

**MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NU "ZAPORIZHZHIA POLYTECHNIC"**

**METHODICAL INSTRUCTIONS
for independent work and implementation of calculation-graphic work
in theoretical electrical engineering
"The transient process in linear electrical circuit" for the students of
specialty 141 for full-time and part-time forms of studying**

2025

METHODICAL INSTRUCTIONS for independent work and implementation of calculation-graphic work in Theoretical Fundamentals of Electrical Engineering "The transient process in linear electrical circuit" for the students of specialty 141 for full-time and part-time forms of studying. /Authors: Kozlov V.V., Nabokova O.V. – NU "Zaporizhzhia Politechnic", 2025. – 22 p.

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Затверджено на засіданні
кафедри ЕАПУ
Протокол №7
від "6" лютого 2025 р.

Затверджено на засіданні
науково-методичної комісії
Електротехнічного факультету
Протокол №11
від "26" червня 2025 р.

THE CONTENT

1 GENERAL INSTRUCTIONS	4
1.1 The aim and the task of C.G.W.	4
1.2 Contents and the volume of the work	4
1.3 Designing and presenting of C.G.W.	4
2 THE TECHNICAL TASK TO C.G.W.	5
3 INSTRUCTIONS ON USING THE WORK	5
3.1 The transient process by the classical method	5
3.2 General Outline of Transient Analysis	7
3.3 The transient process by the operator method	7
3.4. Ohm's Law in Operational Form. Internal EMFs	8
3.5 Kirchhoff's Laws in Operational Form	9
3.6 The equivalent operational circuit	10
4 THE EXAMPLES OF EXECUTION AND FORMATION OF CALCULATION-GRAPHICS WORK	11
4.1 The example of detailed calculations by the classical technique in the case of two different real roots	11
4.2 The example of detailed calculations by the classical technique in the case of complex-conjugate roots	14
4.3 The example of detailed calculations by the operator technique	17
LITERATURE	19
Appendix A The values of active and passive elements	20
Appendix B The diagrams of electric circuits	21

1 GENERAL INSTRUCTIONS

These methodical instructions include the tasks to calculation-graphic work (C.G.W. in abbreviated form) on calculations methods of the transient processes in the linear direct current circuits by the classical and operator methods.

1.1 The aim and the task of C.G.W.

The aim of this work is to assist students in independent study of suitable parts of theoretical engineering. Students must get practical skills in using the method of analysis of the transient processes in the linear electrical circuits with concentrated parameters. The example considered in datum methodical instructions will help the students to fulfil the necessary calculations and investigations when doing calculation-graphical works prescribed in the course of electrical engineering.

1.2 Contents and the volume of the work

This design-graphic work consists of the explanatory report with the volume of about pages including figures' graphs and tables.

The contents of the explanatory report:

- a title-page;
- the technical task to calculation-graphic work;
- an abstract;
- contents;

the essence of the explanatory report:

- explanations to computations and independent calculations;
- checking up of rightness of the calculations;
- conclusions;
- the list of used original sources;
- appendixes.

1.3 Designing and presenting of C.G.W.

When using this work one must carry out the following requirements: - the calculation of errors must be lower than 3%;

- the explanatory report must be designed according to the requirements of the Ukrainian State Standard;
- the main parts of the work must be set by the lecturer.

2 THE TECHNICAL TASK TO C.G.W.

2.1 Every student must do his own assignment. The number of his assignment equals the number of a student's surname in a group register. The values of active and passive elements and resources of voltage are given in appendix A. The diagrams of electrical circuits are given in appendix B.

2.2 An electrical circuit contains the direct current source. The circuit is in the steady-state condition before the shorting of the first key K1. After shorting (the first commutation) it takes:

2.2.1 Calculate the transient value of the given current or voltage by the classical method.

2.2.2 Calculate the transient value of given current or voltage by the operator technique and construct a diagram of transient current or voltage.

3 INSTRUCTIONS ON USING THE WORK

3.1 The transient process by the classical method

The transient appears in the circuits on account of commutations. In circuits containing energy storage elements, the inductor current $i_L(t)$ and the capacitor voltage $u_C(t)$ cannot change value instantaneously; otherwise, their stored energies $Q_L(t) = \frac{1}{2}Li_L^2(t)$ and $Q_C(t) = \frac{1}{2}Cu_C^2(t)$ would change instantaneously. Suppose, for example, that we find the solutions for the element voltages and currents for a particular circuit and then, at some later time, open or close a switch located somewhere in the circuit; activating the switch changes the circuit, and now we must solve this new circuit. The element voltages and currents will have changed due to the fact, that we have a new circuit caused by the switching operation. The object of this work is to determine the behavior of the element voltages and currents in the intermediate or transient time interval while they are adjusting to the new values. There are two laws of commutation. The first switching law: the inductor current cannot sudden change .an the initial (the first) moment after a switching, that is

$$i_L(0_-) = i_L(0_+) \quad (3.1)$$

The second switching law: the capacity voltage cannot sudden change at the initial moment after a switching, i.e.

$$u_C(0_-) = u_C(0_+) \quad (3.2)$$

The values of the inductor current and the capacity voltage at $t = 0$ are called the independent initial conditions.

Thus, the inductor current and the capacitor voltage immediately prior to and immediately after a switching operation must be the same. While calculating of any transient process side by side with the independent conditions we also use dependent initial conditions: resistance and capacity currents, resistance and inductor voltages, values of derivatives of voltages and currents at the first instant (moment) after switching. The independent initial conditions are determined before the switching. Knowing the independent initial conditions we shall be able to define the dependent initial conditions, which can sudden change at the switching. If we have direct current source (dc) all currents and voltages in the circuit must also be constant in value. Since $u_L = L \frac{di_L}{dt}$ if the inductor current i_L is constant, then $u_L = 0$, which is a short circuit. Similarly, if the capacitor voltage u_C is constant, then $i_C = C \frac{du_C}{dt}$ and $i_C = 0$, which is equivalent to an open circuit. If the source is sinusoidal (a.c.), we may use any method of calculation the sine currents in order to find the portion of the steady-state solution due to this type of source. The transient currents are also called the total currents. They may be described by the system of integral-differential equations worked out by Kirchhoff's laws for the after-switching time. The total succession of the calculation by the classical method:

- make up the positive directions of branch currents.
- calculate the circuit before the switching for the purpose of defining the independent initial conditions.
- compose the system of differential equations by Kirchhoff's laws for the after-switching circuit and define dependent initial conditions.
- calculate the steady-state rate after switching.
- define the roots of a characteristic equation. If the roots are real and different, we look for the decision in the form:

$$i = i_{ss} + A_1 e^{p_1 t} + A_2 e^{p_2 t} \quad (3.3)$$

For complex-conjugate roots the decision will be in the form:

$$i = i_{ss} + A e^{\delta t} \sin(\omega_0 t + \psi) \quad (3.4)$$

on the grounds of independent initial conditions define the constants of integration; at last, write down the transient current as the time function in the form (3.3) or (3.4).

3.2 General Outline of Transient Analysis

Transient analysis involves basically the following steps:

1 At first consider the circuit before switching. Choose positive current directions through inductive elements and voltages across capacitive elements. Determine i.i.c. (i.e. the values of currents through inductive elements and voltages across capacitances immediately before switching). I.i.c. are found by some calculation method for direct current circuits, if energy sources are **dc**; or by the calculation method for sinusoidal alternating current, if energy sources are **ac**.

2. Represent an electric circuit after switching. Positive directions are assumed for branch currents and voltages. Positive directions of inductive currents and capacitive voltages must be the same, as before switching. One must work out the integro-differential equations by Kirchhoff's laws.

3. Define the steady-state components of currents and voltages after switching when transient process has been over. The steady-state components are found by some calculation method for direct current circuits, if energy sources are dc; or by the calculation method for sinusoidal alternating current, if energy sources are ac.

4. The characteristic equation is written down and its roots are found. Now we know the type of transient components by form of roots.

5. The value is equal to the sum of steady-state and free components.

6. Then we can calculate the constants of integration which enter into transient components. If the transient component contains two constants of integration, it is required to find not only a value of current or voltage, but also their derivatives after switching.

7. Total current (voltage) is expressed as time function as algebraic sum of steady-state and transient components (taking into account the calculated constants of integration).

3.3 The transient process by the operator method

The total succession of the calculations by the method: at first we define the independent initial conditions by the classical method; then the transient current $i(t)$ is changed by operator image $I(p)$ – is the Laplace transform of a function; resistance R stays without changing; an inductive voltage $u_L = L \frac{di_L}{dt}$ changes by the image $pLI_L(p) - Li_L(0_+)$ and the capacitor

voltage $u_C = \frac{1}{C} \int i_C dt$ - by the image $\frac{1}{pC} I_C(p) + u_C(0_+)$;

draw up the operator circuit for after-switching mode;

write the initial system of operator equations and solve the system of algebraic equations relative to the image of the unknown quantity and pass from the image to the original function by the expansion theorem

$$f(t) = \sum \frac{N(p_k)}{M'(p_k)} e^{p_k t} \quad (3.5)$$

where $N(p_k)$ - is the multinomial of numerator;

$M'(p_k)$ - is the derivative of the multinomial of denominator;

p_k - are the roots of denominator;

draw up a diagram of the current as the function of time.

The operator technique allows defining an integral of a linear equation without defining of constants of integration that usually becomes easier the solving of a problem.

The Laplace transform of the voltage across the inductance is

$$U_L(p) = pLI(p) - Li(0) \quad (3.6)$$

The Laplace transform of the voltage across a capacitor

$$u_C = \frac{1}{c} \int_0^t i dt + u_C(0) = \frac{I(p)}{pc} + \frac{u_C(0)}{p} \quad (3.7)$$

The Laplace transform and the respective inverse Laplace transform are referred to as a transform pair.

The Laplace transform $F(p)$ is found for a specific time function $f(t)$, it is tabulated for future reference just as is done for logarithms or integrals.

Draw up a diagram of the current as the function of time.

The example of the detailed calculation by operator technique at the end of theoretical material.

3.4. Ohm's Law in Operational Form. Internal EMFs

Let's consider the branch consisting of e.m.f. source and resistive, inductive and capacitor elements. It is connected to two nodes a and b .

The voltage between nodes a and b for the circuit after switching

$$u_{ab} = \phi_a - \phi_b = u_R + u_L + u_C - e(t). \quad (3.8)$$

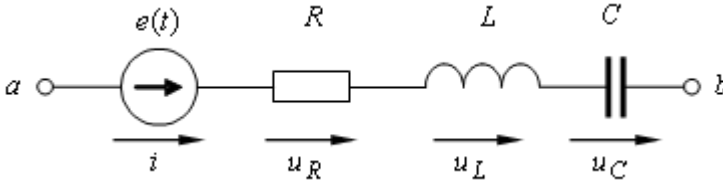


Fig. 4.1. A part of the complex ramified electrical circuit.

Since the Laplace transform is a linear operator, it follows that the transform of a sum is equal to the sum of transforms. Therefore, we may substitute the respective transforms for each of the terms and we get

$$U_{ab}(p) = RI(p) + pLI(p) - Li(0) + \frac{1}{pC}I(p) + \frac{u_C(0)}{p} - E(p) \quad (3.9)$$

From this equation it follows that

$$I(p) = \frac{U_{ab}(p) + E(p) + Li(0) - \frac{u_C(0)}{p}}{R + pL + \frac{1}{pC}}, \quad (3.10)$$

where $Z(p) = R + pL + \frac{1}{pC}$ is the *operation impedance* of the branch ab .

The term $Li(0)$ is the internal e.m.f. due to the energy stored by the magnetic field of an inductive element, produced by the current through the inductance just before a switching.

The term $u_C(0)/p$ is the internal e.m.f. due to the energy stored by the electrostatic field of a capacitive element, produced by the voltage across the capacitor immediately before a switching.

3.5 Kirchhoff's Laws in Operational Form

According to the *first Kirchhoff's law* an algebraic sum of instantaneous values of the currents converging in any junction of an electric circuit is equal to zero

$$\sum_{k=1}^n i_k = 0. \quad (3.11)$$

The current by means of Laplace transform can be presented in the operation form. As the Laplace transform is linear, the transform of the sum is equal to the sum of transforms.

$$\sum_{k=1}^n I_k(p) = 0. \quad (3.12)$$

According to the *second Kirchhoff's law* in any closed loop the sum of the instantaneous voltages across all the sections which enter into this contour, is equal to the algebraic sum of electromotive forces:

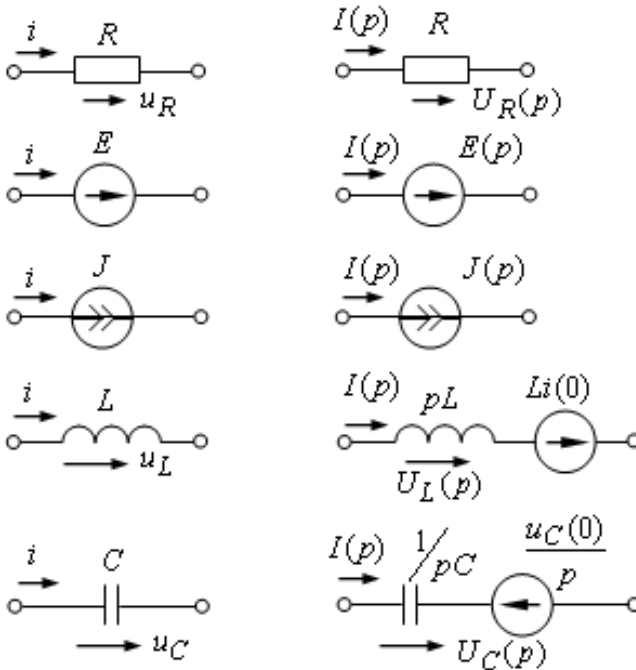
So, we can write the second Kirchhoff's law in the operational form as

$$\sum_{k=1}^n Z_k(p)I_k(p) = \sum_{k=1}^n E_k(p). \quad (3.13)$$

3.6 The equivalent operational circuit

At a transient analysis by operational method, it is convenient to write down the equations by Kirchhoff's laws at once in the operational form and use known calculation methods.

The relationships between circuit elements for instantaneous values of currents and voltages and the elements of the operational circuit are presented in this section:



4 THE EXAMPLES OF EXECUTION AND FORMATION OF CALCULATION-GRAPHICS WORK

4.1 The example of detailed calculations by the classical technique in the case of two different real roots

There is *dc* electrical circuit in fig 4.1 with the next parameters: $E=60$ V; $R_1=10 \Omega$; $R_2=20 \Omega$; $R_3=30 \Omega$; $L=0,1H$; $C=0,002$ F. The key Q is closed at $t=0$. Calculate currents $i_L(t)$ and $i_C(t)$ in the branch with a capacitor.

We choose positive directions of currents, as shown in the figure.

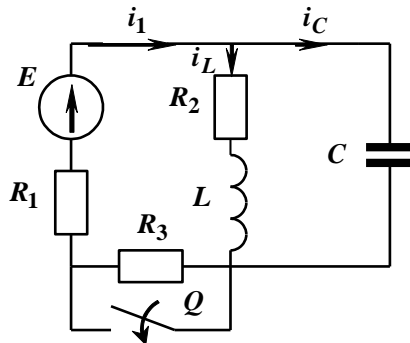


Fig.4.1 - Initial electrical circuit

Let's assume that before the switching there was a steady-state mode in the electric circuit (Fig. 4.2), that is, the currents in all branches and the voltages across elements were constant over time.

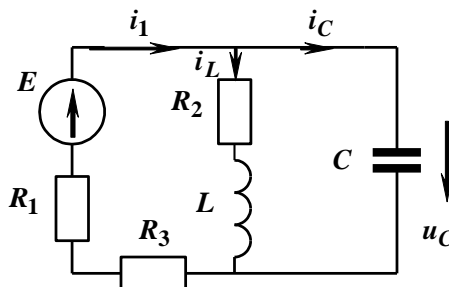


Fig.4.2 – Electrical circuit before switching

We determine independent initial conditions for the circuit in Fig. 4.2. Remember that for *dc* circuit a capacitor is "an open circuit" (branch break). Then we can find the current through inductive element by the next way

$$i_L(0) = i_L(0-) = \frac{E}{R_1 + R_2 + R_3} = \frac{60}{10 + 20 + 30} = 1 \text{ A.}$$

The voltage across capacitive element is

$$u_C(0) = u_C(0-) = R_2 i_L(0-) = R_2 \frac{E}{R_1 + R_2 + R_3} = 20 \frac{60}{10 + 20 + 30} = 20 \text{ V.}$$

So, i.i.c. before a switching

$$\begin{aligned} i_L(0) &= i_L(0-) = 1 \text{ A} \\ u_C(0) &= u_C(0-) = 20 \text{ V} \end{aligned}$$

Let's consider the circuit after a switching (Fig. 4.3). As a result of closing key Q, the resistive element R_3 is shunted.

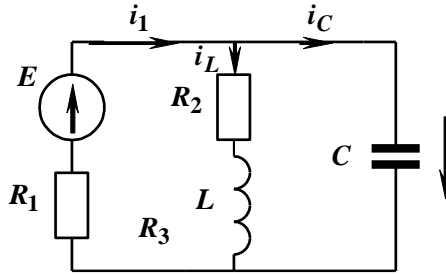


Fig.4.3 – Electrical circuit after switching

Let's calculate the steady state rate of the circuit after switching

$$\begin{aligned} i_{C_{ss}} &= 0 \text{ A;} \\ i_{1_{ss}} = i_{2_{ss}} &= \frac{E}{R_1 + R_2} = \frac{60}{10 + 20} = 2 \text{ A.} \end{aligned}$$

We determine the initial conditions. Since the electrical circuit has two independent energy storage elements, it is necessary to calculate both values of desired currents for the moment $t=0+$ and the value of their first-order derivatives. To do this, we compose a system of integral-differential equations according to Kirchhoff's laws for instantaneous values.

$$\begin{cases} i_1 = i_L + i_C \\ R_1 i_1 + u_C = E \\ R_2 i_L + L \frac{di_L}{dt} - u_C = 0 \end{cases}$$

Taking into account independent initial conditions and the Laws of Switching, we have got

$$\begin{cases} i_L(0) = 1 \text{ A} \\ i_1(0) = \frac{E - u_C(0)}{R_1} = \frac{60 - 20}{10} = 4 \text{ A} \\ i_C(0) = i_1(0) - i_L(0) = 4 - 1 = 3 \text{ A} \\ \frac{di_L}{dt}(0) = \frac{u_C(0) - R_2 i_L(0)}{L} = \frac{20 - 20 \cdot 1}{0.1} = 0 \text{ A/s} \end{cases}$$

Differentiate the system of two equations

$$\begin{cases} \frac{di_1}{dt} = \frac{di_L}{dt} + \frac{di_C}{dt} \\ R_1 \frac{di_1}{dt} + \frac{i_C}{C} = 0 \end{cases}$$

On solving the system we have got

$$\begin{cases} \frac{di_1}{dt}(0) = -\frac{i_C(0)}{R_1 C} = -\frac{3}{10 \cdot 2 \cdot 10^{-2}} = -150 \text{ A/c} \\ \frac{di_C}{dt}(0) = \frac{di_1}{dt}(0) - \frac{di_L}{dt}(0) = -150 - 0 = -150 \text{ A/c} \end{cases}$$

We compose a characteristic equation $Z(p)$ and find its roots using the input resistance method relative to the EMF source after switching.

$$Z(p) = R_1 + \frac{(R_2 + pL) \cdot \frac{1}{pC}}{R_2 + pL + \frac{1}{pC}} = 0$$

After transformation

$$p^2 LCR_1 + p(CR_1 R_2 + L) + (R_1 + R_2) = 0$$

From here

$$p_{1,2} = \frac{-(CR_1 R_2 + L) \pm \sqrt{(CR_1 R_2 + L)^2 - 4LCR_1(R_1 + R_2)}}{2LCR_1}$$

Taking into account given data we have

$$\begin{aligned} p_{1,2} &= \frac{-(0,002 \cdot 10 \cdot 20 + 0,1) \pm \sqrt{(0,002 \cdot 10 \cdot 20 + 0,1)^2 - 4 \cdot 0,1 \cdot 0,002 \cdot 10 \cdot (10 + 20)}}{2 \cdot 0,1 \cdot 0,002 \cdot 10} \\ &= -150; \quad -100 \text{ c}^{-1}. \end{aligned}$$

As we have got two real roots, free (transient) components of currents are next:

$$i_{1f} = A_{1_1} e^{-150t} + A_{1_2} e^{-100t};$$

$$i_{cf} = A_{c_1} e^{-150t} + A_{c_2} e^{-100t}.$$

Write down transient currents

$$i_1 = i_{1ss} + i_{1f} = 2 + A_{1_1} e^{-150t} + A_{1_2} e^{-100t} \quad A;$$

$$i_c = i_{css} + i_{cf} = 0 + A_{c_1} e^{-150t} + A_{c_2} e^{-100t} \quad A.$$

The derivatives of transient currents

$$\frac{di_1}{dt} = \frac{di_{1ss}}{dt} + \frac{di_{1f}}{dt} = 0 - 150A_{1_1} e^{-150t} - 100A_{1_2} e^{-100t} \quad A/s;$$

$$\frac{di_c}{dt} = \frac{di_{css}}{dt} + \frac{di_{cf}}{dt} = 0 - 150A_{c_1} e^{-150t} - 100A_{c_2} e^{-100t} \quad A/s.$$

For the moment of switching for $t = 0_+$

$$\begin{cases} i_1(0) = 2 + A_{1_1} + A_{1_2} = 4 \\ \frac{di_1}{dt}(0) = -150A_{1_1} - 100A_{1_2} = -150; \\ i_c(0) = 0 + A_{c_1} + A_{c_2} = 3 \\ \frac{di_c}{dt}(0) = -150A_{c_1} - 100A_{c_2} = -150. \end{cases}$$

Having solved the system of equations, we determine the value of integration constants and write down the transient currents

$$i_1(t) = 2 - 1 \cdot e^{-150t} + 3 \cdot e^{-100t} \quad A;$$

$$i_c(t) = -3 \cdot e^{-150t} + 6 \cdot e^{-100t} \quad A.$$

4.2 The example of detailed calculations by the classical technique in the case of complex-conjugate roots

An electrical circuit is set with the next parameters: $E=100$ V; $R1=10$ Ω ; $R2=50$ Ω ; $L=10$ mH; $C=10^{-5}$ F. The key S is closed at $t=0$.

Determine transient current $i_2(t)$ in the branch with resistance.

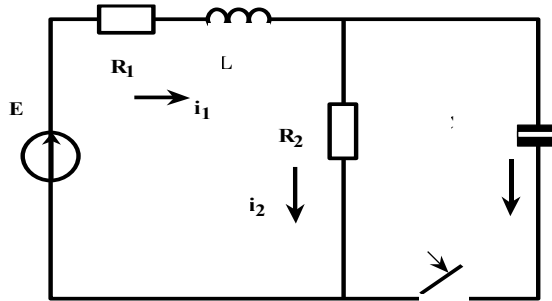
The solution by classical method.

We choose positive directions of currents, as shown in the figure.

Define the independent initial conditions - *i.i.c.*:

$$u_{c(0-)} = u_{c(0+)} = 0;$$

$$i_{L(0-)} = i_{L(0+)} = i_1(0) = \frac{E}{R_1 + R_2} = \frac{100}{10 + 50} = 1.67 \quad A.$$



There are two reactive elements (stored) in this scheme. That is why, the transient process in it is described by differential equations of the second power. Hence, for defining two constants of integration one must also find dependent initial conditions. For that we write the system of integral-differential equations, which describe the after-switching process:

$$i_1 = i_2 + i_3;$$

$$E = R_1 i_1 + L \frac{di_1}{dt} + R_2 i_2;$$

$$R_2 i_2 = \frac{1}{C} \int i_3 dt.$$

After substituting given values and independent initial conditions in the system we calculate it and define dependent initial conditions

$$i_{2(0+)} = \frac{u_{c(0+)}}{R_2} = 0 \text{ A},$$

$$i_{3(0+)} = i_{1(0+)} - i_{2(0+)} = 1.67 - 0 = 1.67 \text{ A}.$$

A current in the branch we define in the following way:

$$i_2 = i_{2ss} + i_{2tr}$$

The steady-state value of the current is particular solution of the system of not uniform integral-differential equations:

. It takes: - define the transient current in the second branch by classical

$$\text{technique } i_{2ss} = \frac{E}{R_1 + R_2} = \frac{100}{10 + 50} = 1.67 \text{ A}$$

Now one must define the roots of the characteristic equation. For that we write $Z_{in}(p)$ and equal it to 0:

$$Z_{in}(p) = pL + R_1 + \frac{R_2 \frac{1}{pC}}{R_2 + \frac{1}{pC}} = 0$$

Reduce this equation to a common denominator, restore similar values and then equate the nominator with zero:

$$p^2 R_2 LC + p(R_1 R_2 C + L) + R_1 + R_2 = 0$$

After substituting numerical values:

$$p^2 + 3000p + 12 \cdot 10^6 = 0$$

Calculated it we define the roots:

$$p_{1,2} = \delta \pm j\omega_0 = -1500 \pm j3120 \text{ s}^{-1},$$

where δ is called the factor of damping of a transient component;

ω_0 is called an angular frequency.

As we have got the complex-conjugate values of roots a transient value of current will be in the following form:

$$i_{2(0+)} = i_{2ss} + Ae^{\delta t} \sin(\omega_0 t + \psi)$$

For defining the constant of the integration A and the initial phase ψ one must differentiate the previous equation:

$$\frac{di_2}{dt} = \delta Ae^{\delta t} \sin(\omega_0 t + \psi) + \omega_0 Ae^{\delta t} \cos(\omega_0 t + \psi)$$

So, we need calculate $\frac{di_2}{dt}(0+)$.

To define the value of derivative we must differentiate the system of integral-differential equations:

$$\begin{aligned} \frac{di_1}{dt}(0+) &= \frac{di_2}{dt}(0+) + \frac{di_3}{dt}(0+) \\ R_2 \frac{di_2}{dt}(0+) &= R_3 \frac{di_3}{dt}(0+) + \frac{du_C}{dt}(0+) \\ R_2 \frac{di_2}{dt}(0+) &= \frac{du_C}{dt}(0+) \end{aligned}$$

On solving this system, we define the unknown value

$$\frac{di_2}{dt}(0+) = \frac{1}{R_2} \frac{du_C}{dt}(0+),$$

where $\frac{du_C}{dt}$ is defined as $\frac{du_C}{dt} = \frac{i_2}{C} = \frac{1.67}{1 \cdot 10^{-5}} = 1.67 \cdot 10^5 \text{ V/s}$

Substitute found values of the current and its derivative in the proper system of two equations at the moment $t = 0$

$$i_{2(0)} = 1.67 + A \sin \psi = 0$$

$$\frac{di_2}{dt}(0) = -1500A \sin \psi + 3120A \cos \psi = 3340$$

Divide the second equation by the first one:

$$-1500 + 3120 \operatorname{ctg} \phi = -2000$$

$$\operatorname{ctg} \psi = -0.1603; \quad \psi = -81^\circ$$

$$A = -\frac{1.67}{\sin(-81^\circ)} = 1.69$$

As a result, we have the solution for the transient current

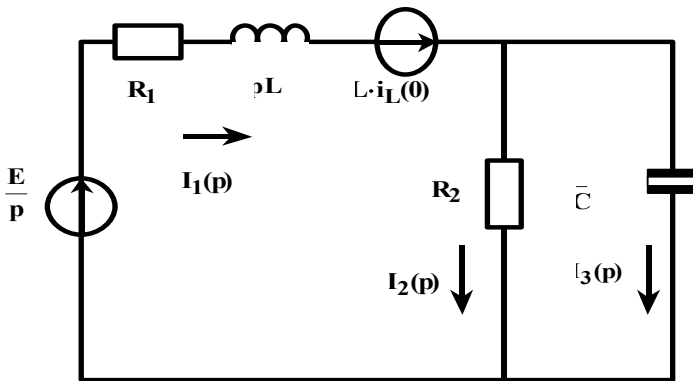
$$i_2(t) = 1.67 + 1.69e^{-1500t} \sin(3120t - 81^\circ) \text{ (A)}$$

4.3 The example of detailed calculations by the operator technique

Let's consider the solution to the problem (fig. 4.2). The initial independent conditions we always determine by classical method. So, i.i.c.:

$$i_L(0) = 1.67 \text{ A}; \quad u_C(0) = 0.$$

After switching the figure has the following form



The term $L \cdot i_L(0)$ is the internal electromotive force due to the energy stored by the magnetic field of inductance, produced by the current $i_L(0)$ through the inductance just before switching.

Let the potential of the node "b" is grounded. According to the method of node-pair potentials a current in a branch can be found if we divide the voltage between two nodes into the resistance of the branch:

$$I_2(p) = \frac{U_{ab}(p)}{R_2} = \frac{\left(\frac{E}{p} \cdot \frac{1}{R_1 + pL} + L \cdot i_L(0) \right)}{\frac{1}{R_1 + pL} + \frac{1}{R_2} + \frac{1}{1/pC}}$$

After necessary conversions we have got the following expression:

$$I_2(p) = \frac{U_{ab}(p)}{R_2} = \frac{E + pl \cdot i_L(0)}{p(p^2 R_2 LC + pCR_1 R_2 + pL + R_1 + R_2)}$$

After substituting known numerical data the operator current has the following form:

$$I_2(p) = \frac{1.67 \cdot 10^{-2} p + 100}{p(5 \cdot 10^{-6} p^2 + 0.015 p + 60)} = \frac{N(p)}{p \cdot M(p)}$$

The roots of the equation $M(p) = 0$ are

$$p_1 = 0; \quad p_2 = -1500 + j3120 \text{ s}^{-1}, \quad p_3 = -1500 - j3120 \text{ s}^{-1}.$$

As the denominator of the current's image has a zero root and two complex-conjugate roots, the original of the current $i_2(t)$ can be found using the next formula

$$\frac{N(p)}{p \cdot M(p)} \Rightarrow f(t) = \frac{N(0)}{M(0)} + 2 \operatorname{Re} \frac{N(p_1)}{p \cdot M'(p_1)} e^{p_1 t}.$$

Define the values of functions incoming into the equation for the current $i_2(t)$.

$$N(0) = 100; \quad M(0) = 60;$$

$$N(p_1) = 100 + 1.67 \cdot 10^{-2}(-1500 + j3120) = 74.95 + j52.10 = 91.28e^{j35^\circ}$$

$$M'(p_1) = 10^{-5}p + 0.015 = 10^{-5}(-1500 + j3120) + 0.015 = 0.03e^{j90^\circ}$$

Then we can calculate the necessary transient current:

$$\begin{aligned} i_2(t) &= \frac{100}{60} + 2 \operatorname{Re} \left[\frac{91.282e^{j35^\circ}}{3461.85e^{j116^\circ} \cdot 0.0312e^{j90^\circ}} e^{(-1500+j3120)t} \right] = \\ &= 1.67 + 1.69e^{-1500t} \cos(3120t - 171^\circ) = \\ &= 1.67 + 1.69e^{-1500t} \sin(3120t - 81^\circ) A \end{aligned}$$

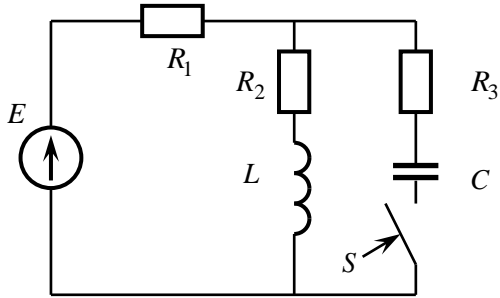
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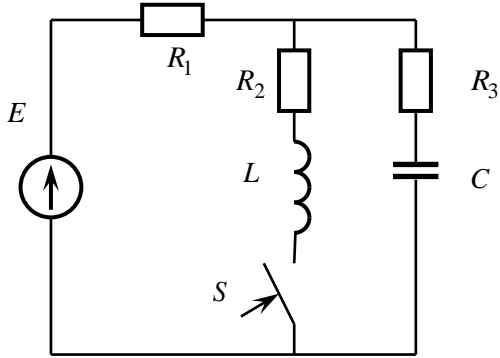
Appendix A
The values of active and passive elements

V	D	E	R_1	R_2	R_3	R_4	L	$C \cdot 10^4$	Wanted value
		V	Ω	Ω	Ω	Ω	H	F	
1	1	50	20	20	30	-	0.1	2	$i_1(t)$
2	2	50	10	15	20	-	0.2	4	$i_1(t)$
3	3	120	40	40	10	20	0.3	6	$i_1(t)$
4	4	90	15	30	30	-	0.4	8	$i_3(t)$
5	5	100	15	25	10	-	0.5	10	$i_3(t)$
6	1	60	10	20	30	-	0.1	2	$u_1(t)$
7	2	120	20	20	20	-	0.2	4	$i_1(t)$
8	3	200	30	30	20	40	0.3	6	$i_3(t)$
9	4	120	12	20	30	-	0.4	8	$i_1(t)$
10	5	70	25	25	10	-	0.5	10	$i_1(t)$
11	1	50	10	5	30	-	0.1	2	$i_1(t)$
12	2	50	20	10	15	-	0.2	4	$i_3(t)$
13	3	60	10	30	15	5	0.3	5	$i_1(t)$
14	4	60	18	30	20	-	0.4	6	$i_3(t)$
15	5	90	30	20	15	-	0.5	8	$i_1(t)$
16	1	80	40	15	40	-	0.1	10	$i_1(t)$
17	2	50	10	25	15	-	0.2	2	$i_3(t)$
18	3	80	15	10	20	15	0.3	4	$i_3(t)$
19	4	120	28	20	30	-	0.4	6	$i_3(t)$
20	5	120	25	40	25	-	0.5	8	$i_1(t)$

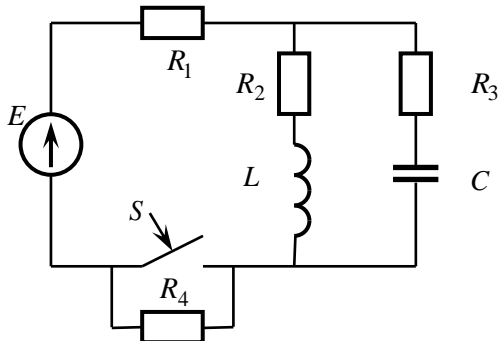
Appendix B
The diagrams of electrical circuits



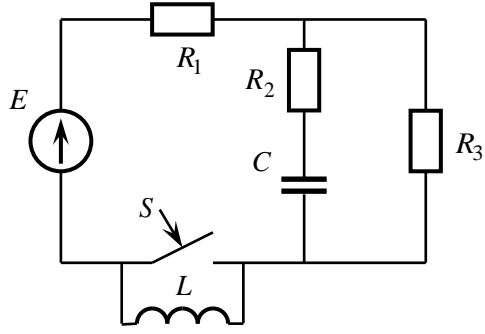
1)



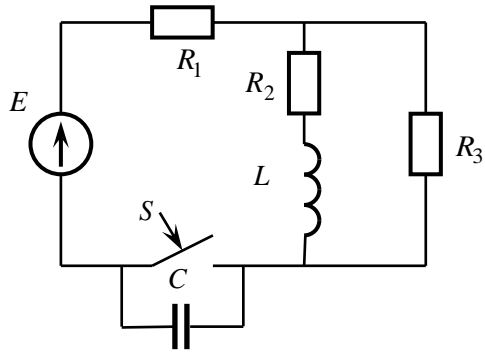
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4)



5)