MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
Zaporizhzhia Polytechnic National University

CALCULATION TASKS ON HIGHER MATHEMATICS (3rd module)

for students majoring in
141 Power Engineering, Electrical Engineering
and Electrical Mechanics

2020

Calculation tasks on higher mathematics (3rd module) for students majoring in 141 Power Engineering, Electrical Engineering and Electrical Mechanics / Compiler: Snizhko Nataliia. – Zaporizhzhia: Zaporizhzhia Polytechnic National University, 2020. – 30 p.

Compiler: Snizhko Nataliia,
candidate of science (physics and mathematics),
associate professor

Executive editor: Snizhko Nataliia,
candidate of science (physics and mathematics),
associate professor

Reviewer: Antonenko Nina.
candidate of science (physics and mathematics),
associate professor

Major expert: Antonov Mykola,
candidate of science (engineering),
associate professor

English language adviser: Khavkina Olena,
candidate of science (philology),
associate professor

Approved
by the Department of Higher Mathematics
(Minutes № 7 of 12.02.2020)

Recommended
by the Scientific and Methodological Council
of the Faculty of Mechanical Engineering
(Minutes № 4 of 25.02.2020)

CONTENTS

INTRODUCTION

One of the types of unaided work of students is the execution of calculation tasks during the semester. Their purpose is to develop practical skills of students. They are designed to help students capture theoretical material more deeply and learn how to apply the acquired knowledge to solve practical problems. The offered typical calculation tasks correspond to the course "Higher Mathematics" taught to students majoring in Power Engineering, Electrical Engineering and Electrical Mechanics full-time in the third module of the two-semester course. This set of tasks can also be used for students in other technical majors who study higher mathematics in a two-semester course.

This task book covers virtually all major sections of the third module of the course on higher mathematics; it contains assignments for all the topics of the course, and offers 15 options for unaided work. A list of recommended literature is also provided. Tasks for solving are selected mainly from task books [6, 7, 18, 20]. Examples of solving such problems are contained in the manual [31]. For ease of use, in this manual the numbering of examples corresponds to the numbering of tasks in the given task book. In addition to the methodological instructions for solving problems and examples of solutions, manual [30] also contains the necessary theoretical information and reference material (tables, literature, etc.).

Calculation tasks are executed during the semester, which provides students with a systematic study of the course. While carrying out these tasks, students work with recommended textbooks and manuals, independently search for necessary literary sources and materials, analyze them and summarize, independently research and make written presentation of practical assignments.

The student chooses the option (the number of the variant) according to his number in the register list. The work is done in English in writing, preferably in a notebook in a cell. Note fields must be left blank. The name of the subject, major, group and course, surname, first name and patronymic of the student, name of the teacher who accepts the work should be indicated on the title page of the work.

While performing the work, the student should solve the offered tasks by the methods specified in the tasks, as well as make all the necessary drawings (graphical solutions). The student must show the acquired theoretical knowledge of the course.

When doing the work, the student can use both the lecture and practical material, as well as the supporting literature listed at the end of this manual. Before starting the work, it is recommended to study the relevant theoretical material, then to understand the solutions of the tasks that were performed in the practical classes, and only after that to start the actual calculation work.

When evaluating a work, the indicator of its quality is, first of all, how the student independently and correctly solved the tasks and understood the content of the obtained solutions. That is why additional questions may be asked to protect the student's work, including the theoretical material presented for the exam.

Completed work is submitted to the teacher for verification and subsequent protection in the form of an interview (usually during modular control). The student must be able to:

– present the content of the tasks and to prove their solutions;
– answer questions about the content of the solutions obtained;
– answer additional questions.

If the work is successfully protected, the student receives a certain number of rating points. If the specified requirements are not fulfilled, then the work is returned to the student for completion, indicating the term of re-protection.

1 FUNCTIONS OF COMPLEX VARIABLE

Task 1.1. Express a complex number in trigonometric and power forms. Make a graph.

1) a) $z=-2+2\sqrt{3}\,i$; b) $z=-\sqrt{29}$; c) $z=i\sqrt{5}$.

2) a) $z=\sqrt{3}$; b) $z=1-\sqrt{3}\,i$; c) $z=-15i\sqrt{2}$.

3) a) $z=\sqrt{2}-\sqrt{2}\,i$; b) $z=-\sqrt[5]{64}$; c) $z=-i\sqrt[3]{31}$.

4) a) $z=\sqrt[3]{5}-\sqrt[3]{5}\,i$; b) $z=-3{,}5i$; c) $z=\sqrt{2}$.

5) a) $z=3+i$; b) $z=-12i$; c) $z=-3^{7}$.

6) a) $z=-3^{15}+i\cdot 3^{15}$; b) $z=-i\sqrt{3}$; c) $z=-\sqrt[15]{3}$.

7) a) $z=2+2\sqrt{3}\,i$; b) $z=i\cdot 2^{-\sqrt{5}}\,4$; c) $z=-25^{\sqrt[3]{3}}$.

8) a) $z=\sqrt{3}-i$; b) $z=\sqrt{26}\,i$; c) $z=-3^{\sqrt{3}}$.

9) a) $z=\sqrt[6]{23}-\sqrt[6]{23}\,i$; b) $z=\dfrac{3}{25}$; c) $z=-6^{\sqrt[7]{6}}$.

10) a) $z=-\sqrt{2}-i\sqrt{2}$; b) $z=\sqrt[5]{6}\,i$; c) $z=15^{\sqrt{6}}$.

11) a) $z=\sqrt[5]{6}-i\sqrt[5]{6}$; b) $z=9{,}9i$; c) $z=\sqrt[5]{26}$.

12) a) $z=-1-i$; b) $z=-4\sqrt{3}$; c) $z=5^{100}$.

13) a) $z=-\sqrt{13}\,i$; b) $z=-2\sqrt{3}-2i$; c) $z=-5^{25}$.

14) a) $z=2-2\sqrt{3}\,i$; b) $z=-\sqrt{26}$; c) $z=i\sqrt{7}$.

15) a) $z = \sqrt{11}$; b) $z = 1 + \sqrt{3}\,i$; c) $z = -13\sqrt{2}$.

Task 1.2. Express a complex number in rectangular form $a + ib$. Make a graph.

1) a) $w = (3 - i)(2 + 3i)$; b) $w = \dfrac{1 + 2i}{\frac{1}{3} + \frac{1}{2}i}$;

c) $w^8 - i = 0$.

2) a) $w = (2 + i)(-1 + 3i)$; b) $w = \dfrac{-1 + 3i}{\frac{1}{4} - \frac{1}{3}i}$;

c) $w^3 - i + 1 = 0.$

3) a) $w = (1 - i)(-3 - 2i)$; b) $w = \dfrac{1 - 2i}{\frac{1}{3} - \frac{1}{4}i}$;

c) $w^4 - 1 + i\sqrt{3} = 0$.

4) a) $w = (3 + i)(1 + 2i)$; b) $w = \dfrac{1 + i}{-\frac{3}{4} - \frac{1}{3}i}$;

c) $w^6 - 1 + i\sqrt{3} = 0$.

5) a) $w = (3 - 2i)(1 + 2i)$; b) $w = \dfrac{-2 + 3i}{-\frac{1}{3} - \frac{1}{2}i}$;

c) $w^5 + 1 = 0$.

6) a) $w=(3-2i)(-3+i)$; b) $w=\dfrac{1-i}{\frac{1}{4}-\frac{3}{4}i}$; c) $w^8+1=0$.

7) a) $w=(1-i)(-2+3i)$; b) $w=\dfrac{-1+i}{\frac{2}{3}-\frac{1}{3}i}$; c) $w^6-1=0$.

8) a) $w=(2-i)(1+2i)$; b) $w=\dfrac{3-i}{\frac{1}{3}-\frac{2}{3}i}$; c) $w^8-1=0$.

9) a) $w=(1+2i)(-1+3i)$; b) $w=\dfrac{-2-3i}{\frac{1}{2}-\frac{1}{3}i}$; c) $w^6+2-2i=0$.

10) a) $w=(1+2i)(3+i)$; b) $w=\dfrac{-1+2i}{\frac{1}{3}+\frac{2}{3}i}$; c) $w^6+i=0$.

11) a) $w=(2+3i)(1-2i)$; b) $w=\dfrac{1+2i}{\frac{1}{4}+\frac{1}{2}i}$; c) $w^8+i=0$.

12) a) $w=(-1-2i)(-1+3i)$; b) $w=\dfrac{-1-i}{\frac{1}{3}-\frac{1}{2}i}$;

c) $w^4-1+i=0$.

13) a) $w=(-2-3i)(1-2i)$; b) $w=\dfrac{1+2i}{\frac{1}{2}+\frac{1}{2}i}$;

c) $w^6-i=0$.

14) a) $w=(3+i)(2+3i)$; b) $w=\dfrac{1+2i}{\frac{1}{3}-\frac{1}{2}i}$;

c) $w^6+1=0$.

15) a) $w=(2+i)(-1-3i)$; b) $w=\dfrac{-1+3i}{\frac{1}{4}+\frac{1}{3}i}$;

c) $w^6-1-i\sqrt{3}=0.$

Task 1.3. Using DeMoivre's law compute the given quantities.

1) $\left(\sqrt{3}+i\right)^{126}$ 2) $(1+i)^{137}$

3) $(2+2i)^{28}$ 4) $\left(-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}\right)^{30}$

5) $\left(\sqrt{3}-3i\right)^{36}$ 6) $\left(1+i\sqrt{3}\right)^{40}$

7 $\left(\sqrt{3}+3i\right)^{48}$ 8) $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{24}$

9) $\left(\sqrt{3}-i\right)^{128}$ 10) $\left(-\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{46}$

11) $\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)^{52}$ 12) $\left(3-i\sqrt{3}\right)^{24}$

13) $\left(3+i\sqrt{3}\right)^{30}$ 14) $\left(1-i\right)^{72}$

15) $\left(1-i\right)^{137}$

Task 1.4. Find the set of points on complex plane defined by the following conditions. Make a graph.

1) a) $|z-8|<\left|1-8\overline{z}\right|$; b) $\begin{cases} |z|=4, \\ \frac{\pi}{2}<arg\,z<\pi. \end{cases}$

2) a) $Im\left(\overline{z}\right)^2<1$; b) $\begin{cases} -\frac{\pi}{4}<Re\,z<\frac{\pi}{4}, \\ -\infty<Im\,z<\infty. \end{cases}$

3) a) $Im\frac{1}{z}<-\frac{1}{2}$; b) $\begin{cases} |z|=4, \\ \pi<arg\,z<\frac{3\pi}{2}. \end{cases}$

4) a) $\frac{1}{4}<Re\frac{1}{z}+Im\frac{1}{z}<\frac{1}{2}$; b) $\begin{cases} 0<Im\,z<\pi, \\ -\infty<Re\,z<9. \end{cases}$

5) a) $|z| = Im\,z - 1$; b) $\begin{cases} |z| = 2, \\ -\dfrac{\pi}{2} < arg\,z < 0. \end{cases}$

6) a) $|z - 4| > |1 - 4\bar{z}|$; b) $\begin{cases} 0 < Re\,z < \dfrac{\pi}{2}, \\ 0 < Im\,z < \infty. \end{cases}$

7) a) $Re(\bar{z})^2 = 1$; b) $\begin{cases} -\pi < Re\,z < \pi, \\ -\infty < Im\,z < 0. \end{cases}$

8) a) $Re(1 + z) = |z|$; b) $\begin{cases} |z| < \infty, \\ -\dfrac{\pi}{4} < arg\,z < \dfrac{\pi}{4}. \end{cases}$

9) a) $|z - 3| < |1 - 3\bar{z}|$; b) $\begin{cases} 0 < Im\,z < \pi, \\ 0 < Re\,z < \infty. \end{cases}$

10) a) $|z| - 3\,Im\,z = 6$; b) $\begin{cases} Im\,z > 0, \\ Re\,z > 0. \end{cases}$

11) a) $3|z| - Re\,z = 12$; b) $\begin{cases} |z| < \infty, \\ \dfrac{\pi}{2} < arg\,z < \pi. \end{cases}$

12) a) $|z| = Re\,z + 1$; b) $\begin{cases} -\pi < Im\,z < 0, \\ 0 < Re\,z < \infty. \end{cases}$

13) a) $Im\,\overline{\left(z^2 - \bar{z}\right)} = 2 - Im\,z$; b) $\begin{cases} |\bar{z}| = 3, \\ 0 < arg\,z < \dfrac{\pi}{2}. \end{cases}$

14) a) $|z-1| < |1-\overline{z}|$; b) $\begin{cases} |z| = 5, \\ \dfrac{\pi}{2} < arg\, z < \pi. \end{cases}$

15) a) $Im(\overline{z})^2 > 2$; b) $\begin{cases} -\dfrac{3\pi}{4} < Re\, z < \dfrac{3\pi}{4}, \\ -\infty < Im\, z < \infty. \end{cases}$

Task 1.5. Find the analytic function having its real or imaginary part $u(x,y)$, $v(x,y)$.

1) $u(x,y) = 2^x \cos(y \ln 2)$

2) $v(x,y) = 2^x \sin(y \ln 2)$

3) $u(x,y) = x^2 - y^2 + 1$

4) $v(x,y) = -2y - 2xy$

5) $u(x,y) = x^3 - 3xy^2 + y$

6) $v(x,y) = 2xy - 17y$

7) $u(x,y) = x^2 - y^2 - 7x$

8) $u(x,y) = \cos x\, ch y$

9) $v(x,y) = -2 \sin x\, sh\, y$

10) $v(x,y) = 3x^2 y - y^3$

11) $u(x,y) = 2x + x^2 - y^2$

12) $v(x,y) = -2xy + 8x$

13) $v(x,y) = -e^{-2x} \sin 2y$

14) $u(x,y) = e^{-2x} \cos 2y$

15) $v(x,y) = 2 \cos x\, ch y$

2 DIFFERENTIAL EQUATIONS

Task 2.1. Find the general solution of the separable differential equation.

1) $\dfrac{dx}{dt} = 4t^3\sqrt{x}$

2) $3x^2\dfrac{dy}{dx} - 1 = cos\,2y$

3) $sin\,x \cdot ctg\,y\,dx - \dfrac{dy}{sin\,x} = 0$

4) $2x^2yy' - y^2 = 8$

5) $\left(1 + e^y\right)dx - e^{2y}\,sin^3\,xdy = 0$

6) $y'ctgx + y = 2$

7) $\dfrac{dz}{dx} = 5^{x-z}$

8) $2y\dfrac{dy}{dt} + 5t^4 = 1$

9) $\dfrac{dy}{dx} = \dfrac{2(y+1)}{x+1}$

10) $xy' + y = 2y^2$

11) $x\left(1 + y^2\right) + y\left(1 + x^2\right)\dfrac{dy}{dx} = 0$

12) $x^3y^2y' + 2 = 2y$

13) $\left(x^2 - 1\right)\dfrac{dy}{dx} + 2xy = 0$

14) $\dfrac{dy}{dx} = x^2y^2 + 2xy^2$

15) $sin\,y\,dx + x\,cos\,y\,dy = 0$

Task 2.2. Find the general solution of the homogeneous differential equation.

1) $x^2y' = y^2 - 2xy$

2) $ydx + \left(2x^2/y + x\right)dy = 0$

3) $2xdy = (x + 2y)dx$

4) $xy' = y - 5xe^{5y/x}$

5) $ln(y/x)xy' = y$

6) $\dfrac{dy}{dx} = \dfrac{y+6x}{x}$

7) $xy' - y = (x+y)ln\dfrac{x+y}{x}$

8) $(5x+2y)dx - xdy = 0$

9) $(x-y)dx + (x+y)dy = 0$

10) $xy' - y = 5xctg\dfrac{y}{x}$

11) $\dfrac{dy}{dx} = \dfrac{xy + y^2 e^{-x/y}}{x^2}$

12) $2x^3 y' = y(2x^2 - y^2)$

13) $y^2 + x^2 y' = xyy'$

14) $(x^2 + y^2)y' = 2xy$

15) $(y^3 - 2x^2 y)dx + x^3 dy = 0$

Task 2.3. Find the general solution of the linear differential equation of the first order or Bernoulli's equation.

1) $y' = \dfrac{x}{y}e^{2x} + y$

2) $\dfrac{dx}{x} = \left(\dfrac{1}{y} - 2x\right)dy$

3) $\dfrac{dy}{dx} = y^4 \cos x + ytgx$

4) $x\dfrac{dy}{dx} + y = \sin x$

5) $x\dfrac{dy}{dx} + y = ln\,x + 1$

6) $x^2\dfrac{dy}{dx} + xy + 1 = 0$

7) $xy' - 2y + x^2 = 0$

8) $\dfrac{dy}{dx} - 2xy = 3x^2 - 2x^4$

9) $\dfrac{dy}{dx} - yctgx = 2x\sin x$

10) $y' + 2y = y^2 e^x$

11) $\frac{dy}{dx}=\frac{y}{2x}+\frac{x^2}{2y}$

12) $\left(xy+e^x\right)dx-xdy=0$

13) $x^2y'+xy+1=0$

14) $\frac{dy}{dx}-y\cos x=e^{-\sin x}$

15) $xy'-2x^2\sqrt{y}=4y$

Task 2.4. Find the general solution of the exact differential equation.

1) $\left(3y^2+2xy+2x\right)dx+\left(6xy+x^2+3\right)dy=0$

2) $(x+y+1)dx+\left(x-y^2+3\right)dy=0$

3) $e^{-y}dx-\left(2y+xe^{-y}\right)dy=0$

4) $\left(4-\frac{y^2}{x^2}\right)dx+\frac{2y}{x}dy=0$

5) $(\sin 2x+2\cos(x+y))dx-2\cos(x+y)dy=0$

6) $xy^2dx+y\left(x^2+y^2\right)dy=0$

7) $\left(xe^x+\frac{y}{x^2}\right)dx-\frac{1}{x}dy=0$

8) $\frac{2x}{y^3}dx+\frac{y^2-3x^2}{y^4}dy=0$

9) $3x^2e^ydx+\left(x^3e^y-1\right)dy=0$

10) $\left(2x+3x^2y\right)dx+\left(x^3-3y^2\right)dy=0$

11) $\left(2xy+3y^2\right)dx+\left(x^2+6xy-3y^2\right)dy=0$

12) $\left(2-9xy^2\right)xdx+\left(-6x^3+4y^2\right)ydy=0$

13) $2xydx+\left(x^2-y^2\right)dy=0$

14) $3x^2(1+ln\,y)dx-\left(2y-\dfrac{x^3}{y}\right)dy=0$

15) $2x\left(1+\sqrt{x^2-y}\right)dx-\sqrt{x^2-y}dy=0$

Task 2.5. Find the general solution of the differential equation of order higher than the first.

1) $x^2y''=y'^2$

2) $y''\left(e^x+1\right)+y'=0$

3) $y'''=2(y''-1)ctgx$

4) $y''=\dfrac{y'}{x}+\dfrac{x^2}{y'}$

5) $xy''+xy'^2-y'=0$

6) $y'''=y''-xy''$

7) $y''\left(x^2+1\right)=2xy'$

8) $2xy''+y'''=0$

9) $y''(2y'+x)=1$

10) $\left(1-x^2\right)y''+xy'=2$

11) $2xy'y''=y'^2-1$

12) $\left(1+x^2\right)y''+y'^2+1=0$

13) $xy''=y'+x\left(y'^2+x^2\right)$

14) $xyy''-xy'^2=yy'$

15) $xy'''=y''+xy''$

Task 2.6. Solve the Cauchy's problem.

1). $yy''-y'^2=y^2\,ln\,y,\ y(0)=y'(0)=1$

2) $2y'' = 3y^2$, $y(-2) = 1$, $y'(-2) = 1$

3) $yy'' = y'^2 - y'^3$, $y(1) = 1$, $y'(1) = -1$

4) $y^3 y'' = -1$, $y(1) = 1$, $y'(1) = 0$

5) $y^4 - y^3 y'' = 1$, $y(0) = \sqrt{2}$, $y'(0) = \dfrac{\sqrt{2}}{2}$

6) $y'' = e^{2y}$, $y(0) = 0$, $y'(0) = 1$

7) $y'' = 2\sqrt{y'}$, $y(0) = 1$, $y'(0) = 1$

8) $2y'^2 = y''(y-1)$, $y(1) = 2$, $y'(1) = -1$

9) $3y'y'' = e^y$, $y(-3) = 0$, $y'(-3) = 1$.

10) $y'' \cos y + y'^2 \sin y - y' = 0$, $y(-1) = \dfrac{\pi}{6}$, $y'(-1) = 2$

11) $y'y''' - 3y''^2 = 0$, $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$

12) $\dfrac{y''}{y'} = \dfrac{2yy'}{(1+y^2)}$, $y(0) = 0$, $y'(0) = 1$

13) $yy'' - y'^2 = y^2$, $y(0) = 1$, $y'(0) = 0$

14) $2yy'' - 3y'^2 = 4y^2$, $y(0) = 1$, $y'(0) = 0$

15) $y''^2 = 4(y' - 1)$, $y(0) = 0$, $y'(0) = 2$

Task 2.7. Find the general solution of the linear differential equation using method of undetermined coefficients.

1) $y'' - 5y' = 3x^2 + \sin 5x$ 2) $y'' - 2y' + y = 3xe^x$

3) $y'' - 2y' + y = 2xe^{x}$ 4) $y'' - 5y' = 4x^{2} - e^{5x}$

5) $y'' + 2y' + y = xe^{-x} + cos\,x$ 6) $y'' - 2y' + y = 4e^{x}$

7) $y^{V} - 2y^{IV} + y''' = 3x + 5e^{x}$ 8) $y^{IV} - 8y' = xe^{2x} + x$

9) $y'' - 2y' + 2y = e^{x} + x\,cos\,x$ 10) $y'' + y = 4xe^{x}$

11) $y'' - y = 2\,sin\,x - 4\,cos\,x$ 12) $y'' + 4y = sin\,2x$

13) $y'' + y = cos\,x + cos\,2x$ 14) $y'' - 3y' = e^{3x} - 18x$

15) $y'' - 4y' + 3y = xe^{x} - 3e^{3x}$

Task 2.8. Find the general solution of the linear differential equation using method of undetermined coefficients (without finding the numerical values of the coefficients).

1) $y'' + 2y' + 5y = e^{x}\left((2x+1)cos\,2x + sin\,2x\right)$

2) $y'' - 4y' = (x-1)^{2} + x^{2}e^{4x}$

3) $y^{IV} - y = (x+2)cos\,x + e^{x}$

4) $y''' - y = e^{-x/2}\,cos\dfrac{\sqrt{3}}{2}x + sin\,x$

5) $y^{VI} - y^{V} = 2xe^{x} + x$

6) $y'' + 4y = e^{2x}\left(x^{2}\,sin\,2x + cos\,2x\right)$

7) $y''' - 3y'' + 3y' - y = 2xe^{x} + 4$

8) $y''' - 2y'' + y' = 4x + e^{x} + cos\,4x$

9) $y'' + 3y' + 2y = e^{-2x} + 2$

10) $y^{IV} + y'' = 7x - 3\cos x$

11) $y'' + 4y' = 2x + e^{-2x} + x\cos 2x$

12. $y''' - 9y' = 3 + e^{3x} + e^{3x}\sin 3x$

13. $y''' - 5y'' + 4y' = x^2 + 2x^2e^x + \cos x$

14. $y^{IV} - 5y''' + 6y'' = 6\sin x + xe^{2x} + e^{2x}\cos 3x$

15. $y^{IV} - 4y''' + 5y'' = x^2\cos 2x + 2e^x\sin 2x$

3 OPERATIONAL CALCULUS

Task 3.1. Find a Laplace transforms for a given function.

1) $f(t) = sin^2 t$
2) $f(t) = cos^3 t$
3) $f(t) = sin^4 t$
4) $f(t) = sin\, 2t\, cos\, 3t$
5) $f(t) = sin\, 2t\, sin\, 3t$
6) $f(t) = cos\, 2t\, cos\, 3t$
7) $f(t) = cos^2 t$
8) $f(t) = sin^3 t$
9) $f(t) = cos^4 t$
10) $f(t) = t\, sin\, 3t$
11) $f(t) = t\, cos\, 2t$
12) $f(t) = te^t$
13) $f(t) = t^2 cos\, t$
14) $f(t) = t \cdot sh3t$
15) $f(t) = e^{5t} cos\, 7t$

Task 3.2. Find an original function for a given Laplace transforms.

1) $F(p) = \dfrac{e^{-p}}{p^2 - 1}$
2) $F(p) = \dfrac{e^{-p/3}}{p(p^2 + 1)}$
3) $F(p) = \dfrac{e^{-p/2}}{p(p+1)(p^2+4)}$
4) $F(p) = \dfrac{e^{-p}}{p^2} + \dfrac{2e^{-2p}}{p^3} + \dfrac{6e^{-3p}}{p^4}$
5) $F(p) = \dfrac{e^{-p}}{p^2 - 2p + 5}$
6) $F(p) = \dfrac{e^{-2p} + 2e^{-3p} + 3e^{-4p}}{p^2 + 1}$

7) $F(p)=\dfrac{pe^{-2p}}{p^2-4}$ 8) $F(p)=\dfrac{e^{-p}}{p(p-1)}$

9) $F(p)=\dfrac{e^{-3p}}{(p+1)^2}$ 10) $F(p)=\dfrac{pe^{-2p}}{p^2+9}$

11) $F(p)=\dfrac{e^{-p/3}}{p^2+4p+3}$ 12) $F(p)=\dfrac{e^{-2p}}{p^2}$

13) $F(p)=\dfrac{2e^{-p}}{p^3}$ 14) $F(p)=\dfrac{3pe^{-2p}}{p^2+9}$

15) $F(p)=\dfrac{pe^{-4p}}{(p+1)^2}$

Task 3.3. Solve the Cauchy's problem.

1) $x''+2x'-3x=e^{-t}$, $x(0)=0$, $x'(0)=1$

2) $x''+x=1$, $x(0)=-1$, $x'(0)=0$

3) $x''+x=2\,sin\,t$, $x(0)=1$, $x'(0)=-1$

4) $x'''+x'=e^{t}$, $x(0)=0$, $x'(0)=2$, $x''(0)=0$

5) $x''+2x'=t\,sin\,t$, $x(0)=x'(0)=0$

6) $x'''+x'=cos\,t$, $x(0)=0$, $x'(0)=-2$, $x''(0)=0$

7) $x''+x'=1$, $x(0)=0$, $x'(0)=1$

8) $x''+4x=t$, $x(0)=1$, $x'(0)=0$

9) $x'' + x = 0$, $\quad x(0) = 1$, $x'(0) = 0$

10) $x'' - 2x' + 5x = 1 - t$, $\quad x(0) = x'(0) = 0$

11) $x'' = 1$, $\quad x(0) = 0$, $x'(0) = 1$

12) $x'' + x' = \cos t$, $\quad x(0) = 2$, $x'(0) = 0$

13) $x'' + 3x' = e^t$, $\quad x(0) = 0$, $x'(0) = -1$

14) $x''' + 3x'' + 3x' + x = 1$, $\quad x(0) = x'(0) = x''(0) = 0$

15) $x'' - 2x' = e^{2t}$, $\quad x(0) = x'(0) = 0$

REFERENCES

1. Анпілогов, Д.І. Диференціальні рівняння [Текст]: навч. посібник / Д.І.Анпілогов, Н.В.Сніжко. – Запоріжжя: НУ "Запорізька політехніка", 2019. – 176 с.
2. Берман Г.Н. Сборник задач по курсу математического анализа / Г.Н. Берман. – М. : Наука. – 2005. – 416 с.
3. Бугров Я.С. Высшая математика. Дифференциальное и интегральное исчисление : учебник для студентов инженерно-технических специальностей вузов / Я.С. Бугров, С.М. Никольский. – М. : Наука, 1980. – 432 с.
4. Данко П.Е. Высшая математика в упражнениях и задачах : в 2 ч. / Данко П.Е., Попов А.Г., Кожевникова Т.Я. – М. : Высшая школа, 2003. – Ч. 2. – 2003. – 416 с.
5. Данко П.Е. Высшая математика в упражнениях и задачах : в 2 ч. / Данко П.Е., Попов А.Г., Кожевникова Т.Я. – М. : Высшая школа, 2003. – Ч. 3. – 2003. – 415 с.
6. Краснов М.Т., Киселев А.И., Макаренко Г.И. Функции комплексного переменного. Операционное исчисление. Теория устойчивости. / Краснов М.Т., Киселев А.И., Макаренко Г.И. – М.: Наука, 1981
7. Кузнецов Л.А. Сборник задач по высшей математике (типовые расчёты) / Кузнецов Л.А. – М.: Высшая школа, 1983. – 175с.
8. Кузьмин Ю. А. Краткий англо-русский технический словарь / Кузьмин Ю. А., Владимиров В. А., Гельман Я. Л. и др. – Издательство: Московская международная школа переводчиков, 1992. – 416 с.
9. Лаврентьев М.А., Шабат Б.В. Методы теории функций комплексного переменного / Лаврентьев М.А., Шабат Б.В. – М. : Наука, 1987. – 688 с.
10. Овчинников П. П., Михайленко В. М. Вища математика: Підручник. У 2 ч. Ч. 2: Диференціальні рівняння. Операційне числення. Ряди та їх застосування. / За ред. П.П. Овчинникова. – К.: Техніка, 2004. – 792 с.

11. Пискунов Н.С. Дифференциальное и интегральное исчисление для ВТУЗов : в 2 т. / Н.С. Пискунов. – М. : Наука, 1985. – Т. 1. – 1985. – 526 с.
12. Пискунов Н.С. Дифференциальное и интегральное исчисление для ВТУЗов : в 2 т. / Н.С. Пискунов. – М. : Наука, 1985. – Т. 2. – 576 с.
13. Письменный Д.Т. Конспект лекций по высшей математике: полный курс / Д.Т. Письменный. – М. : Айрис-пресс, 2006. – 608 с.
14. Сборник задач по курсу высшей математики / под ред. Г.И.Кручковича. – М.: Высш. Шк.., 1973
15. Сборник задач по математике для вузов. Ч. 1, 2 / под ред. А.Е.Ефимова, В.П. Демидовича. М.: Высш. Шк.., 1978.
16. Сборник задач по математике для ВТУЗов. Ч.II. Специальные разделы математического анализа / под ред. В.А. Ефимова, Б.П. Демидовича. – М. : Наука, 1986. – 615с.
17. Сборник индивидуальных заданий по высшей математике (в трёх частях), Ч. 1 / под ред. А.П.Рябушко. – Минск: Вышейшая школа, 1990. – 271с.
18. Сборник индивидуальных заданий по высшей математике (в трёх частях), Ч. 2 / под ред. А.П.Рябушко. – Минск: Вышейшая школа, 1991. – 352с.
19. Сборник индивидуальных заданий по высшей математике (в трёх частях), Ч. 3 / под ред. А.П.Рябушко. – Минск: Вышейшая школа, 1991. – 288 с.
20. Филиппов А.Ф. Сборник задач по дифференциальным уравнениям / А.Ф.Филиппов. – М. : Наука. – 1965. – 100 с.
21. Ahlfors L.V. Complex Analysis / L.V. Ahlfors. – McGrah Hill, 1966. – 317 p.
22. Beerends R. J. Fourier and Laplace Transforms / Beerends R. J., et al. – Cambridge University Press, 2003. – 458 p.
23. Chasnov J. R. Introduction to Differential Equations. Lecture notes for MATH / Chasnov J. R. – The Hong Kong University of Science and Technology, 2009. – 128 p.
24. Efimov A.V. Mathematical Analysis (Advance Topics). Part 1. General Functional Series and Their Application / Efimov A.V. – Moscow: Mir Publishers, 1985. – 356 p.

25. Kodaira K. Introduction to Complex Analysis / K. Kodaira. – Cambridge University Press, 2007. – 400 p.
26. Kuznetsov B. Russian-English Polytechnic Dictionary / Kuznetsov B. – Moscow: Russian Language Publishers, 1980. – 723 p.
27. Smirnov V.I. A course of Higher Mathematics / V.I. Smirnov; transl. from Russian by D.E. Brown. – Pergamon Press, 1964. – 543 p.
28. Trench W. F. Elementary Differential Equations / Trench W. F. – Texas, USA, 2013. – 662 p.
29. http://www.ets.ru/pg/r/dict/math.html
30. http://slovar-vocab.com/russian-english/mathematical-sciences-vocab.html
31. Workbook on higher mathematics (3rd module) for students majoring in 141 Power Engineering, Electrical Engineering and Electrical Mechanics / Compiler: Snizhko Nataliia. – Zaporizhzhia: Zaporizhzhia Polytechnic National University, 2020. – 50 p.

Appendix A
Table of the derivatives

In the table $u = u(x)$ is a differentiable function

1	$(C)' = 0$, $C = const$	11	$(tgu)' = \dfrac{1}{cos^2 u} \cdot u'$
2	$(x)' = 1$	12	$(ctgu)' = -\dfrac{1}{sin^2 u} \cdot u'$
3	$(u^n)' = n \cdot u^{n-1} \cdot u'$	13	$(\arcsin u)' = \dfrac{1}{\sqrt{1-u^2}} \cdot u'$
4	$(\sqrt{u})' = \dfrac{1}{2\sqrt{u}} \cdot u'$	14	$(\arccos u)' = -\dfrac{1}{\sqrt{1-u^2}} \cdot u'$
5	$(a^u)' = a^u \cdot ln\, a \cdot u'$, $a = const$	15	$(arctgu)' = \dfrac{1}{1+u^2} \cdot u'$
6	$(e^u)' = e^u \cdot u'$	16	$(arcctgu)' = -\dfrac{1}{1+u^2} \cdot u'$
7	$(log_a u)' = \dfrac{1}{u \cdot ln\, a} \cdot u'$	17	$(sh\, u)' = ch\, u \cdot u'$
8	$(\ln u)' = \dfrac{1}{u} \cdot u'$	18	$(ch\, u)' = sh\, u \cdot u'$
9	$(sin\, u)' = cos\, u \cdot u'$	19	$(th\, u)' = \dfrac{1}{ch^2 u} \cdot u'$
10	$(cos\, u)' = -sin\, u \cdot u'$	20	$(cth\, u)' = -\dfrac{1}{sh^2 u} \cdot u'$

Appendix B
Table of the basic indefinite integrals

In the table $u = u(x)$ is a differentiable function

1	$\int du = u + C$	10	$\int ctgu\, du = ln\left\|sin\, u\right\| + C$
2	$\int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1} + C$, $\alpha \neq -1$	11	$\int \frac{du}{cos^2 u} = tg\, u + C$
3	$\int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$	12	$\int \frac{du}{sin^2 u} = -\, ctg\, u + C$
4	$\int \frac{du}{u} = ln\left\|u\right\| + C$	13	$\int \frac{du}{sin\, u} = ln\left\|tg\, \frac{u}{2}\right\| + C$
5	$\int a^u du = \frac{a^u}{ln\, a} + C$	14	$\int \frac{du}{cos\, u} = ln\left\|tg\left(\frac{u}{2} + \frac{\pi}{4}\right)\right\| + C$
6	$\int e^u du = e^u + C$	15	$\int sh\, u\, du = ch\, u + C$
7	$\int \sin u\, du = -\cos u + C$	16	$\int ch\, u\, du = sh\, u + C$
8	$\int cos\, u\, du = sin\, u + C$	17	$\int \frac{du}{ch^2 u} = th\, u + C$
9	$\int tgu\, du = -\, ln\left\|cos\, u\right\| + C$	18	$\int \frac{du}{sh^2 u} = -\, cth\, u + C$

19	$\int \frac{du}{u^2 + a^2} = \frac{1}{a} arctg \frac{u}{a} + C$
20	$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} ln \left\| \frac{u-a}{u+a} \right\| + C$
21	$\int \frac{du}{\sqrt{a^2 - u^2}} = arcsin \frac{u}{a} + C$
22	$\int \frac{du}{\sqrt{u^2 \pm a^2}} = ln \left\| u + \sqrt{u^2 \pm a^2} \right\| + C$
23	$\int \sqrt{a^2 - u^2} du = \frac{1}{2} u \sqrt{a^2 - u^2} + \frac{1}{2} a^2 arcsin \frac{u}{a} + C$
24	$\int \sqrt{u^2 \pm a^2} du = \frac{1}{2} u \sqrt{u^2 \pm a^2} \pm \frac{1}{2} a^2 ln \left\| u + \sqrt{u^2 \pm a^2} \right\| + C$

Appendix C
Properties of the Laplace transformation

Property	The calculation formula
Linearity	$\sum_{k=1}^{n} \lambda_k f_k(t) \leftrightarrow \sum_{k=1}^{n} \lambda_k F_k(p)$
Delay theorem	$f(t-\tau)\cdot\eta(t-\tau) \leftrightarrow e^{-\tau p} F(p)$
Shifting theorem	$e^{at} f(t) \leftrightarrow F(p-a)$
Similarity theorem	$f(at) \leftrightarrow \frac{1}{a} F\left(\frac{p}{a}\right)$
Differentiation of original	$f'(t) \leftrightarrow pF(p) - f(0)$, $f^{(n)}(t) \leftrightarrow p^n F(p) - \sum_{k=1}^{n} p^{k-1} f^{(k-1)}(0)$
Differentiation of image	$t^n f(t) \leftrightarrow (-1)^n \frac{d^n}{dp^n} F(p)$
Integration of original	$\int_0^t f(\tau)d\tau \leftrightarrow \frac{F(p)}{p}$
Integration of image	$\frac{f(t)}{t} \leftrightarrow \int_p^{\infty} F(p)dp$
Convolution theorem	$f_1(t) * f_2(t) = \int_0^t f_1(t) f_2(t-\tau)d\tau \leftrightarrow F_1(p)\cdot F_2(p)$

Appendix D
Table of Laplace transforms

Original function $f(t)$	Laplace transform $F(p)$	Original function $f(t)$	Laplace transform $F(p)$
$\eta(t)$	$\frac{1}{p}$	$\mathrm{sh}\,\omega t$	$\frac{\omega}{p^2-\omega^2}$
$t^n, n \in N$	$\frac{n!}{p^{n+1}}$	$e^{\alpha t}\cdot\cos\omega t$	$\frac{p-\alpha}{(p-\alpha)^2+\omega^2}$
$t^\beta, \beta > -1$	$\frac{\Gamma(\beta+1)}{p^{\beta+1}}$	$e^{\alpha t}\cdot\sin\omega t$	$\frac{\omega}{(p-\alpha)^2+\omega^2}$
$e^{\alpha t}$	$\frac{1}{p-\alpha}$	$e^{\alpha t}\cdot\mathrm{ch}\,\omega t$	$\frac{p-\alpha}{(p-\alpha)^2-\omega^2}$
$t^n e^{\alpha t},\ n \in N$	$\frac{n!}{(p-\alpha)^{n+1}}$	$e^{\alpha t}\cdot\mathrm{sh}\,\omega t$	$\frac{\omega}{(p-\alpha)^2-\omega^2}$
$t^\beta e^{\alpha t},\ n \in N$	$\frac{\Gamma(\beta+1)!}{(p-\alpha)^{\beta+1}}$	$t\cdot\cos\omega t$	$\frac{p^2-\omega^2}{(p^2+\omega^2)^2}$
$\sin\omega t$	$\frac{\omega}{p^2+\omega^2}$	$t\cdot\sin\omega t$	$\frac{2p\omega}{(p^2+\omega^2)^2}$
$\cos\omega t$	$\frac{p}{p^2+\omega^2}$	$t\cdot\mathrm{ch}\,\omega t$	$\frac{p^2+\omega^2}{(p^2-\omega^2)^2}$
$\mathrm{ch}\,\omega t$	$\frac{p}{p^2-\omega^2}$	$t\cdot\mathrm{sh}\,\omega t$	$\frac{2p\omega}{(p^2-\omega^2)^2}$