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AN APPROACH TO EVALUATION RELIABILITY OF MCDM METHODS

Multi-criteria decision-making methods (MCDM) support selecting the most suitable alternative from a set of competing alternatives $A = (A_1, \dots, A_i, \dots, A_m)$. The suitability of alternative Q or an overall performance measure, is assessed using the MCDM methods algorithm and procedures based on criteria $C_i = (c_{i1}, c_{i2}, \dots, c_{ij}, \dots, c_{in})$ with specific values for each alternative. Criteria are usually normalized, so their values are $0 < c_{ij} < 1$. Each criterion has a certain significance (weight) for the decision-maker (DM):

$$W = (w_1, w_2, \dots, w_n), \quad 0 < w_j < 1, \quad \sum_{j=1}^n w_j = 1 \quad (1)$$

Thus, the overall performance of the alternative can be written as a function:

$$Q_i = f(C_i, W), \quad i = 1, \dots, m, \quad 0 < Q_i < 1 \quad (2)$$

The performance measure of alternatives significantly depends on the selected criteria weights. Therefore, the weights' selection errors significantly influence on positions of alternatives in the final ranking. In [1], the accuracy of the weight estimation process by evaluators was experimentally studied and was shown that in the case of a uniform distribution of errors, the range of change in weights is 8,66% for 60% of evaluators.

Many researchers of the MCDM methods use the Monte Carlo simulation method to assess the impact of changes in input criteria, weights, and other parameters of a decision-making problem on the results of ranking alternatives [2, 3]. Figure 1 shows the result of such a simulation, which we obtained for assessing the ranks of seven alternatives when changing the weights of the criteria in the range of 10%. The popular 'Technique for Order Preference by Similarity to Ideal Solution' (TOPSIS) MCDM method [4] was used. As can be seen from the Fig. 1, when determining the positions of alternatives once, two ranking options can be obtained, namely $(A_7 > A_2 > A_6 > A_1 > A_5 > A_3 > A_4)$ or $(A_7 > A_2 > A_6 > A_5 > A_1 > A_3 > A_4)$. Changing the order of items in a list of alternatives is known as the phenomenon of rank reversal [3, 5]. The rank reversal indicates that the DM risks receiving an incorrect ranking of alternatives when the alternatives' criteria values are close and there is an error range in the values of the criteria weights. From this, one can interpret the reliability of the MCDM method as the probability that it will produce a correct ranking of alternatives when their performance measures are evaluated once.

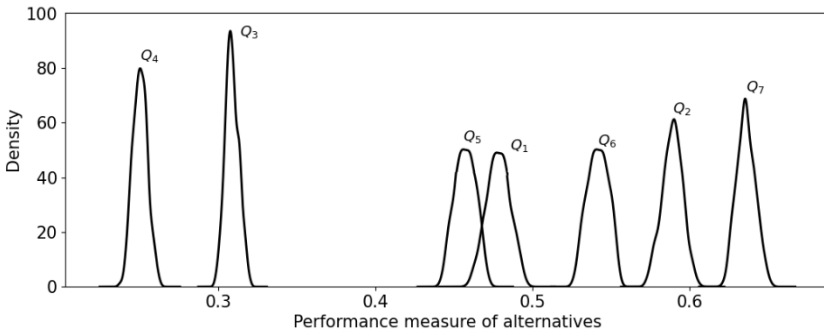


Figure 1 – Distributions of alternatives' overall performance measure

Let us denote the perturbed weights as $W' = (w'_1, w'_2, \dots, w'_n)$, where $w'_j = w_j \pm \delta w_j$; δw_j is a uniformly distributed random variable of error, which is generated at each repetition of the Monte Carlo simulation process. Then the performance measure of each alternative is a random variable

$$X_i = f(C_i, W'), \quad (3)$$

taking values $x_{i1}, x_{i2}, \dots, x_{iN}$, where N is the sample size.

Thus, a Monte Carlo simulation results in a set of samples $X = (X_1, X_2, \dots, X_m)$. Let us introduce into consideration a set of samples $Y = (Y_1, Y_2, \dots, Y_m)$, which is the set X with the reordered in descending order of values of means $\bar{x}_1, \bar{x}_2, \dots, \bar{x}_m$, $\bar{x}_i = (x_{i1} + \dots + x_{iN}) / N$. Therefore, the set Y is such that the samples' mean values $\bar{y}_1 > \bar{y}_2 > \dots > \bar{y}_{m-1} > \bar{y}_m$. Let us make samples of differences $\Theta_1 = Y_1 - Y_2$, $\Theta_2 = Y_2 - Y_3$, ..., $\Theta_{m-1} = Y_{m-1} - Y_m$. Each of these samples Θ_i consists of values $\theta_{i1}, \theta_{i2}, \dots, \theta_{iN}$, $\theta_{ik} = y_{ik} - y_{i+1,k}$. Since $0 < Q_i < 1$, it is evident that $\theta_{ik} < 1$. An example of the difference distribution is shown in Fig. 2. Positive values of the Θ_i sample elements correspond to the probability of non-intersection of the distributions Y_i and Y_{i+1} . Thus, the probability of non-intersection of these distributions can be calculated as the area under the Θ_i distribution curve that covers the values $\theta_{ik} \geq 0$ (shaded area in Fig. 2).

Let us assume that the distributions Θ_i are normal (Gaussian). Then the probability of non-intersection distributions Y_i and Y_{i+1} can be calculated as

$$\Pr(Y_i > Y_{i+1}) = \frac{1}{s_i \sqrt{2\pi}} \int_0^1 \exp\left(-\frac{(x - \bar{\theta}_i)^2}{2s_i^2}\right) dx, \quad (4)$$

where $\bar{\theta}_i$ and s_i are the sample mean and sample standard deviation of Θ_i .

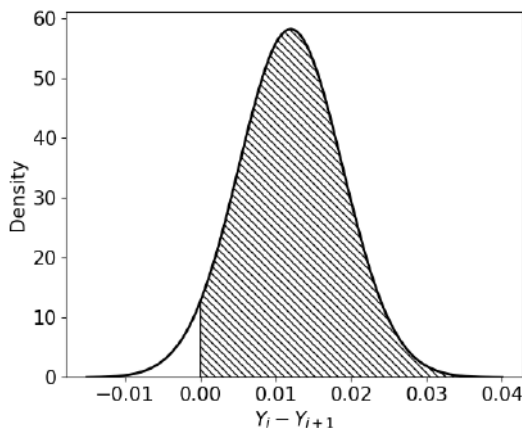


Figure 2 – Example distribution of difference distributions Y_i and Y_{i+1}

The probability that all adjacent distributions Y_i , Y_{i+1} do not intersect simultaneously is the product $\Pr(Y_1 > Y_2) \times \dots \times \Pr(Y_{m-1} > Y_m)$. This product of probabilities is proposed to be used as a lower limit of the MCDM method reliability:

$$R = \prod_{i=1}^{m-1} \Pr(Y_i > Y_{i+1}) \quad (5)$$

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