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HOW CAN SPACE BE CURVED?

One of the most paradoxical-sounding phrases in mathematics is ‘curved space’.. Even if one could somehow make sense of the idea of three-dimensional curviness, the analogy with a curved surface suggests that we would not be able to see for ourselves whether space was curved unless we could step out into a fourth dimension to do so. Perhaps we would then discover that the universe was the three-dimensional surface of a four-dimensional sphere, which at least *sounds* curved.

One way is to go up in a space shuttle, look back, and see that it is approximately spherical. However, the following experiment, which is much more two-dimensional, would also be very persuasive. Start at the North Pole and travel due south for about 6,200 miles, having marked your initial direction. Then turn to your left and go the same distance again. Then turn to your left and go the same distance one more time. 6,200 miles is roughly the distance from the North Pole to the equator, so your journey will have taken you from the North Pole to the equator, a quarter of the way round the equator, and back to the North Pole again. Moreover, the direction at which you arrive back will be at right angles to your starting direction. It follows that on the earth’s surface there is an equilateral triangle with all its angles equal to a right angle. On a flat surface, the angles of an equilateral triangle have to be 60 degrees, as they are all equal and add up to 180, so the surface of the earth is not flat. Thus, one way of demonstrating that a two-dimensional surface is curved, from within that surface, is to find a triangle whose angles do not add up to 180 degrees, and this is something that can be attempted in three dimensions as well. If we measure the angles of triangles in space and find that they add up to more than 180 degrees, then that will suggest that space is more like a three dimensional version of the surface of a sphere than like the sort of space that can be described by three Cartesian coordinates.

If this happens, then it seems reasonable to say that space is positively curved. Another feature that one would expect of such a space is that lines that started off in the same direction would converge and eventually meet. Still another is that the circumference of a circle of radius r would not be $2\pi r$, but a little less.

In 1919, one of the most famous scientific experiments of all time showed that the idea of curved space was not just a fantasy of mathematicians, but a fact of life. According to Einstein's general theory of relativity, which was published four years earlier, space is curved by gravity, and therefore light does not always travel in a straight line, at least as Euclid would understand the term. The effect is too small to be detected easily, but the opportunity came in 1919 with a total eclipse of the sun, visible from Principe Island in the Gulf of Guinea. While it was happening, the physicist Arthur Eddington took a photograph that showed the stars just next to the sun in not quite their expected places, exactly as Einstein's theory had predicted.

Though it is now accepted that space (or, more accurately, space time) is curved, it could be that the curvature that we observe is just a small perturbation of a much larger and more symmetrical shape. One of the great open questions of astronomy is to determine the *large-scale* shape of the universe, the shape that it would have if one ironed out the curves due to stars, black holes, and so on. Would it still be curved, like a large sphere, or would it be flat, as one more naturally, but quite possibly wrongly, imagines it?

A third possibility is that the universe is *negatively* curved. This means more or less the opposite of positively curved. Thus, evidence for negative curvature would be that the angles of a triangle added up to *less* than 180 degrees, that lines starting in the same direction tended to diverge, or that the circumference of a circle of radius r was *larger* than $2\pi r$. This sort of behavior occurs in the hyperbolic disc.