frac{b_r}{a_r^2} \text{ by } r \in \{p, q\}, \qquad (18)$$

$$\frac{1-a_1-a_2-a_3}{\left(1-a_p-a_q-b_k\right)^2} > \frac{1}{b_k} > \frac{b_r}{a_r^2} \text{ by } r \in \{p, q\}. \qquad (19)$$

The inequalities (16)–(18) are united into the single inequality

$$\frac{1}{b_k} \geqslant \frac{1-a_1-a_2-a_3}{\left(1-a_p-a_q-b_k\right)^2} \qquad (20)$$

that is going to be the last pre-condition for finding the projector optimal strategy (7).

THEOREM ON CONTINUUM OF PROJECTOR OPTIMAL STRATEGIES (7) UNDER (10)–(12) BY (20)

Theorem. In the game with kernel (3) on the hyperparallelepiped (4) by the conditions (10)–(12) and (20) the components of the projector optimal strategy (7) are

$$y_r^* \in [a_r; b_r] \text{ by } r \in \{p, q\} \qquad (21)$$

at

$$b_p + b_q \leqslant 1 - b_k - \sqrt{b_k\left(1-a_1-a_2-a_3\right)} \qquad (22)$$

or

$$y_r^* \in \left[a_r; y_r^{\langle\max\rangle}\right] \text{ by } r \in \{p, q\} \qquad (23)$$

at

$$b_p + b_q > 1 - b_k - \sqrt{b_k\left(1-a_1-a_2-a_3\right)} \qquad (24)$$

for

$$y_p^{\langle\max\rangle} + y_q^{\langle\max\rangle} = 1 - b_k - \sqrt{b_k\left(1-a_1-a_2-a_3\right)}, \qquad (25)$$

where

$$y_k^* = b_k. \qquad (26)$$

In the case of the inequality

$$\frac{1}{b_k} > \frac{1-a_1-a_2-a_3}{\left(1-a_p-a_q-b_k\right)^2} \qquad (27)$$

the projector has a continuum of its optimal strategies (7).

Proof. As the inequality (20), being the corollary of the conditions (10)–(15), is true then the second player cannot make its payoff less than $\frac{1}{b_k}$. This payoff value is the optimal game value $v_* = \frac{1}{b_k}$, which is reached at the right endpoint of the k-th rib of the parallelepiped (6). If $y_k < b_k$ then the payoff is greater than v_*, what is unacceptable for the projector, so (26) is the single possible k-th component of the projector optimal strategy (7). Going further, the inequality (20) may be overstated as

$$\left(1-a_p-a_q-b_k\right)^2 \geqslant b_k\left(1-a_1-a_2-a_3\right) \qquad (28)$$

or

$$1-a_p-a_q-b_k \geqslant \sqrt{b_k\left(1-a_1-a_2-a_3\right)}, \qquad (29)$$

where it has been root-extracted due to positiveness of the fourth unit-normed cross-section square. So, the projector should select such y_p^* and y_q^* that the inequality

$$1-y_p^*-y_q^*-b_k \geqslant \sqrt{b_k\left(1-a_1-a_2-a_3\right)} \qquad (30)$$

would turn true. The inequality (30) is overstated as the condition

$$y_p^* + y_q^* \leqslant 1 - b_k - \sqrt{b_k\left(1-a_1-a_2-a_3\right)} \qquad (31)$$

for selecting the components y_p^* and y_q^*. Then if (22) is true then (31) is true $\forall\ y_r^* \in [a_r; b_r]$ by $r \in \{p, q\}$ and therefore the r-th component of the projector optimal strategy (7) is (21). Otherwise, if (24) is true then (31) is true for such y_p^* and y_q^* that their sum is not greater than the right side of (31). So, the r-th component is selected as (23) for (25). The continuum of projector optimal strategies with the inequality (27) is subsequent to that $y_r^* > a_r$ at $r \in \{p, q\}$ for the case with (22) and either $y_p^{\langle\max\rangle} > a_p$ or $y_q^{\langle\max\rangle} > a_q$ for the case with (24). The theorem has been proved.

It remains only to constate that in the case of the equality

$$\frac{1}{b_k} = \frac{1-a_1-a_2-a_3}{\left(1-a_p-a_q-b_k\right)^2} \tag{32}$$

the projector does not have more than the single optimal strategy (7) as here its components are $y_r^* = a_r$ by $r \in \{p, q\}$ and, undoubtedly, (26).

Consider an example of applying the proved theorem. Let

$$a_1 = 0,24\,,\ b_1 = 0,26\,,\ a_2 = 0,26\,,$$

$$b_2 = 0,29\,,\ a_3 = 0,1,\ b_3 = 0,2\,. \tag{33}$$

Having calculated the values (9), here (10)–(12) are true:

$$\frac{\sqrt{b_1}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} <$$

$$< 0,23961 < a_1 = 0,24\,, \tag{34}$$

$$\frac{\sqrt{b_2}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} <$$

$$< 0,2531 < a_2 = 0,26\,, \tag{35}$$

$$\frac{\sqrt{b_3}}{\sqrt{b_1}+\sqrt{b_2}+\sqrt{b_3}+\sqrt{1-a_1-a_2-a_3}} >$$

$$> 0,2101 > b_3 = 0,2\,. \tag{36}$$

Also the inequality (17) is true, that is (20) is the precondition for finding the projector optimal strategy (7). So, as

$$b_1 + b_2 = 0,55 > 0,5172 >$$

$$> 1 - b_3 - \sqrt{b_3\left(1-a_1-a_2-a_3\right)} \tag{37}$$

is true, that is the condition (24) is true, then here the projector optimal strategy (7) components are (23) for (25) and (26):

$$y_1^* \in \left[0,24;\ y_1^{\langle\max\rangle}\right],$$

$$y_2^* \in \left[0,26;\ y_2^{\langle\max\rangle}\right],\ y_3^* = 0,2\,, \tag{38}$$

where

$$y_1^{\langle\max\rangle} + y_2^{\langle\max\rangle} = 0,8 - 0,2\sqrt{2}\,. \tag{39}$$

CONCLUSION AND WORKING FURTHER PERSPECTIVE

The disclosed continuum of the projector optimal strategy (7) under strictness in the inequality sign must not delude an explorer or a real constructor, because the summed unit-normed cross-section squares are fixed at unit, and it may select y_p^* and y_q^* whatever, just satisfying (21) or (23) with (25). Nevertheless, this selection could have been rational due to some criterion with respect to y_p^*, y_q^*, $y_4^* = 1 - y_p^* - y_q^* - b_k$. One of suchlike criterions is equalization of values y_p^*, y_q^*, y_4^*, that is solving the problem

$$\min_{\left[a_p;\, b_p\right]\times\left[a_q;\, b_q\right]} \left(\max\left\{\frac{y_p^*}{y_q^*}, \frac{y_q^*}{y_p^*}, \frac{y_p^*}{y_4^*}, \frac{y_4^*}{y_p^*}, \frac{y_q^*}{y_4^*}, \frac{y_4^*}{y_q^*}\right\}\right) \tag{40}$$

or

$$\min_{\left[a_p;\, y_p^{\langle\max\rangle}\right]\times\left[a_q;\, y_q^{\langle\max\rangle}\right]} \left(\max\left\{\frac{y_p^*}{y_q^*}, \frac{y_q^*}{y_p^*}, \frac{y_p^*}{y_4^*}, \frac{y_4^*}{y_p^*}, \frac{y_q^*}{y_4^*}, \frac{y_4^*}{y_q^*}\right\}\right). \tag{41}$$

In perspective, there should be worked the situation with the inequality (19), which seems to be more complicated in finding the projector optimal strategy (7).

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Романюк В. В.

КОНТИНУУМ ОПТИМАЛЬНЫХ СТРАТЕГИЙ ДЛЯ ПРОЕКТИРОВАНИЯ ЧЕТЫРЕХОПОРНОЙ КОНСТРУКЦИИ В УСЛОВИЯХ ИНТЕРВАЛЬНЫХ НЕОПРЕДЕЛЕННОСТЕЙ С ПРЕДВАРИТЕЛЬНО НЕКОРРЕКТНО ОЦЕНЕННЫМИ ДВУМЯ ЛЕВЫМИ И ОДНИМ ПРАВЫМ КОНЦАМИ

Исследуется игровая модель двух лиц в оптимизации площадей поперечного сечения четырехопорной конструкции, где модельное ядро определяется на шестимерном гиперпараллелепипеде как произведении трех замкнутых интервалов единично нормированных нагрузок и трех замкнутых интервалов единично нормированных площадей поперечного сечения. Для случая предварительно некорректно оцененных двух левых и одного правого концов этих интервальных неопределенностей доказано, что проектировщик может получать континуум оптимальных стратегий. Предложен критерий для выделения единственного элемента из этого континуума.

Ключевые слова: оптимизация площадей поперечного сечения, игровая модель двух лиц, четырехопорная конструкция, некорректное предварительное оценивание.

Романюк В. В.

КОНТИНУУМ ОПТИМАЛЬНИХ СТРАТЕГІЙ ДЛЯ ПРОЕКТУВАННЯ ЧОТИРЬОХОПОРНОЇ КОНСТРУКЦІЇ В УМОВАХ ІНТЕРВАЛЬНИХ НЕВИЗНАЧЕНОСТЕЙ З ПОПЕРЕДНЬО НЕКОРЕКТНО ОЦІНЕНИМИ ДВОМА ЛІВИМИ Й ОДНИМ ПРАВИМ КІНЦЯМИ

Досліджується ігрова модель двох осіб в оптимізації площ поперечного перерізу чотирьохопорної конструкції, де модельне ядро задається на шестивимірному гіперпаралелепіпеді як добутку трьох замкнених інтервалів одинично нормованих навантажень і трьох замкнених інтервалів одинично нормованих площ поперечного перерізу. Для випадку попередньо некоректно оцінених двох лівих й одного правого кінців цих інтервальних невизначеностей доведено, що проектувальник може отримувати континуум оптимальних стратегій. Запропоновано критерій для виокремлення єдиного елемента з цього континуума.

Ключові слова: оптимізація площ поперечного перерізу, ігрова модель двох осіб, чотирьохопорна конструкція, некоректне попереднє оцінювання.