

УДК 517.547.7

Nataliia Snizhko¹

¹PhD (Physics and Mathematics),

associate professor NU “Zaporizhzhia Polytechnic”

APPROXIMATION OF FUNCTIONS OF TWO VARIABLES ON LYAPUNOV SKELETONS

The approximative properties of interpolational Lagrange polynomials, Faber – Laurent series of functions of two variables which are defined at an arbitrary closed Lyapunov skeleton, is stated. For functions from generalized Hölder spaces H_ω the estimations are conducted for norms of C , H_ω , L_p , $p > 1$ spaces [1].

Let $\gamma = \gamma_1 \times \gamma_2$ is skeleton which is formed by arbitrary simple closed Lyapunov contours γ_1 and γ_2 (origin is contained inside them). Let $\omega(\delta_1, \delta_2)$ be the some modulus of continuity, and $\Omega_1(\delta)$, $\Omega_2(\delta)$ are the simple moduli of continuity, which respect to that one; $\omega_{1,1}$ denotes the mixed modulus of continuity of the second order [2]. Let H_ω denotes the space of continuous functions $x(t, \tau)$ at γ_0 , which satisfy the conditions:

$$H(x; \omega) = \sup_{\delta_1^2 + \delta_2^2 \neq 0} \frac{\omega(\delta_1, \delta_2; x)}{\omega(\delta_1, \delta_2)} \leq c_1, \quad H^{t\tau}(x; \omega) = \sup_{\delta_1^2 + \delta_2^2 \neq 0} \frac{\omega_{1,1}(\delta_1, \delta_2; x)}{\Omega_1(\delta_1) \Omega_2(\delta_2)} \leq c_2.$$

In the space H_ω the norm is defined as:

$$\|x(t, \tau)\|_{H_\omega} = \|x(t, \tau)\|_C + H(x; \omega) + H^{t\tau}(x; \omega).$$

$$\text{Let } (L_{mn}x)(t, \tau) = \sum_{p=0}^{2m} \sum_{q=0}^{2n} x(t_p, \tau_q) \gamma_p(t) \gamma_q(\tau) = \sum_{k=-m}^m \sum_{l=-n}^n \lambda_{kl} t^k \tau^l,$$

$$l_p(t) = \prod_{\substack{k=0 \\ k \neq p}}^{2m} \frac{t-t_k}{t_p-t_k} \left(\frac{t_p}{t} \right)^m = \sum_{k=-m}^m \lambda_{1,k}^{(p)} t^k, \quad l_q(\tau) = \prod_{\substack{j=0 \\ j \neq q}}^{2n} \frac{\tau-\tau_j}{\tau_q-\tau_j} \left(\frac{\tau_q}{\tau} \right)^n = \sum_{j=-n}^n \lambda_{2,j}^{(q)} \tau^j,$$

denotes interpolational Lagrange polynomial of the function $x(t, \tau)$ with respect to the system of Fejér points $\{t_p; \tau_q\}$, $p=0, \dots, 2m$, $q=0, \dots, 2n$ at the skeleton $\mathcal{V}$ [1]. Let us denote by F_{mn} the operator which assigns to any twice continuously differentiable function $x(t, \tau)$ at skeleton $\mathcal{V}$ (whose partial derivatives $\frac{\partial x}{\partial t}$, $\frac{\partial x}{\partial \tau}$, $\frac{\partial^2 x}{\partial t \partial \tau}$ belong to the space H_ω) the mn -th partial sum of its double Faber – Laurent series.

Theorem 1. Let $x(t, \tau) \in H_\omega$ (its partial derivatives $\frac{\partial x}{\partial t}$, $\frac{\partial x}{\partial \tau}$, $\frac{\partial^2 x}{\partial t \partial \tau}$ belong to the space H_ω), the points $\{t_p\}_{p=0}^{2m}$, $\{\tau_q\}_{q=0}^{2n}$ form the system of Fejér points at the skeleton $\mathcal{V}$ and $(L_{mn}x)(t, \tau)$ is the interpolational Lagrange polynomial which is formed with respect to this system of points. Then the estimation is true:

$$\|x - L_{mn}x\|_C \leq c \ln m \ln n H(x; \omega) \omega\left(\frac{1}{m}, \frac{1}{n}\right)$$

$$(\|x - F_{mn}x\|_C \leq c(\ln m + \ln n) H(x; \omega) \omega\left(\frac{1}{m}, \frac{1}{n}\right)).$$

Theorem 2. Let $x(t, \tau) \in H_{\omega^{(1)}}$ (its partial derivatives $\frac{\partial x}{\partial t}$, $\frac{\partial x}{\partial \tau}$, $\frac{\partial^2 x}{\partial t \partial \tau}$ belong to the space $H_{\omega^{(1)}}$); $\omega^{(2)}(\delta_1, \delta_2)$ is such that $H_{\omega^{(1)}} \subset H_{\omega^{(2)}}$ and $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ are increasing functions. Then:

$$\|x - L_{mn}x\|_{H_{\omega^{(2)}}} \leq c \ln m \ln n \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right) \Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}}$$

$$\left(\|x - F_{mn}x\|_{H_{\omega^{(2)}}} \leq c \ln mn \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_f^{(2)}\left(\frac{1}{m}\right) \Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}} \right).$$

Theorem 3. Let $x(t, \tau) \in H_{\omega^{(1)}}$, $\gamma \in C_{(2, \omega)}$ [1]. Then the estimation is fulfilled ($p > 1$):

$$\|g - L_{mn}g\|_{L_p} \leq (1 + c(p)) H\left(g; \omega^{(1)}\right) \omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right),$$

where $\|L_{mn}\|_{C \rightarrow L_p} \leq c(p)$.

REFERENCES

1. Snizhko N. Approximation of functions of two variables at Lyapunov skeletons in generalized Hölder spaces [Электронный ресурс] / N. Snizhko // Information and its impact on social processes. Abstracts of XIII International Scientific and Practical Conference. Florence, Italy, 2023. – P. 230–234.
2. Timan A. F. Theory of Approximation of Functions of a Real Variable/ A. F. Timan. – Oxford-London-New York-Paris : Pergamon Press, 1963. – 655 p.