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THERMAL BEHAVIOR ANALYSIS OF POWER ELECTRIC AND ELECTRONIC EQUIPMENT

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Analysis and calculation methods of thermal behavior of power electric and electronic equipment components are represented. Recommendations in respect to optimize theirs thermal behavior with consideration of constructive peculiarities as well as cooling technologies are proposed.

The textbook is intended for higher educational students as well as may be useful for electromechanical engineers.

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PREFACE

When creating economical and reliable power electric and electronic equipment as well as its proper servicing an engineer is forced to deal with necessity to solve numerous problems, among which the task of heating its components is great importance. Thermal processes are essential factors defining rationality of power electric and electronic equipment construction. The less thermal energy will be released by the equipment components, all factors being equal, as well as the better withdrawing this energy to environment or special-purpose cooling mediums is provided, the higher technical-and-economical indexes the equipment in whole will have.

In spite of paramount importance and great number of works on consideration of thermal processes and analysis of thermal behavior of power electric and electronic equipment components, these questions now remain ones of least developed. It is not the new that accuracy of engineering thermal calculations and at the present time leaves something to be desired. Various electric and magnetic calculations can be performed with errors by a few percents, while the errors of thermal calculations is in specific cases tens percents.

Therefore, thermal calculations, thermal investigations, thermal tests of power electric and electronic equipment is given much attention in educational process as well. At the present time electromechanical engineer must in full measure to understand not only the questions of heat exchange, but to have practice experience to analyze and optimize thermal behavior of power electric and electronic equipment.

By this why educational schedule on specialty “Electrical machines and apparatuses” includes respective subjects, such as: “Electrical machines”, “Theory fundamentals of electrical apparatuses”, “Power electronics fundamentals”, “Electrical apparatuses for high voltage applications”, “Designing of electrical apparatuses” etc., which consider the questions of analysis and optimization of thermal behavior of power and instrument transformers, electrical reactors, switching apparatuses and switchgear panels for high and low voltage applications, power semiconductor-based devices etc. With that, at the present time educational editions including combined consideration of the problems to analyze thermal behavior of various components of electric and electronic equipment is absent in Ukraine.

Existing textbooks usually consider thermal behavior of particular equipment components, having ones or others certain cooling technologies

[4, 7–10, 12, 13, 24, 25]. A number of educational editions consider solely theory of heat exchange [3, 11, 14–16] that is very unsuitable for the students to study the noted subjects. Moreover, the publications proposed advanced approaches to solve denoted problems have been come into existence in recent years [10, 27–29].

Creation of this textbook is purposed another aim as well. Namely, it is to study English technical terminology on the questions of heat exchange and heat calculations of power electric and electronic equipment. The textbook is first and foremost intended for the students studying in English, but it will be useful for another students studying on specialty “Electrical machines and apparatuses” as well as for engineers working in the field of investigation, design, testing and servicing of power electric and electronic equipment.

1. THERMAL PHENOMENA IN POWER EQIPMENT COMPONENTS

1.1 General

The power electric and electronic equipment, such as, electrical machines, power and instrument transformers, electric and electronic apparatus for electricity distribution and technological applications are complex electrotechnical devices containing many various components. While functioning, some of them carry current load; others provide passage of a magnetic flux; and the others are used for electrical isolation. In all cases mentioned above the power losses and its conversion to heat are necessary to dissipate heat to environment (i.e. cooling components for the purpose to ensure their design thermal behavior). Therefore, thermal phenomena are very important for electrical power equipment and its components. However, it should be noted they may play both negative and positive roles. Foremost, thermal phenomena applications for particular equipment are exemplified as follows:

- thermal expansion of bodies: bending a bimetallic strip due to its heating by electrical current is basic for electrical apparatus functioning, such as, thermal relays, circuit breakers etc.;

- melting materials: fuse link is melted under thermal action of electrical current and by that the circuit is interrupted in the event of overload or short circuit.

Thermal phenomena play also positive role in the circuit interruption process performed with contact-based switching apparatus. In the process, electromagnetic energy stored by inductive circuit components, converses to thermal energy of an arc discharge. It is well known that an electric arc is the most concentrative absorbent of thermal energy. When an electric circuit is interrupted with no arcing, electromagnetic energy storage converses to electrostatic energy with high overvoltages (e.g. interruption of small inductive currents). However, in most cases thermal phenomena play negative role for power equipment and their components.

As it was noted, the power electric and electronic equipment contains current carrying components providing the passage of electrical energy. A certain part of energy is lost in the current-carrying component resulting in its heating. Alternatively, heated component gives up (dissipates) thermal energy to environment. Hence, the equipment or its separate component can be considered as thermodynamic object, wherein thermal energy is generated and driven out to environment. If generated capacity is equal to driven out capacity, the component operates in steady state. It usually takes place, as the equipment continuously operates under rated conditions. If generated and driven out capacities are not equal, the component operates in unsteady state. The negative role of thermal phenomena is in the fact that the problem of thermal energy driving out (dissipating) in the process of electrical equipment functioning is quite complex. It is especially topic problem for power electronic (semiconductor-based) equipment, wherein great heat generation occurs. In such cases the following special cooling technologies are used:

- *finned heat sinks***:** devices with extended surface of cooling;

- *liquid cooling***:** using a liquid (water or transformer oil) as coolant media;

- *forced cooling***:** cooling the components with the help of movable medium (the flow of air, water or transformer oil);

- *cooling with phase change***:** usage of thermosyphons or heat pipes.

1.2 Heat specification for equipment components

It is well known that the object heat degree is defined by the value of *temperature*. It is time-constant in steady state and time-varying in unsteady state. The temperature of an object is defined by the capacity released or by the intensity of heat rejection. It is very important to establish the *maximum allowable temperatures*, which must not be exceeded while the equipment is in operation. If the allowable temperature of the component is low enough, it results in non-rational expenditure of materials in equipment design. It leads to non-competitiveness of the equipment products. If the temperature is far high, then it affects such important performances (characteristics) of the equipment as reliability, life span and in some cases functional ability. Allowable heat temperatures of the equipment components are defined by the following factors:

1. The most important factor limiting the allowable temperature is thermal stability of solid insulating materials, which in contact with heat generating components. It should be noted that solid insulating materials can continuously save insulating properties over certain temperature depending on the material. All electrical insulating materials for electrical power equipment and other electro-technical applications in respect to thermal stability are divided into classes as shown in table 1.1 [30]. Allowable temperature above 250°C should be increased by interval 25 K with assignment of corresponding classes.

Table 1.1 – Continuously allowable temperatures for the insulating materials of different classes as per ГОСТ 8865–93 (IEC 85-84)

Class	Material	Allowable temperature, °C
Y	Fibrous materials of cellulose, silk etc. non-impregnated and non-immersed into liquid insulator	**90**
A	The same, but immersed into liquid insulator	**105**
E	Particular synthetic and organic films etc.	**120**
B	Materials based on mica (on organic backup plates included), asbestos and glass-fiber applied with organic binders and impregnators	**130**
F	The same, but applied in combination with synthetic binders and impregnators	**155**
H	The same, but applied in combination with silicon-organic binders and impregnators as well as silicon-organic elastomers	**180**
200		**200**
220		**220**
250		**250**

2. Allowable temperatures are also defined by behavior of metals mechanical properties used for making current-carrying components of equipment; when the temperature comes as high as 165°C and above, tensile strength and other mechanical parameters of most metals significantly decrease; in this case duration of heat is very important (on long-time heat deterioration of mechanical properties occurs at lower temperatures than on short-time one); therefore, allowable heat temperatures are differently specified for steady state (rated current load) and unsteady state (overload, short-circuit conditions);

3. Allowable temperatures of equipment are also defined by allowable heat of electrical contacts (see subsection 5.5). Contacts of electrical apparatus usually limit the allowable heat of current-carrying parts, since increase in temperature of the contact interface causes the intensification of oxidizing processes, which lead to growth of contact resistance and increase in heat generation.

4. The basic parameter for power electronic equipment that defines its maximal current load is ultimate load capability of power semiconductor devices (PSCD). In steady state it is defined by maximal value of forward current passing through the device, at which its operational ability will be ensured. It is defined by *maximum allowable temperature* of PSCD *p-n*

junction that must not be exceeded in long-term duty of service (at steady state conditions). It depends on the type of the device and its design. Specific magnitudes of the maximal allowable temperature are represented in nameplate data for each type of device.

Considering these factors, heat is specified for the elements of the power electrical and electronic equipment that is expressed by the maximum allowable temperatures. They must not be exceeded while equipment is in operation. Allowable temperature magnitudes are set up by engineering specifications: national and international (e.g. International Electrotechnical Commission) standards, other specifications for particular types of equipment and its components. It should be pointed out that allowable temperature values are also defined by equipment's operation mode. For nominal (steady state) conditions allowable temperatures are specified by ГОСТ12434-84 (power equipment for high voltage applications) and ГОСТ8024-90 (power equipment for low voltage applications).

For unsteady state (overload, short-circuit), when duration of current flowing does not exceed a few seconds, higher allowable temperatures are set up. However, it should be noted that theirs magnitudes are not specified by standards. There are only some recommendations represented in special textbooks and handbooks. For example, for copper conductors non-covered or covered with organic insulation, allowable temperature comprises 250°C; for copper conductors covered with non-organic insulation – 350°C; for aluminium conductors non-covered – 350°C [5, 17].

Draw your attention that the operating temperature of equipment components significantly depends on ambient temperature since losses in equipment define only temperature-rise over the ambient temperature. It is a common fact that ambient temperature (free atmosphere) is not constant. It depends on season and daily time, climate conditions. In engineering process most ambient temperatures are accepted. For example, for power electric and electronic equipment for applications in template climate it comprises +40°C. Compliance with specifications for power electric and electronic equipment is checked by testing for its heating produced by continuous load current and by momentary short circuit current.

1.3 Generation of thermal energy in power electric and electronic equipment

Thermal energy in power electric and electronic equipment is at first generated by *current carrying components* due to *Joule's losses*. Amount of heat generated by a conductor volume V for the time span t_1–t_2 is generally expressed as:

$$Q = \int_{t_1}^{t_2} \int_{V} j^2 \rho dV dt\,, \tag{1.1}$$

where j is current density within the volume element dV; ρ is resistivity of the material.

Note that using this expression is rather difficult. Let's consider simpler case. It is a wire having invariable cross-sectional area flowed by direct non-time-varying current I. Quantity of thermal energy released in such case is calculated by the following expression:

$$Q = I^2 Rt\,, \tag{1.2}$$

where R is active resistance of the wire expressed as:

$$R = \rho \frac{l}{q}\,, \tag{1.3}$$

where l, q is length and cross-sectional area of the wire, respectively.

Remember that resistivity of the conductor materials is complex function of temperature:

$$\rho = \rho_0 \left(1 + \alpha_1 \vartheta + \alpha_2 \vartheta^2 + \alpha_3 \vartheta^3 + \cdots\right), \tag{1.4}$$

However, only two terms may be taken into account for the considered temperature range:

$$\rho = \rho_0 (1 + \alpha \vartheta), \tag{1.5}$$

where α is the temperature coefficient of resisitivity of the conductor material.

When alternating current flows through a wire, additional losses conditioned by *skin effect* (the current is forced towards the outer surface of the wire, and away from the center) and *proximity effect* (effect of nearby wires with current) take place. In such case power losses in a conductor will be determined as follows:

$$P = I^2 R \kappa_{д}\,, \tag{1.6}$$

where $\kappa_д$ is additional losses coefficient defined by the following production

$$\kappa_{д} = \kappa_{пэ} \kappa_{бл}\,, \tag{1.7}$$

where $\kappa_{пэ}$ is skin-effect coefficient taking into account the influence of the skin-effect on the conductor resistance, defined by the relation of the conductor resistance while passing through it an alternating current to its resistance at direct current)

$$\kappa_{sk} = \frac{R_{\sim}}{R_{=}} > 1\,, \tag{1.8}$$

where $\kappa_{бл}$ is proximity effect coefficient taking into account influence of the proximity effect on the conductor resistance defined by the relationship:

$$\kappa_{pr} = \frac{R_{\sim}}{R_{\sim s}}\,, \tag{1.9}$$

where $R_{\sim}$ is ac resistance of the conductor with available nearby other conductors; $R_{\sim s}$ is ac resistance of the single conductor (with no available other conductors nearby).

Skin-effect. The skin effect is the tendency of an alternating current to distribute itself within a wire so that the current density near the surface of the wire is greater than that at its core. Thus, the alternating current tends to flow at the “skin” of the conductor. It can be pointed out that in this case we consider a single conductor not affected by magnetic field produced by other wires. Action of the wire’s magnetic field results in skin effect.

When alternating current flows through the conductor, the alternating magnetic field excited induces counter-emf within it. The values of counter-emf in different points of the cross-section will be distinctive since they are linked with different magnetic fluxes. The most counter-emf will be on the core of the wire since it is linked with most magnetic flux. At the “skin” of the wire counter-emf will be smaller since it is linked with smaller magnetic flux. Thus, the nearer elemental wire is placed to “skin” of the wire, the higher is its inductive impedance. That is why the current density will grow with increase in distance from the wire’s core.

AC density distribution from outer surface to deeper layers of massive round wire is expressed by the following equation [9]:

$$j = j_{max}\, e^{-x/\Delta_{эм}} \Big/ \sqrt{1 - x/r_0}\,, \tag{1.10}$$

where j is current density at the depth x from the wire surface; j_{max} is maximal current density on the wire surface; r_0 is the wire radius; $\Delta_{эм}$ is equivalent penetration depth of electromagnetic wave (current) from the wire surface to its depth or *skin depth*; it is defined by substitution of an actual inhomogeneous distribution of current density by homogeneous one equal to maximal current density j_{max}.

The skin depth of a massive conductor is expressed as:

$$\Delta_{эм} = \sqrt{\frac{\rho}{\omega\mu\mu_0}}\,, \tag{1.11}$$

where ω is ac angle frequency; $\mu\mu_o$ is absolute magnetic permeability of conductor material.

Hence, influence of the skin effect increases with growth of ac frequency and also is defined by electrical and magnetic properties of the conductor material. It can be pointed out also substantial influence of geometry shape of the wire cross-section on its resistance due to the skin effect. From expression (1.11) it follows also that the skin effect shows itself in significantly higher degree when alternating current flows through ferromagnetic conductors and by contrast in lower degree for non-magnetic conductors. Thus there are different approaches to determination of additional losses due to skin effect for ferromagnetic and non-magnetic conductors.

The skin effect factor for non-magnetic conductors with relatively great skin depth is determined with the help of the parameter: $\sqrt{f/R}$, where f is ac frequency; R is dc pure resistance of the wire of definite length (mostly 100 m).

Simple calculations show that for Cu 25-mm radius wire at ac frequency 50 Hz the skin effect factor is 1,45. That is why application of solid cylindrical wires with diameter 25–30 mm is beside the purpose due to enhanced heat release caused by skin effect. In this case the wire cross-section is not used properly. Thin-wall profiles bounded by smooth curves closed with round or ellipse are usually used for better use of cross-section of wires for high (2–12 kA) and ultrahigh applications. It is recommended to use thickness of the wall in the range (0.4–0.8)$\Delta_{эм}$.

To determine the losses in ferromagnetic conductors with taking into account skin effect is more difficult due to dependence of magnetic permeability on magnetic field intensity. It is very important that even at power frequency 50 or 60 Hz within a few millimeters from the outer surface of the conductor electromagnetic wave (at fairly high magnetic intensity on the surface) almost fully decays. Hence, it can be accepted that in this region practically all losses by the eddy current and remagnetization are concentrated. Thus, skin effect factor in a ferromagnetic wire will be expressed as:

$$\kappa_{\text{sk}} = \frac{R_{\sim}}{R_{=}} = \frac{\rho \dfrac{l}{p\Delta}}{\rho \dfrac{l}{q}} = \frac{q}{p\Delta} = \frac{q}{p}\sqrt{\frac{\omega\mu\mu_0}{\rho}}, \quad (1.12)$$

where q, p are cross-sectional area and perimeter of the ferromagnetic wire, respectively.

In the expression obtained, relative permeability μ depends on magnetic intensity H, which, in turn, depends on the current flowing through the wire and its cross-sectional perimeter:

$$H = \frac{I}{p}, \quad (1.13)$$

Accordingly, the power losses in the wire will be expressed as:

$$P = I^2 R = H^2 p^2 \rho \frac{l}{p\Delta} = H^2 pl\sqrt{\omega\rho\mu_0\mu(H)}, \quad (1.14)$$

Thus, thermal power generated by unit of ferromagnetic component surface is:

$$p_{\text{sp}} = \frac{P}{pl} = H^2 \sqrt{\omega\rho\mu_0\mu(H)}, \quad (1.15)$$

It should be noted that using expression (1.15) is difficult in practical engineering because of dependence of relative permeability μ on the magnetic intensity H. Therefore, thermal losses in ferromagnetic elements are frequently determined with the help of empirical formulas and calculating curves. For example, simplified calculations of thermal losses power at alternating current I with frequency f flowing through ferromagnetic conductor can be performed as follows [6]

$$P = (2.9...3.25)\cdot 10^{-4}\left(\frac{I}{p}\right)^{5/3} F\sqrt{f}, \quad (1.16)$$

where p is the perimeter of the conductor cross-section; F is the conductor side surface.

Proximity effect. In a conductor carrying current, if currents are flowing through one or more nearby conductors, the distribution of current within one conductor will be constrained to smaller regions. The resulting *current crowding* is called the proximity effect. The closer conductor is relatively to the considered one, the stronger the proximity effect exhibits.

The value of proximity effect coefficient as well as skin effect one is significantly dependent on ac frequency, electrical and magnetic properties of the conductor material, geometry and mutual arrangement of the wires and values and phases of currents through the wires. It should be noted that proximity effect factor can be both higher and lower than 1. In certain cases it results in leveling off current density of wire's cross-section. Quantifying proximity effect factor is very difficult; for particular cases it has been represented in handbooks [1, 5, 6, 17, 21].

It is observed that along with Joule's heat produced by current carrying parts thermal energy is also generated by the equipment components free from current carrying, such as, *ac magnetic circuits* (including short-circuited turns) as well as *constructional components of equipment*: casing pieces, covers, flanges, arms etc. All denoted components are usually made of ferrous materials and are frequently affected by ac magnetic field. It results in heat generation due to eddy currents. Thermal power generated by ferrous material components free from current carrying affected by ac magnetic field may be quantified by the following simplified formulas [6]:

- for closed one-piece magnetic circuit having magnetizing coil,

-

$$P = (2.9...3.25)\cdot 10^{-4}\left(\frac{Iw}{l_{\mathrm{m}}}\right)^{5/3} F\sqrt{f} \quad , \qquad (1.17)$$

where Iw is magneto-motive force of the coil; l_{m} is length of the average magnetic force line, m; F is side surface area of the magnetic circuit, m^2;

- for closed magnetic circuit laminated of electrotechnical steel sheets

$$P = p_{\mathrm{sp}} m \, , \qquad (1.18)$$

where m is the mass of the magnetic circuit; p_{sp} is specific losses.

- for ferromagnetic ring enclosing a wire with current I

$$P = (2.9...3.25)\cdot 10^{-4}\left(\frac{I}{l_{\mathrm{m}}}\right)^{5/3} F\sqrt{f} \quad , \qquad (1.19)$$

where l_{m} is length of middle magnetic line, m; F is the surface area of the ring, m^2;

To reduce power losses and heat generation of the components free from current carrying such precautions are taken:

1) making these components of non-ferrous materials such as aluminum,

brass, bronze, non-magnetic cast iron, nonmagnetic steel etc.;

2) application of nonmagnetic gaps on the magnetic flux path: air, copper, brass;

3) application of short-circuited components enclosing its cross-sectional area;

4) application of electromagnetic shields.

The electrical insulation, especially of equipment for high voltage applications, generates thermal energy and subsequently is the cause heating its components. Under voltage applied to solid insulation, leakage current flows through it. This current has active I_a and reactive (capacitive) I_C components. The ratio between them is so-called *dielectric loss tangent*

$$tg\delta = \frac{I_a}{I_C}, \qquad (1.20)$$

Its magnitude indicates the quality of insulating materials: its values of order 10^{-4}…10^{-3} corresponds to high quality of insulation material; of order 10^{-2}…10^{-1}, it corresponds to low quality.

Capacitive component of the current is expressed as

$$I_C = U\omega C, \qquad (1.21)$$

where C is the capacitance across the insulation gap.

Substitution of the expression (1.21) into (1.20) as well as multiplication of both numerator and denominator of (1.20) into the value of voltage applied across insulation gap gives:

$$tg\delta = \frac{P}{U^2\omega C}, \qquad (1.22)$$

Hence, the dielectric losses defining heating of solid insulation components will be expressed as:

$$P = U^2\omega C tg\delta, \qquad (1.23)$$

This expression shows that dielectric losses significantly grow with increase in voltage. Therefore, consideration of the dielectric losses is especially important for high-voltage electrical equipment. Most important problem in this case is in homogeneity of electrical field, as dielectric losses concentrate in small regions of insulating constructional components. It causes local heating of insulation and its thermal breakdown. That is why in the engineering process of high voltage insulating structures significant efforts are concentrated to ensure that electrical field will be as homogeneous as possible. It is very complex problem that leads to substantial increase in cost of high-voltage electrical equipment.

Significant thermal source in switching apparatuses is the *electric arc* which occurs as electrical power circuit is interrupted. As it was mentioned, it plays positive role since limits switching overvoltages. But, it is quite powerful heat source due to high temperature (3000…20000 K). That is why duration of electric arc must be restricted. It is especially important for switching apparatuses applied for frequent switching operations.

Thermal sources in electric and electronic power equipment are also *electrical contacts* carrying current load and performing great variety of functions. In some instances *friction forces* in movable components are significant for thermal behavior of the power equipment.

Main thermal sources in power electronic equipment are *power semiconductor devices* **(**PSCD**).** It should be noted that thermal losses occurred with the passage of electric current through a semiconductor device is by some orders more than in contacts carrying the same current load. In the process, heat is generated in very small volume of semiconductor structure (*p*-*n*-junction). Losses in PSD in steady state are defined by electrical parameters and load duty. They are aggregated of losses at the passage of current in forward and reverse directions and losses by switching. For arbitrary wave voltage applied to a device

$$P_{\text{пр}} = \int i_{\text{пр}} du_{\text{пр}}\,; \qquad P_{\text{обр}} = \int i_{\text{обр}} du_{\text{обр}}\,, \tag{1.24}$$

where $i_{\text{пр}}$, $i_{\text{обр}}$ are instantaneous values of currents (as function of $u_{\text{пр}}$ and $u_{\text{обр}}$, respectively) defined by the wave-form of voltage-current characteristic (for slowly enough time-varying voltage).

In most low-frequency operational duties of PSCD the losses in the time of current passage in reverse direction and losses due to switching can be neglected since they are not comparable with losses at the passage of forward current. Current-voltage characteristic of a PSCD is complex enough and cannot be described by simple analytical dependence $I_{\text{пр}} = f(U_{\text{пр}})$. To simplify thermal calculations, it is linearized so that relationship between current and losses in forward direction is expressed as

$$P_{\text{пр}} = I_{\text{пр}} U_0 + \kappa_{\text{ф}}^2 R_{\text{д}} I_{\text{пр}}^2\,, \tag{1.25}$$

where $I_{\text{пр}}$ is average value of forward current expressed for arbitrary wave current as follows:

$$I_{\text{пр}} = \frac{1}{\tau_{\text{и}}} \int_{0}^{\tau_{\text{и}}} i d\tau \,, \tag{1.26}$$

where $\tau_{\text{и}}$ is duration of current impulse; U_0, $R_{\text{д}}$ are parameters of linearized current-voltage characteristic: cut-in voltage and dynamic resistance, respectively; $\kappa_{\text{ф}}$ is the form coefficient of current reducing average value to rectangular impulse of equivalent capacity.

Computation formula to calculate form factor for the typical case sinusoidal waveform is

$$\kappa_{\text{ф}} = \frac{\sqrt{\pi}\sqrt{2(\pi - \alpha) + \sin 2\alpha}}{2(1 + \cos\alpha)} \,, \tag{1.27}$$

where α is cut off angle; it is in the range of 0 to 180 el. degrees.

2. WITHDRAWING HEAT FROM EQUIPMENT COMPONENTS

2.1 General

Withdrawing thermal energy from physical (thermodynamic) object (solid, liquid, gaseous, plasma) is generally carried out by means of the heat transfer.

Heat transfer is the passage of thermal energy from a hot to a colder body. It is a complex process caused by the temperature difference. Where there is a temperature difference between objects in proximity, heat transfer between them can never be stopped; it can only be slowed down. Classical transfer of thermal energy occurs only through conduction, convection, radiation or any combination of these. In each case, the driving force for heat transfer is a difference of temperature.

Conduction is the transfer of thermal energy through direct molecular communication within a medium or between mediums in direct physical contact without a flow of the material medium. In metals this process is more intensive due to free electron diffusion.

Convection is a mechanism of heat transfer occurring because of bulk motion (observable movement) of fluids. The transport of heat in this case is intimately associated with the transport of substance and leveling off density (pressure) of the fluid.

Radiation – transfer of heat by electromagnetic waves or, equivalently, by photons. Internal energy of the body (emitter) converses to the energy of the electromagnetic radiation that is absorbed by another body or dissipates to environment.

It should be noted that foregoing mechanisms of the heat transfer rarely occur by itself from each other. In most practical cases they are observed simultaneously; this case is named *combined heat exchange*.

2.2 Heat transfer by conduction

Heat is transferred by conduction when adjacent atoms vibrate against one another, or as electrons move from atom to atom. Conduction is greater in solids, where atoms are in constant contact. In liquids (except liquid metals) and gases, the molecules are usually further apart, giving a lower chance of molecules colliding and passing on thermal energy.

Heat conduction is directly analogous to diffusion of particles into a fluid, in the situation where there are no fluid currents. Metals (e.g. copper) are usually the best conductors of thermal energy. This is due to the way that metals are chemically bonded: metallic bonds (as opposed to covalent or ionic bonds) have free-moving electrons and form a crystalline structure, greatly aiding in the transfer of thermal energy. Fluids (except liquid metals and gasses) are not typically good conductors. This is due to the large distance between atoms in a gas: fewer collisions between atoms means less conduction. As density decreases so does conduction. Conductivity of gases increases with temperature but only slightly with pressure near and above atmospheric. Conduction does not occur at all in a perfect vacuum.

So, this way of heat transfer is not typical for liquids (except liquid metals) and gases. In solid materials heat transfer occurs only through conduction both within body and between bodies in proximity. Therefore, in further studying the heat conduction we will deal with heat transfer in an *isotropic solid*, that is, a body having adequate properties along all directions.

Conductive heat transfer process is characterized by certain notions, such as:

- *temperature field* is a set of the temperature values in different points of the object considered at given time moment; it is mathematically expressed as following function: $\vartheta = f(x, y, z, t)$. The temperature field features *thermal state* **(***thermal behavior***)** of the object at the predetermined time instant. If the temperature field is constant in time, it is *stationary* one; otherwise, it is *non-stationary* one. The main property of the temperature field is continuity, that is, the temperature in any point of considered object (medium) can not vary by step;

- *isothermal surface* is geometrical set of the temperature field points with equable temperature;

- *heat flow* **(***heat flux***)** is amount of thermal energy transferred through the isothermal area in time unit (heat flow per unit of isothermal surface). The vector of heat flux is always normal to the isothermal surface.

To describe the process of conductive heat transfer is the *law of heat conduction,* also known as *Fourier's law* is used. It establishes relationship between temperature field and heat flux at any point of the object considered. It states that the heat flux through a material is proportional to negative gradient in the temperature:

$$q = -\lambda grad\vartheta \text{ or } d^2Q = -\lambda \frac{\partial \vartheta}{\partial n} dSdt, \qquad (2.1)$$

where λ is the material conductivity.

Sign minus in this expression indicates that heat flux is always directional as decrease in temperature.

Heat conductivity is most important thermal-and-physical characteristic of the material in respect to ability to conduct heat. It is amount of thermal energy passing through the unit of area per time unit at the temperature difference 1 degree. It is revealed that for the most metals within relatively small range of temperatures the conductivity is linearly variable with temperature:

$$\lambda = \lambda_0 \left[1 + \beta (T - T_0)\right], \tag{2.2}$$

where β is the temperature coefficient of conductivity; λ_0 is the conductivity at certain temperature $T = T_0$.

To determine the amount of heat passing through the isothermal area S for the time span 0–t, expression of the Fourier's law is to be integrated with respect to time over the area S:

$$Q = -\int_0^t \int_S \lambda grad \vartheta dS dt, \tag{2.3}$$

This expression is the integral representation of the Fourier's law.

Basing upon Fourier's law the heat conduction equation for an isotropic body with uniform internal heat generation is derived. It is as follows:

$$\frac{\partial \vartheta}{\partial t} = a \cdot div \; \overline{grad \vartheta} + \frac{q_V}{c\gamma}, \tag{2.4}$$

where; c, γ is specific heat and density of the substance, respectively; q_V is the volumetric internal heat generation in the body; a is the thermal diffusivity expressed as

$$a = \frac{\lambda}{c\gamma}, \tag{2.5}$$

In stationary (steady) state (when $\frac{\partial \vartheta}{\partial t} = 0$) heat conduction equation is reduced to Poisson's equation. If considered body has not internal heat generation $(q_V = 0)$, then it is reduced to Laplace's equation. Heat conduction equation can be represented:

- in Cartesian coordinate system, then

$$\overline{grad\vartheta} = \boldsymbol{i}\frac{\partial\vartheta}{\partial x} + \boldsymbol{j}\frac{\partial\vartheta}{\partial y} + \boldsymbol{k}\frac{\partial\vartheta}{\partial z}, \qquad (2.6)$$

- in spherical coordinate system, then

$$\overline{grad\vartheta} = \boldsymbol{e}_r\frac{\partial\vartheta}{\partial r} + \boldsymbol{e}_\varphi\frac{1}{r\cdot\cos\theta}\cdot\frac{\partial\vartheta}{\partial\varphi} + \boldsymbol{e}_\theta\frac{1}{r}\frac{\partial\vartheta}{\partial\theta}, \qquad (2.7)$$

- or in cylindrical coordinate system, then

$$\overline{grad\vartheta} = \boldsymbol{e}_r\frac{\partial\vartheta}{\partial r} + \boldsymbol{e}_\varphi\frac{1}{r}\frac{\partial\vartheta}{\partial\varphi} + \boldsymbol{e}_z\frac{\partial\vartheta}{\partial z}, \qquad (2.8)$$

In order to solve the heat conduction equation the conditions of uniqueness for each specific task must be predetermined. They may be predetermined by some means, such as: *initial conditions* prescribing the temperature field of given body at zero-time moment $\vartheta = f(x, y, z, 0)$; *boundary conditions* prescribing anyhow conditions on the body's surface at any time moment. Boundary conditions are usually predetermined in such manners:

- temperature field on the body's surface, known as *boundary conditions of the first kind*;
- heat flow through the surface of given body, known as *boundary conditions of the second kind*;
- ambient temperature and conditions of heat exchange of given body with environment, known as *boundary conditions of the third kind*;
- when heat transfer on the boundary of given body occurs trough conduction, then the boundary condition is expressed as follows:

$$\lambda_1\left(\frac{\partial\vartheta}{\partial t}\right)_1 = \lambda_2\left(\frac{\partial\vartheta}{\partial t}\right)_2, \qquad (2.9)$$

where the indexes accord to first and second bodies having common boundary.

To solve heat conduction equation the number of methods are used, such as, classic, operational, numerical, sources and absorbents etc. The method of sources and absorbents is applied in the cases when highly concentrative thermal sources are available, for example, heating the contacts with electrical arc via its roots. Heat conduction equation for stationary and non-stationary conditions is generally 3-dimensinal problem. However, for much practical problems it is significantly simplified and is reduced to 2-dimensional and even one-dimensional problems.

As an example, let us consider the practical task to determine steady state temperature field of infinitely long cylindrical conductor coated with even solid insulation layer shown in Figure 2.1. Direct current *I* flows through the conductor with diameter d_1; outer diameter of the wire is d_2, (see Figure 2.1).

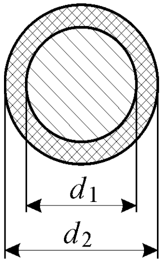


Figure 2.1

Thermal energy generated by the conductor is transferred by conduction in radial direction to internal and further to outer surface of insulation layer. From outer surface of the wire thermal energy is dissipated to its surrounding by other heat transfer mechanisms (convection and/or radiation), which in this task is not considered.

The temperature field within the conductor is governed by the heat conduction equation, which in this case is most suitable to represent in cylindrical coordinates:

$$\frac{\partial \vartheta}{\partial t} = \frac{\lambda}{c\gamma}\left(\frac{\partial^2 \vartheta}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta}{\partial r} + \frac{\partial^2 \vartheta}{\partial \varphi^2} + \frac{\partial^2 \vartheta}{\partial z^2}\right) + \frac{q_V}{c\gamma}, \tag{2.10}$$

In steady-state conditions $\frac{\partial \vartheta}{\partial t} = 0$. Since the heat transfer conditions are axially and circularly not variable, then

$$\frac{\partial^2 \vartheta}{\partial \varphi^2} = \frac{\partial^2 \vartheta}{\partial z^2} = 0, \tag{2.11}$$

At direct current in single conductor the skin and proximity effects does not occur, accordingly uniform internal heat generation within the conductor is available

$$q_V = j^2 \rho, \tag{2.12}$$

Hence, heat conduction equation, which describes temperature field within the conductor, is:

$$\frac{\partial^2 \vartheta}{\partial r^2} + \frac{1}{r}\frac{\partial \vartheta}{\partial r} + \frac{j^2 \rho}{\lambda} = 0, \tag{2.13}$$

Its solution with boundary condition of first kind $\vartheta(d_1/2) = \vartheta_1$ will be expressed as:

$$\vartheta = \vartheta_1 + \frac{j^2\rho}{4\lambda}\left(\frac{d_1^2}{4} - r^2\right), \qquad (2.14)$$

Derived expression enables to find temperature difference between the conductor's surface and its core:

$$\vartheta(0) - \vartheta_1 = \frac{j^2\rho d_1^2}{16\lambda}, \qquad (2.15)$$

The calculations by this expression for continuously loaded copper conductor at following data:

$$j = 4\,\frac{\text{A}}{\text{mm}^2};\, d_1 = 20\ \text{mm};\ \ \rho = 1.62\cdot 10^{-8}\ \text{Ohm}\cdot\text{m};\ \ \lambda = 390\,\frac{\text{Wt}}{\text{m}\cdot\text{K}},$$

give this difference:

$$\Delta\vartheta = \frac{\left(4\cdot 10^6\right)^2\cdot 1.62\cdot 10^{-8}\cdot 0.02}{16\cdot 390} \approx 0.17\ \text{K}.$$

The result obtained indicates that the temperature field immediately within the conductor is practically uniform.

The solid insulation layer is in this case hollow cylinder with no internal heat generation. Accordingly, the heat conduction equation to describe the temperature field within it is as follows:

$$\frac{\partial^2\vartheta}{\partial r^2} + \frac{1}{r}\frac{\partial\vartheta}{\partial r} = 0, \qquad (2.16)$$

Solution of this equation with same boundary condition of first kind as well as boundary condition of second kind

$$\vartheta\left(\frac{d_1}{2}\right) = \vartheta_1 \quad p = -\pi d_1 \lambda_{\text{из}}\frac{\partial\vartheta}{\partial r}, \qquad (2.17)$$

will be as follows:

$$\vartheta = \vartheta_1 - \frac{p}{2\pi\lambda_{\text{из}}}\ln\frac{r}{d_1/2}, \qquad (2.18)$$

where p is thermal power generated by the conductor per unit of its length.

The expression derived enables to determine the temperature difference between internal and outer surfaces of the insulation layer:

$$\vartheta_1 - \vartheta_2 = p \frac{1}{2\pi\lambda_{из}} \ln \frac{d_2}{d_1}, \tag{2.19}$$

This expression can be written with respect to generated power:

$$p = \frac{\vartheta_1 - \vartheta_2}{\frac{1}{2\pi\lambda_{из}} \ln \frac{d_2}{d_1}} \Rightarrow p = \frac{\vartheta_1 - \vartheta_2}{r_{th}}, \tag{2.20}$$

From this expression vital tenet in respect to analogy between electrical and thermal processes follows. The temperature difference $\vartheta_1 - \vartheta_2$ is analogy in respect to fall of potential in electric circuit. Heat flow p in this expression is analogy in respect to current in electrical circuit. From this analogy the notion of *thermal resistance* and respectively analogy between electrical and thermal conductivities follows. Therefore, expression (2.20) is frequently named *thermal Ohm's law*. In given case it is valid for the insulation layer as the section of thermal circuit.

Hence, for unit length of hollow cylinder with no internal heat generation thermal resistance is expressed as

$$r_{th} = \frac{1}{2\pi\lambda} \ln \frac{d_2}{d_1}, \tag{2.21}$$

where λ is heat conductivity of the wall material; d_1, d_2 is inner and outer diameter of the wall, respectively.

By the same a way, using Fourier's law and heat conduction equation thermal resistance of a plane wall with no internal heat generation is determined [6, 30, 31].

$$R_{th} = \frac{\delta}{\lambda S}, \tag{2.22}$$

where δ is the wall thickness; S is the wall cross-sectional area.

When it is dealt with multilayer walls total thermal resistance is determined as sum of thermal resistances of the separate walls.

The notion of thermal resistance enables to solve more complex tasks without need to solve immediately the heat conduction equation, but by means of constructing the equivalent thermal circuits.

2.3 Convective heat transfer: fundamentals

As already noted, convective heat transfer (convection) is carried out by bulk movement (observable movement) of fluid. In more detail, convection is a combination of conduction and the transfer of thermal energy by fluid circulation or movement of the hot particles in bulk to cooler areas in material medium. That is, unlike the case of pure conduction, now currents in fluids are additionally involved in convection. This movement occurs into a fluid or within a fluid, and cannot happen in solids because here molecules are rigidly held in space and any currents cannot occur.

Hence, convective heat transfer is most typical for the case of heat exchange between solid surface of a body and fluid (liquid or gaseous medium). Heat transfer by conduction in this case occurs in narrow fluid interlayer, which is in direct contact with solid surface named *boundary layer*. Convective heat transfer occurs in two forms:

- *natural* or *free convection* in which the fluid motion is not generated by any external source. In natural convection, fluid surrounding a heat source receives heat, becomes less dense and rises. The surrounding, cooler fluid then moves to replace it. This cooler fluid is then heated and the process continues, forming a convection current;

- *forced convection*, by contrast, occurs when pumps, fans or other means are used to propel the fluid and create an artificially induced convection current. Forced convection can be also naturally caused by the forces of wind.

Behavior of the fluid motion both at the free and at the forced convection depends on the fluid physical properties, difference in temperature of the heated body and the fluid as well as volume of the space wherein there is heat exchange. Two basic variations of the fluid motion (flow) along a heated surface are distinguished:

- *laminar*: such motion is usually observed at low velocities of the fluid motion; in the process the fluid particles move practically in parallel with the heated surface;

- *turbulent*: with increase in velocity of the fluid motion the vortices of certain fluid particles happen; transition from laminar to turbulent motion occurs at certain velocity named *critical* one.

Convective heat transfer is described with complex set of the differential equations:

1. *Heat conduction equation* under conditions of convective heat transfer is added by the term depending on velocity of bulk movement. When uniform internal heat generation is available, it is as follows:

$$\frac{\partial \vartheta}{\partial t} + \mathbf{v} \cdot \nabla \vartheta = a \nabla^2 \vartheta + \frac{q_V}{c\gamma} \; ; \qquad (2.23)$$

2. *Equation of fluid motion*, known as *Navier-Stokes equation*:

$$\gamma \frac{d\mathbf{v}}{dt} = \gamma \mathbf{g} - \nabla \mathbf{p} + \mu \nabla^2 \mathbf{v} \; ; \qquad (2.24)$$

3. *Continuity equation* characterizing continuity of fluid motion:

$$\frac{\partial \gamma}{\partial t} + div(\gamma \mathbf{v}) = 0 \, . \qquad (2.25)$$

If γ is a constant, as in the case of incompressible flow, the continuity equation is simplified as follows: $div(\gamma \mathbf{v}) = 0$.

In the expressions represented: ϑ is the temperature in arbitrary point; *t* is time; *a* is the fluid diffusivity; q_V is the volumetric internal heat generation; γ, μ is density and dynamic viscosity of the fluid, respectively; p is pressure; g, **v** is the vector of gravitational acceleration and fluid velocity, respectively; ∇, ∇^2 is the “nabla” (Hamilton) and Laplace operators, respectively.

A large number of the variables, which the equations describing convective heat exchange are included as well as complexities to assign starting and boundary conditions (see subsection 2.2) result in extreme difficulties to solve these equations. It is usually possible only under much simplifying conditions. However, such solutions are frequently inaccurate and are in poor agreement with experience. The importance of these equations for the most part is they point to the values that influence on the convective heat exchange process. Such values are at first thermo-physical quantities of the fluid, such as, heat conductivity, specific heat, density, volumetric expansion coefficient and viscosity.

Heat conductivity defines amount of heat, which can be withdrawn through the boundary layer. Since the temperature drop have been concentrated just in boundary layer, influence of the fluid heat conductivity on convective heat exchange is very significant. *Specific heat* comparatively little effects on the convective heat exchange in steady state, however in transients, for example when sudden rapid heating conductors, its influence is very substantial. *Density* and *volumetric expansion coefficient* define buoyancy force defining, in turn, velocity of convective currents. *Viscosity* of the fluid is equally important; it also defines the convective currents velocity at all factors being equal.

The convective heat exchange is strongly affected by the shape and dimensions of the heated body as well as by the physical properties of the fluid. It is caused by that the shape and dimensions define the conditions of the fluid flow around the bodies both at the free and at the forced convection.

One is forced to accept the fact that in spite of intensive development of mathematical knowledge and considerable possibilities of computer technologies rigorous solution of foregoing set of differential equations is for the most part impossible. By this why to analysis convective heat transfer the simplified methods that enable to avoid its immediate solution are used in engineering practice. One way is application of *Newton's law of cooling* combined with the scaling law. In XVIII century Newton and Russian scientist, academic Georg Richmann independently of one other revealed that the heat flow from the surface S is proportional to the difference in temperatures between the surface and its surroundings:

$$P = \alpha_{\text{к}} \left(\vartheta - \vartheta_0 \right) S \,, \tag{2.26}$$

where $\alpha_{\text{к}}$ is the convective heat transfer coefficient.

The *heat transfer coefficient* **(**htc) is most important parameter for the convective heat transfer. It defines the power dissipated (or absorbed) by solid surface unit through convection when the difference in temperatures between the surface and its surrounding is one degree. This quantity concentrates all sophistication of convective heat transfer and is very complex function of many quantities that significantly influences on the process.

In general case the heat transfer coefficient is the function of much number of quantities:

$$\alpha_{\text{к}} = f\left(\vartheta,\, \vartheta_0,\, \beta,\, \lambda,\, c_{\text{p}},\, \nu,\, a,\, g,\, v_{\text{f}}\,, \Phi\right), \tag{2.27}$$

where ϑ, ϑ_0 is the temperature of heated body and coolant media, respectively; v_{f} is velocity of the fluid flow; Φ is the complex of quantities that feature shape, structure, orientation of the surface in space and its dimensions.

Thermo-physical parameters of a fluid: β is the volumetric thermal expansion coefficient; for gases its value is determined as follows:

$$\beta = \frac{1}{T_0} = \frac{1}{\vartheta_0 + 273}; \tag{2.28}$$

λ is the heat conductivity; c_{p} is the specific heat at constant pressure; ν is the cinematic viscosity.

To use immediately complex function (2.27) is very inconvenient because of much number of variables that effect on the convective heat exchange. Therefore, taking into account degree of influence of ones or others parameters and basing on the findings of experiments the htc is usually expressed in simpler manner. It is to be taken into consideration here that each experiment yields the results that valid only for the investigated particular case. Extension of the findings obtained to another experiment conditions and investigate object is usually intolerable. However, basing upon foregoing set of differential equations and certain manner of processing the experiment findings suitable to noted equations the findings of experimental investigations can be extended on other cases and htc can be uniformly expressed for the bodies having various configurations and dimensions. It for the most part follows with the help of so-called *scaling law* or *similarity* **(***similitude***)** *theory*. Its fundamental statements are formulated in three similarity (similitude) theorems:

1. *Newton's theorem*: phenomena, which are similar to each other, offer identical dimensionless parameters.

2. *Buckingham's pi theorem***:** differential equations describing phenomena can be represented as dependence between dimensionless quantities (dimensionless equation).

3. *Kirpichev-Gukhman theorem*: two phenomena are similar, if they are described with identical set of differential equations and have similar conditions of uniqueness.

Scaling law enables to describe complex processes not with many separate quantities, but with theirs certain combinations that represent *dimensionless parameters* saving invariable magnitude for all similar phenomena. Thus, the process of convective heat exchange is mostly described with following parameters (numbers):

1) *Reynolds number* that gives a measure of the ratio of inertial forces to viscous forces and consequently quantifies the relative importance of these two types of forces for given flow conditions:

$$Re_{\text{f}} = \frac{vl}{\nu_{\text{f}}}, \tag{2.29}$$

where l is the dimension of the surface along direction of the flow;

2) *Nusselt number*, which defines thermal similarity and quantifies convective heat transfer from a surface:

$$Nu = \frac{\alpha_{\text{к}} L}{\lambda}; \tag{2.30}$$

3) *Grashof number*, which approximates the ratio of the buoyancy to viscous force acting on a fluid

$$Gr = \beta g \frac{L^3}{\nu^2}(\vartheta - \vartheta_0); \tag{2.31}$$

4) *Prandtl number*, which measures the relative importance of viscosity and heat conduction (in the sense of their relative abilities to produce diffusion of vorticity and of heat):

$$Pr = \frac{\nu}{a}. \tag{2.32}$$

In these expressions the quantity L is *characteristic dimension* (geometrical factor), that is, the geometrical dimension being typical for the body of given configuration (e.g. it is diameter for cylindrical surface oriented horizontally; it is height for the plane oriented vertically etc.). The answers to the questions: how dimensionless parameters are composed; how geometrical factors are determined, are to be found in special handbooks [11, 14–16, 30, 31].

Dependence (2.27) for the general case of convection is usually represented as the equation including all dimensionless numbers as follows:

$$Nu = f(Re \cdot Gr \cdot Pr). \tag{2.33}$$

Its specific form is defined by the heat transfer conditions.

2.4 External natural convection

External natural convection (convection in infinite space) takes place in practice when the fluid volume wherein there is a heat exchange so great that the temperature of a heated surface does not effect on the fluid temperature. In such case equation (2.33) will be represented in following form:

$$Nu = C(Gr \cdot Pr)_{\mathrm{m}}^{n}, \tag{2.34}$$

where C and n are the empirical coefficients defined by the magnitude of the product $(Pr{\cdot}Gr)_{\mathrm{m}}$ according to table 2.1; m is the index indicating that the magnitudes of physical quantities of the fluid λ, a, ν must correspond to *characteristic (middle) temperature* between the body ϑ and its surrounding ϑ_0.

The magnitude of the product $(Gr \cdot Pr)_{\mathrm{m}}$ indicates the regime of fluid motion (flow):

Table 2.1 – Magnitudes of *C* and *n*

$(Pr \cdot Gr)_m$	*C*	*n*
Less than 10^{-3}	0.5	0.0
$10^{-3}...5 \cdot 10^{2}$	1.18	0.125
$5 \cdot 10^{2}...2 \cdot 10^{7}$	0.54	0.25
$2 \cdot 10^{7}...10^{13}$	0.135	0.33

1. *Unstable flow (film regime)* (*Nu* = 0.5): such regime may take place, for example, in the neighborhood of the body with smooth outlines at low temperature heads; in this case almost stationary fluid layer forms in the immediate vicinity of the surface and heat exchange is mainly provided by the conduction; the rate of heat exchange is very low.

2. *Laminar motion (law of the 1/8 power)*: the rate of heat transfer in this case is low as well, however, moderate fluid motion along the surface is available; it is typical for the medium surrounding, for example, thin wires; the difference between unstable and laminar regimes is not qualitative, but quantitative.

3. *Intensive laminar motion (the law of 1/4 power)*: here the rate of heat transfer significantly increases; the paths of the fluid jets begin to loss parallelism with the heated surface, but the motion yet not take random behavior; it is transient between laminar and turbulent regimes; it takes place in the vicinity of medium-sized flat and cylindrical surfaces, plane fins of cooling radiators etc.

4. *Turbulent motion (the law of 1/3 power)*: in this case the heat exchange is highly intensive; analysis of formula (2.34) leads to the conclusion that dimensions of the body does not influence on intensity of the process; such regime occurs in the neighborhood of large-sized surfaces at high temperature heads.

It should be once again emphasized that basic regimes of the fluid motion at the free convection are laminar and turbulent. Unstable and intensive laminar are not always observed. Conversing equation (2.34), the formulae suitable for engineering calculations of natural convective htc for bodies distinctive shaped can be derived:

Heat transfer from infinite cylinders. The task to calculate convective htc arises when thermal behavior of cylindrical shaped wires is analyzed. Such calculations with reasonable accuracy can be performed by the following formula at $(Gr \cdot Pr) = 10^{-3} \ldots 5 \cdot 10^{2}$ [8]:

$$\alpha_c = A_1 \left(\frac{\vartheta - \vartheta_0}{d^5} \right)^{1/8}, \tag{2.35}$$

where d is the wire diameter in meters; A_1 is the quantity including all physical parameters of the medium; its magnitudes are represented in table 2.2.

Table 2.2 – The magnitudes of A_1 for the cooling mediums

Medium	The magnitudes A_1 at the temperature ϑ_m, °C						
	0	20	40	60	80	100	120
Air	0.291	0.295	0.300	0.306	0.310	0.315	0.320
Water	9.35	13.1	15.7	17.6	19.0	20.0	–
Transformer oil	1.8	2.06	2.27	2.43	2.57	2.66	2.73

Heat transfer from flat and cylindrical surfaces. The engineering formulae considered below are suitable to calculate convective htc for flat and cylindrical surfaces in conditions of natural convection in bulk. Analyzing formula (2.34) as well as taking the magnitudes of the coefficients C and n represented in table 2.1 into account, the inequality that enables to determine the regime of fluid flow in the vicinity of heated surface is derived

$$(\vartheta - \vartheta_m) \leq \left(\frac{840}{L} \right)^3, \tag{2.36}$$

where L is the geometrical factor in millimeters.

If inequality (2.36) is valid, then the fluid flow comply with the law of 1/4 power. Otherwise, the fluid flow comply with the law of 1/3 power.

In the case the law of 1/4 powers the following formula for calculations of convective htc for the flat vertically oriented with the height h or cylinder with diameter d (in meters) is proposed [8]

$$\alpha_c = A_2 \left(\frac{\vartheta - \vartheta_m}{h} \right)^{1/4}, \tag{2.37}$$

where A_2 is the quantity including all physical parameters of the medium; its magnitudes are represented in table 2.3.

If the heat transfer complies with the law of 1/4 powers, then the formula for calculations of convective htc for the flat vertically oriented, cylindrical or spherical surface is as follows [8]:

Table 2.3 – The magnitudes of A_2 for the cooling mediums

Medium	The magnitudes A_2 at the temperature ϑ_m, °C						
	0	20	40	60	80	100	120
Air	1.40	1.38	1.34	1.31	1.29	1.27	1.26
Water	90	105	149	178	205	227	–
Transformer oil	13.1	14.9	18.3	21.3	23.9	26.2	28.3

$$\alpha_c = A_3(\vartheta - \vartheta_m)^{1/3}, \qquad (2.38)$$

where A_3 is the quantity including all physical parameters of the medium; its magnitudes are represented in table 2.4.

It should be noted that the rate of heat exchange is significantly affected by the orientation of cooled surface. If it is horizontally oriented and is faced up, then calculated value of heat transfer coefficient is to be increased by 30%; if it is faced down, then – reduced by 30%.

Table 2.4 – The magnitudes of A_3 for the cooling mediums

Medium	The magnitudes A_3 at the temperature ϑ_m, °C						
	0	20	40	60	80	100	120
Air	1.69	1.61	1.53	1.45	1.39	1.33	1.26
Water	102	198	290	363	425	480	–
Transformer oil	17.0	24.6	32.2	39.8	47.4	56.0	–

2.5 Internal natural convection

Heat exchange in the case of internal natural convection or convection in limited (constrained) space of cooling medium is more complicated than in the case of external one. It is at first resulted from mutual influence of temperatures of the heated surface and cooling medium. This process is also affected by the shape and dimensions of the limited space. Such form of heat exchange for the most part takes place in the case of heat transfer through fluid interlayer (annulus).

At relatively small thickness of the interlayer (annulus), heat transfer occurs only through conduction. In this case, the entire temperature drop is concentrated within the laminar motion layers near the hot and the cold surfaces, and these drops continue each other. At relatively great thickness of the interlayer (annulus), the turbulent motion of the fluid is observed in the middle its region and here heat transfer occurs by convection. The temperature drop in this region is small. In this case, the convective component must be taken into account. For this purpose complex process of heat exchange through a fluid interlayer (annulus) is substituted by elementary conductive heat transfer (see subsection 2.2) with use of notion of the *equivalent heat conductivity*. Its value is expressed as follows:

$$\lambda_{eq} = \lambda_f \varepsilon_c , \tag{2.39}$$

where λ_f is the fluid heat conductivity at the temperature corresponding to midvalue between temperatures of the surfaces bounding the interlayer (annulus); ε_c is the convective factor determined with the following equation:

$$\varepsilon_c = A(Gr \cdot Pr)_m^r , \tag{2.40}$$

where A and r are the empirical coefficients defined by the magnitude of the product $(Pr \cdot Gr)_m$ according to table 2.5.

For infinite flat, cylindrical and spherical interlayer (annulus) the following approximate dependence may be recommended [6]:

$$\varepsilon_c = \begin{cases} 1 \quad \text{at } (Gr \cdot Pr) \le 1000; \\ 0.18 (Gr \cdot Pr)^{0.25} \quad \text{at } (Gr \cdot Pr) > 1000. \end{cases} \tag{2.41}$$

Table 2.5 – Magnitudes of A and r

$(Gr \cdot Pr)_m$	A	r
Less than 10^{-3}	1.0	0.0
$10^{-3}\ldots10^{6}$	1.105	0.3
$10^{6}\ldots10^{10}$	0.4	0.2

The last expression can be reduced to the form:

$$\varepsilon_c = A_4 \delta \sqrt[4]{\frac{\Delta\vartheta}{\delta}} , \tag{2.42}$$

where A_4 is the quantity including all physical parameters of the fluid expressed by the following formula:

$$A_4 = 0.18 \frac{(\beta \cdot g \cdot Pr)^{0,25}}{\nu^{0,5}} . \tag{2.43}$$

Thus, thermal resistances for cylindrical and plane fluid interlayer (annulus) are quantified by formulas (2.21) and (2.22), respectively. However, in this case the heat conductivity λ must be substituted by the equivalent heat conductivity λ_{eq}. Another example of internal natural convection is heat transfer in open channel formed by two vertical parallel surfaces as shown in Figure 2.2.

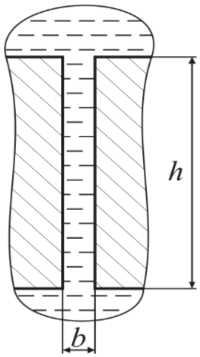


Figure 2.2

In this case convective htc is defined by the following equation [19]:

$$Nu = \frac{b}{24h}\left(Gr \cdot Pr\right)\cdot\left(1 - e^{24}\,\frac{0.5h}{b \cdot Gr \cdot Pr}\right)^{0.75}. \qquad (2.44)$$

In this equation, as the characteristic dimension is the gap value between the surfaces *b*, and the characteristic temperature is the one of the surfaces giving up the heat. The volumetric thermal expansion coefficient of the fluid β in equation (2.44) is accepted at the middle temperature of the fluid between considered surfaces. Equation (2.44) is valid only when the convective component in the range:

$$\left(Gr \cdot Pr\right)\cdot\frac{b}{h} = 2...10^{5}. \qquad (2.45)$$

If coolant media is the air, then substantial role in total content of the heat transfer may play component of radiant heat exchange. Equation taking into account the component of radiant heat exchange is [19]:

$$Nu_{\mathrm{m},\frac{b}{2}} = 0.5\left(Gr_{\mathrm{m},\frac{b}{2}} \cdot \frac{b}{2h}\right)^{0.294}. \qquad (2.46)$$

This equation is valid for the range:

$$Gr_{\mathrm{m}} \cdot \frac{b}{h} = 20...1.2 \cdot 10^{6}. \qquad (2.47)$$

2.6 Heat transfer by forced convection

As it was mentioned, forced convection occurs, when fluid motion along heated surface (convection currents) is produced by extraneous factors (either artificially with the help of pumps, fans etc., or naturally, for example, by wind). In this case convective heat transfer is also quantified with the help of scaling and dimensionless parameters. The basic equation for the forced convection represented in general form is:

$$Nu = f\left(Re \cdot Pr\right), \qquad (2.48)$$

At forced convection two basic regimes of the fluid motion usually occur: laminar and turbulent. Transition from laminar to turbulent motion happens at so-called *critical velocity*, which is indicated by the magnitude of Reynolds number. Considering the forced convection in electric and electronic power equipment, two basic cases can be identified, such as;

1) heat transfer at the fluid motion in tubes or channels;

2) heat transfer of solids passed over by the fluid in unbounded space.

In the case heat transfer of a *flat surface* passed over by the fluid flow the dimensionless equations are [6]:

– at laminar flow ($Re_{\mathrm{f}} < 4 \cdot 10^{-4}$)

$$Nu_{\mathrm{f}} = 0.66 Re_{\mathrm{f}}^{0.5} Pr_{\mathrm{f}}^{0.43} \left(\frac{Pr_{\mathrm{f}}}{Pr} \right)^{0.25}, \tag{2.49}$$

– at turbulent flow (if $Re_{\mathrm{f}} \geq 4 \cdot 10^{4}$)

$$Nu_{\mathrm{f}} = 0.037 Re_{\mathrm{f}}^{0.8} Pr_{\mathrm{f}}^{0.43} \left(\frac{Pr_{\mathrm{f}}}{Pr} \right)^{0.25}. \tag{2.50}$$

Index “f” in equations (2.49) and (2.50) and following equations points to respective quantities are to be accepted at the average fluid temperature; absence of the index points to respective quantities are to be accepted at the surface temperature.

In the case heat transfer of single cylinder transversally passed over by the fluid flow the equations will be [6]:

– at laminar flow ($Re_{\mathrm{f}} = 5 \ldots 10^{3}$)

$$Nu_{\mathrm{f}} = 0.5 Re_{\mathrm{f}}^{0.5} Pr_{\mathrm{f}}^{0.38} \left(\frac{Pr_{\mathrm{f}}}{Pr} \right)^{0.25} \tag{2.51}$$

– at turbulent flow ($Re_{\mathrm{f}} = 10^{3} \ldots 2 \cdot 10^{5}$)

$$Nu_{\mathrm{f}} = 0.25 Re_{\mathrm{f}}^{0.6} Pr_{\mathrm{f}}^{0.38} \left(\frac{Pr_{\mathrm{f}}}{Pr} \right)^{0.25} \tag{2.52}$$

In order to calculate convective htc for the fluid flowing into smooth tubes the following equations are used [6]:

– at laminar flow ($Re_{\mathrm{f}} \leq 2200$)

$$Nu_{\mathrm{f}} = 0.15 Re_{\mathrm{f}}^{0.33} Pr_{\mathrm{f}}^{0.43} \left(\frac{Pr_{\mathrm{f}}}{Pr} \right)^{0.25} \varepsilon_l , \qquad (2.53)$$

where the inner diameter of the tube *d* is used as the characteristic dimension; ε_l is the constant determined from table 2.6, in which *l*/*d* is relation of the length of the tube to its inner diameter.

Table 2.6 – Dependency constant ε_l vs ratio *l*/*d*

l/*d*	1	2	5	10	15	20	30	40	50
ε_l	1.90	1.7	1.44	1.28	1.18	1.13	1.05	1.02	1

– at turbulent flow of the fluid ($Re_{\mathrm{f}} \geq 10^4$)

$$Nu_{\mathrm{f}} = 0.021 Re_{\mathrm{f}}^{0.8} Pr_{\mathrm{f}}^{0.43} \left(\frac{Pr_{\mathrm{f}}}{Pr} \right)^{0.25} \varepsilon_l \varepsilon_R , \qquad (2.54)$$

where ε_R is the coefficient taking into account rounding of the tube expressed as follows:

$$\varepsilon_R = 1 + 1.8 d/R , \qquad (2.55)$$

where *R* is radius of the tube curvature.

The magnitude of ε_l for equation (2.54) is determined from table 2.7.

Table 2.7 – Dependency the Reynolds number vs ratio *l*/*d*

Re_{f}	*l*/*d*								
	1	2	5	10	15	20	30	40	50
10^4	1.65	1.50	1.34	1.23	1.17	1.13	1.07	1.03	1
$2\cdot10^4$	1.51	1.40	1.27	1.18	1.13	1.10	1.05	1.02	1
$5\cdot10^4$	1.34	1.27	1.18	1.13	1.10	1.08	1.04	1.02	1
10^5	1.28	1.22	1.15	1.10	1.08	1.06	1.03	1.02	1
10^6	1.14	1.11	1.08	1.05	1.04	1.03	1.02	1.01	1

In case of forced flow of coolant media in a gap between parallel surfaces (see Figure 2.2) at laminar boundary layer $\left(Re_h = 2 \cdot 10^2 \ldots 1.2 \cdot 10^5 \right)$ the following equation is used [6]:

$$Nu_h = 1.35 \left(\frac{h}{b \cdot Re_b} \right)^{0.1} Re_h^{0.5} Pr^{0.33} \left(\frac{Pr_{\mathrm{f}}}{Pr} \right)^{0.25} \qquad (2.56)$$

At turbulent flow ($Re_h \geq 1.2 \cdot 10^5$) equation (2.54) is to be used.

To quantify htc for other conditions and surface shapes the special handbooks are to be used [11, 13–16, 30, 31].

2.7 Heat transfer by radiation

Thermal (temperature) radiation is the transmission of thermal energy by means of electromagnetic waves. As well-known hot or cold, all objects radiate electromagnetic and hence thermal energy.

Electromagnetic radiation has common nature. Its basic parameter is the *wave length*, which defines qualitatively distinctive types of radiation: cosmic radiation – $0.05 \cdot 10^{-12}$ м; γ-radiation – $(0.5–1,0) \cdot 10^{-12}$ m; X-radiation – 10^{-12}–$20 \cdot 10^{-9}$ m; ultra-violet – $20 \cdot 10^{-9}$–$0.4 \cdot 10^{-6}$ m; visible – $(0.4–0.8) \cdot 10^{-6}$ m; infrared emission – $(0.8–200) \cdot 10^{-6}$ m; radio-waves – more than $200 \cdot 10^{-6}$ m.

Infrared rays are in most degree carriers of thermal energy. Since infrared emission has wide enough range of wave length, emissive energy, as differentiated from other ways of heat transfer, is featured not only quantitatively (by the power of radiation), but qualitatively (by the spectrum). Depending on the wave-length range the *monochromatic emission* (narrow spectral range) and *complicated emission* (wide spectral range) are distinguished.

A body situated in the zone of thermal radiation action receives thermal energy

$$Q = Q_A + Q_R + Q_D, \tag{2.57}$$

where Q_R is the energy reflected from the body; Q_A is the energy absorbed by the body; Q_D is the energy passed through the body.

If to divide both sides of the equation by Q, we shall obtain:

$$1 = \frac{Q_A}{Q} + \frac{Q_R}{Q} + \frac{Q_D}{Q} \text{ or } 1 = A + R + D, \tag{2.58}$$

where A is the absorption factor; R is the reflection factor; D is the transition factor.

All coefficients noted may assume the magnitude from zero to one. Here three extreme cases are distinguished:

1. $A = 1; R = 0; D = 0$, that is, energy received by the body is fully absorbed. Such a body is termed as the *blackbody*. When heated, the blackbody radiates the same amount of thermal energy as it absorbs. In the state of radiative equilibrium its temperature is maintained by constant

value. Emission of the blackbody depends only on its temperature, but not on its nature.

2. $A = 0; R = 1; D = 0$, that is, energy received by the body is fully reflected. When the reflection is diffusive, it is the *whitebody*; when the reflection is directive (complies with optics laws), it is the *specula body*.

3. $A = 0; R = 0; D = 1$, that is energy received by the body is fully passed through the one. Such body is *absolutely transparent* one.

A blackbody radiates at given temperature most amount of thermal energy, as differentiated from other bodies. The bodies having the properties of the blackbody are in nature few and far between. Real bodies do not correspond mostly to extreme states considered above, and combine the properties of mentioned types of ideal bodies. Certain of the bodies, such as, lamp black, corrugated asbestos sheets, black dull paint etc approach to a blackbody.

The behavior of thermal radiation is described by different relationships. The *Stefan-Boltzmann law* plays very important role in the theory of thermal radiation. It states relation between density of integral emission and temperature of the blackbody:

$$q_{\text{em}} = \varepsilon \sigma T^4, \tag{2.59}$$

where ε is the emissivity factor of the body; σ is the Stefan-Bolzmann constant; T is the temperature of emitting surface.

Consequently, the emitting energy density is proportional to the forth power of the emitting body temperature. In this case, density of emission is the amount of radiative energy that falls on the volume unit of a body.

Thermal energy emitted by a heated body is received by surrounding bodies, a part of which reflects from these bodies and, in turn, partially returns to the body considered. Hence, practically always one is forced to deal not with the emission of a single body, but with radiative heat exchange between bodies. To solve the problem about of radiative heat exchange between several bodies is a formidable task.

Let's consider the simplest, but very important task for practical engineering calculations about radiative heat exchange between two bodies, namely, between the body with temperature T_1 and enclosure, fully enveloping it, with temperature T_2, as shown in Figure 2.3.

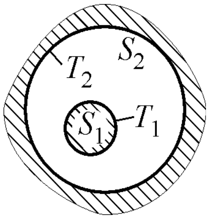


Figure 2.3

It is supposed that $T_1 > T_2$, then the resulting heat flow from the body to the enclosure under mutual radiation is represented as difference:

$$P_{12} = P_{ef1}\varphi_{12} - P_{ef2}\varphi_{21}, \qquad (2.60)$$

where $P_{эф1}$ and $P_{эф2}$ is effective emission of the body and enclosure, respectively, which constitutes the sum of self emission and reflected emission; φ_{12} and φ_{21} are the degrees of irradiancy indicating the part of heat flow emitted by one object is received by another one; in this case:

$$\varphi_{12} = 1; \quad \varphi_{21} = \frac{S_1}{S_2}, \qquad (2.61)$$

where S_1 and S_2 the area of the body and enclosure, respectively;

Using Stefan-Boltzmann law, the expression of resulting heat flux from the body surface will be as follows [7,8]:

$$p_{12} = 5.67 \cdot \varepsilon_{red}\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] S_1, \qquad (2.62)$$

where ε_{red} is so-called reduced emissivity factor determined by the expression:

$$\varepsilon_{п} = \frac{1}{\frac{1}{\varepsilon_1} + \left(\frac{1}{\varepsilon_2} - 1\right)\varphi_{21}}. \qquad (2.63)$$

ε_1 and ε_2 is the emissivity factor of the body and enclosure, respectively.

If expression (2.62) is to be divided by the difference in temperature of the body and enclosure, we shall have derived expression for thermal resistance of radiative heat transfer between them:

$$R_{т.и} = 5.67 \cdot \frac{\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4}{\left[\frac{1}{S_1\varepsilon_1} + \frac{1}{S_2}\left(\frac{1}{\varepsilon_2} - 1\right)\right](T_1 - T_2)}. \qquad (2.64)$$

In the case when a heated body is far enough from other bodies in surrounding with temperature T_0, radiative htc from its surface will be defined by the following expression [6]:

$$\alpha_{и} = 5.67 \cdot \varepsilon \frac{\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_0}{100}\right)^4}{T_1 - T_0}, \qquad (2.65)$$

where ε is the heated body emissivity factor.

It should be noted that both in the first and second cases combined heat exchange including the convective heat transfer apart from radiation can occur as well. If between the body and its enclosure the gaseous or liquid interlayer is available, then total thermal resistance between them will be defined by the following expression:

$$R_{т} = \frac{R_{т.и} \cdot R_{т.к}}{R_{т.и} + R_{т.к}}, \qquad (2.66)$$

where $R_{т.к}$ is the convective component of thermal resistance for the fluid interlayer between the body and enclosure enveloping it (see subsection 2.5).

For the second case, that is, when considered body is far from other bodies in fluid surrounding, then total htc is defined by the sum of the respective heat transfer coefficients:

$$\alpha_{т} = \alpha_{к} + \alpha_{и}, \qquad (2.67)$$

where $\alpha_{к}$ convective htc from the body surface.

Other cases of heat transfer by radiation are represented in special handbooks dealt with heat exchange [11, 14–16, 30, 31].

2.8 Heat exchange with phase change of coolant media

Heat exchange with the use of the coolant media phase change is usually named evaporative cooling. Such cooling technology is featured by considerably higher values of heat transfer coefficients in comparison with free and even forced convection. Cooling the elements of power electric and electronic equipment, two kinds of phase change is used: boiling and condensation.

Boiling heat transfer. Boiling is the process of vapor formation, when new free interfaces between liquid and vapor phases appear within liquid. The temperature of vapor formation named *saturation temperature* $\vartheta_{н}$ is defined by the pressure under which boiling the liquid is kept. In the immediately adjacent to the heated surface layers the liquid temperature is accepted as equal to the surface temperature ϑ while in the far points – to the saturation temperature $\vartheta_н$.

The mechanism of heat exchange with boiling a liquid is very complex and has not adequately studied now. Two basic regimes of boiling are distinguished: nucleate (bubble) and film. When nucleate boiling occurs, vapor forms on the heated surface as separate periodically appearing bubbles, which rapidly increase in dimensions and go out through the liquid thickness to the vapor space above it. When film boiling occurs, continuous vapor film forms on the heated surface periodically force into liquid space. It should be noted that intensity of heat exchange with film boiling is ten times less than with bubble one**.** The reason is that heat conductivity of vapor is considerably less than the liquid one. The existence of this or that regime of boiling is defined by the value of the heat flux *q* or temperature drop $\tau = \vartheta - \vartheta_н$, physical properties of the liquid, material and condition of the heated surface and hydrodynamic of the flow as whole.

Let’s consider the process of boiling by the classic example of heat transfer from the surface of horizontally oriented wire immersed into a vessel with saturated liquid. Figure 2.4 illustrates typical curve representing htc from the wire surface as function of the constant pressure temperature drop.

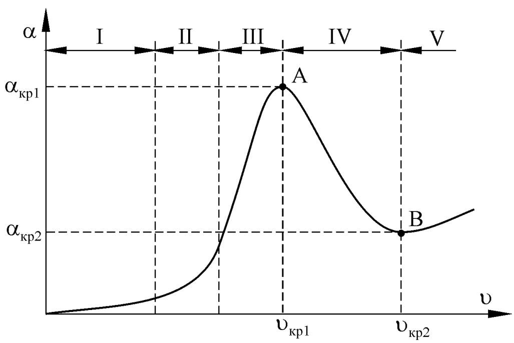


Figure 2.4 – Dependency the htc vs temperature drop on boiling

Boiling does not occur in the low temperature drops (region *I*). Thermal energy from heated surface is driven out through free convection of the liquid that occurs with evaporation from free interface liquid-vapor. Within given region the laws for free convection of non-boiling liquids are valid.

As the temperature drop increases (region *II*) vapor bubble nucleation occurs on the heated surface, however their number is small and they appear irregularly. Given region is usually named as the one of developed nucleate boiling, where heat transfer is resulted from free convection and nucleate boiling.

Further increase in temperature drop (region *III*) leads to increase in number of the regularly appearing bubbles on the heated surface. Distinctive feature for this region is drastic growth of heat transfer intensity with increase in temperature drop. In this region, the regime of developed nucleate boiling takes place.

In the *IV* region increase in temperature drop leads to decrease in intensity of heat transfer. This region is usually qualified as nucleate boiling transition to film one, wherein bubbles and unstable film boiling simultaneously exist. On the heated surface and close to it bubbles coalesce between them forming great vapor cavities. Region *IV* is very unstable and has not much interest for technical applications.

Region *V* is corresponded by stable regime of film boiling. Distinctive feature for this regime is existence of vapor film over all surface heated. With increase in temperature drop, htc increase due to thermal radiation through the vapor film.

The boiling curve (see Figure 2.4) has two inflection points corresponding to qualitative change of heat transfer mechanism. These phenomena are named heat transfer crisis on boiling. The heat flux $q_{кр1}$ (point *A*, Figure 2.4) that is maximally possible under given conditions on nucleate boiling is named first critical heat flux. The heat flux $q_{кр2}$, that minimally possible under given conditions on nucleate boiling is named second critical heat flux (point *B*). The values of temperature drops and heat transfer coefficients corresponding to crisis of heat transfer on boiling are as well commonly named critical ones.

It should be pointed out that htc on boiling is depended on a great number of various parameters. The most important among them are the following: thermo-physical properties of the liquid, thermo-chemical properties of the system liquid–heated surface, roughness of the heated surface, pressure and temperature of saturation, temperature drop or heat flux. Establishing analytic relation between htc and just enumerated factors is very complex task. It is caused by both much number of the factors, influence of which upon htc is not clear, and statistic nature of boiling.

Now quantitative relation between htc and influencing factors is experimentally deduced. To extent derived findings to other conditions, they are generalized by various methods. In so doing, either theory of thermal or thermodynamic similitude and law of corresponding states are most frequently used.

For water boiling at atmosphere and reduced pressures

(6.4–100.0 kPa), experimental findings are generalized by Rohsenow's equation [10]:

$$\frac{\alpha}{\lambda_ж}\sqrt{\frac{\sigma}{g(\rho_ж-\rho_п)}}=\frac{1}{B}\left[\frac{q}{\mu_ж r}\sqrt{\frac{\sigma}{g(\rho_ж-\rho_п)}}\right]^{2/3}, \qquad (2.68)$$

where $\rho_ж$, $\rho_п$ are the density of liquid and vapor, respectively, at saturation temperature; σ is surface tension; $\lambda_ж$, $\mu_ж$ are heat conductivity and cinematic viscosity of the liquid, respectively; r is the heat of vaporization; B is numerical coefficient depending upon properties of the system liquid–heated surface as well as roughness of the surface.

When boiling occurs on copper surface B = 0.0091–0.0142, depending on the surface roughness [10].

In the case developed nucleate boiling water (region *III*) relation between htc and heat flux is most frequently represented as following power law dependence:

$$\alpha=0.43p^{0.18}q^{2/3}\left(1-4.5\cdot10^{-8}p\right),\quad 10^5<p\le2\cdot10^7, \qquad (2.69)$$

where p is pressure of saturated water vapors.

When boiling occurs on horizontal tubes and plates under conditions of free movement in bulk the first critical heat flux can be determined with the help of Kutateladze equation [10]:

$$q_{кр1}=Kr\frac{\alpha}{\lambda_ж}\sqrt{\rho_п}\sqrt[4]{\sigma g(\rho_ж-\rho_п)}, \qquad (2.70)$$

where K is hydrodynamic stability criterion.

This equation is obtained basing upon hydrodynamic theory of boiling crisis derived from analysis of stability conditions of two-phase boiling layer on the surface.

With theoretical analysis it is established that $K=0.12\div0.157$ [10]. It depends on pressure, dimensions of the heated surface, its roughness and orientation in space. For water boiling at reduced pressures, the magnitude K grows with increase in pressure. Increase in roughness of the heated surface leads to increase in stability criterion.

Heat exchange on condensation. Condenser is one of main components of evaporative cooling system. In it the heat is transferred from vapor (intermediate heat transfer agent) to internal cooling medium (air, water). The heat transfer from vapor to internal surface of the condenser occurs with condensation.

When vapor comes into contact with solid surface at the temperature less that saturation one, its condensation occurs. Condensation occurs in two forms: dropwise, when the condensate deposits on a surface by separate drops, and film-type, when deposition occurs as persistent film. The form of condensation depends on the angle of wetting. Experience shows that condensation of the water and liquid dielectric vapors on a pure surface occurs with film-type one [10, 11, 13–16, 30, 31].

When film-type condensation takes place, under action of self-gravity force the condensate flows down. All heat released during vapor condensation is transferred to the wall via the film. In the process laminar and turbulent regimes of the film flow are possible. For example, in the case of vertical surface, beginning at the top the film has laminar flow, but as condensate flows down the film increase in thickness and the flow transits into turbulent one. The laminar flow occurs with heat transfer through conduction, however when turbulent flow takes place, heat is mostly transferred by liquid particles.

To calculate heat transfer by condensation of dry saturated stationary vapor, theoretical dependences derived by Nusselt are used with following assumptions simplified actual physical pattern:

1) the film flow is laminar;

2) inertia forces produced by the film are negligibly small compared with forces due to viscosity and gravity;

3) convective heat transfer within the film and heat conduction along the film are negligibly small in comparison with heat conduction across the film;

4) friction and drastic temperature change at the interface between vapor and liquid phases are absent;

5) the temperature on outer surface of the film equal to the vapor temperature;

6) thermophysical characteristics of the condensate are not dependent upon temperature;

7) the vapor density is small compared with the liquid density.

Average htc in the case of condensation on the surface of vertical wall or tube with height *l* [10] is

$$\bar{\alpha} = 0.943 \cdot \sqrt[4]{\frac{rg\rho_{ж}^2 \lambda_{ж}^3}{\mu_{ж} l \left(\vartheta_{н} - \vartheta\right)}}; \qquad (2.71)$$

on the surface of horizontal tube with diameter *d* [10] will be

$$\bar{\alpha} = 0.728 \cdot \sqrt[4]{\frac{rg\rho_{ж}^{2}\lambda_{ж}^{3}}{\mu_{ж} d\left(\vartheta_{н} - \vartheta\right)}}, \tag{2.72}$$

Motion of the condensate film can be wave-type on retention of laminar flow regime that is involved with surface tension forces between the film surface and vapor as well as accident disturbances on the film surface. When condensation on horizontal tubes with moderate diameter wave-type flow has no time to develop and experimental findings confirm that formula (2.72) is valid.

2.9 Withdrawing heat by means of extended surfaces

As it was pointed out above a number of components of power electrical and electronic equipment are concentrative thermal sources. Ones of that are power semiconductor devices (PSCD). It is to be pointed out here that there are strong enough temperature ranges for its normal operation. In such cases the complex enough task to withdraw great amount of thermal energy from the bodies with relatively small dimensions and respectively small heat transfer surface is raised.

Analysis of Newton's law of cooling (2.26) shows that value of the heat flow related to average temperature-rise between the heated body and its surroundings is proportional to the product of the surface area and htc αS. Consequently, in order to result in high values of the heat flow at the same temperature-rise it is necessary either to increase htc or to extend heat transferring surface.

Increase in htc can be brought about following ways:

1. Application of effective coolant media. In so doing it is most frequently implied application of liquid cooling instead of air one. It may be pointed out here that liquid cooling requires installing additional devices (pump, pipe-lines, valves, surge tank etc.) that results frequently in considerable growth of mass-dimension indexes and constructional complication of the equipment

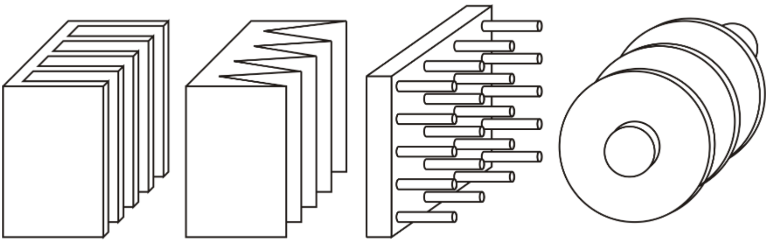
2. Using increased velocities of coolant media circulation. In this case also mass-dimensional indexes growth as well as the power consumed to transfer the coolant media in the system increases. For example, to twice htc it is required to increase velocity of coolant media circulation more than two times. At the same

time to twice circulation velocity, it is required to increase the head four times and power consumed-8 times.

3. Turbulization of coolant media flow, when contributory means for destruction of bound layer is used. It can be achieved by different techniques. However all, as a rule, necessarily result in significant increase of the head and, accordingly, power consumed to force the coolant media.

4. Application of transpiration cooling, when thermal energy is driven out by the change of the coolant media aggregative state, allows substantially increase htc in comparison with liquid (including force) cooling. Nevertheless, efficiency due to application of transpiration cooling for power electrical and electronic equipment has not been yet established [10].

Hence, to increase product αS by only increase in htc is not always advisable. To reach this objective the effective enough method is to increase (to extend) heat exchange surface by means of fining, that is making fins or spines on it. Figure 2.5 shows basic variations of heat sinks with extended surfaces using for cooling PSCD.



a) *b*) *c*) *d*)

Figure2.5 – Finned sinks for power semiconductor devices: *a*) straight fins of uniform cross section; *b*) straight fins of non-uniform cross section; *c*) pin fins of uniform cross section; *d*) annular fins

To determine heat removing capability of fined surfaces let us consider the task of heat transfer through single uniform strip (fin or spine). The assumptions accepted here are as follows:

1. The ambient temperature is constant and lower to any point of the strip.
2. Heat transfer coefficient from the strip surface is invariable.
3. The temperature field over cross-section of the strip is homogeneous.

The strip may have generally variable cross-sectional area. Let us consider at first simplest case the strip with invariable cross-sectional area shown in Figure 2.6.

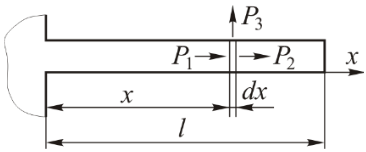


Figure 2.6 – Straight strip

It is required to find the character of temperature distribution along the strip and its heat flow driven out.

For this purpose let us consider thermal balance of element *dx* at distance *x* from the strip root (see Figure 2.6). It will be as follows:

$$P_1 + P_2 = P_3 . \qquad (2.73)$$

where P_1 is the equalizing heat flow coming into the element *dx* (it is here dealt with heat conductance); P_2 is the equalizing heat flow outgoing from the element *dx* (it is conductance as well); P_3 is the heat flow dissipated by the side surface of the strip.

The terms of this equation with considering the fact that heat transfer in axial direction occurs through conduction are expressed as:

$$P_1 = -\lambda q \frac{d\tau}{dx}\,;\ P_2 = -\lambda q \frac{d}{dx}(\tau - d\tau) = -\lambda q \frac{d\tau}{dx} + \lambda q \frac{d^2\tau}{dx^2} dx\,;\ P_3 = \alpha_T p dx\,. \qquad (2.73a)$$

Substituting expressions (2.73*a*) into heat balance equation (2.73) and reducing it to normal form we will have derived differential equation that describes temperature-rise distribution along the strip:

$$\frac{d^2\tau}{dx^2} + b^2\tau = 0\,, \qquad (2.74)$$

where *b* is the coefficient including the strip parameters determined as follows:

$$b = \sqrt{\frac{p\alpha_T}{\lambda q}}\,, \qquad (2.75)$$

where α_T is total htc from the strip surface; *p*, *q* is the perimeter and cross-sectional area of the strip, respectively; λ is the material heat conductivity of the strip.

When heat transfer from the strip tip surface is taken into consideration, then boundary conditions for solution of equation (2.74) will be as follows:

$$\tau(0) = \tau_0; \quad \left.\frac{d\tau}{dx}\right|_{x=l} = -\frac{\alpha_T}{\lambda}\tau(l)\,, \qquad (2.76)$$

where *l* is the length of the strip.

It should be noted that in most cases heat transfer from the strip tip surface can be neglected. In this case solution of equation (2.74) is performed at the following boundary conditions:

$$\tau(0) = \tau_0; \quad \left.\frac{d\tau}{dx}\right|_{x=l} = 0\,. \tag{2.77}$$

Using these boundary conditions gives expression for temperature distribution along the strip

$$\tau = \frac{\tau_0 \operatorname{ch} b(l - x)}{\operatorname{ch} bl}\,. \tag{2.78}$$

Thermal flow through the strip root

$$P_0 = -\lambda q \left.\frac{d\tau}{dx}\right|_{x=0} = \tau_0 \lambda q b \operatorname{th} bl\,. \tag{2.79}$$

Thermal resistance of the strip

$$R_{th} = \frac{\tau_0}{P_0} = \frac{1}{\lambda q b \operatorname{th} bl}\,. \tag{2.80}$$

Basing upon the expressions derived let us consider a few specific cases. To drive out thermal energy from PSCD finned surfaces with straight fins of square profile (side cross section) are extensively used (see Figure. 2.5*a*).

Expression for thermal resistance of straight square profile fin of wide h that in far excess of its thickness δ, that is $h >> \delta$, will be as follows:

$$R_{th\,f} = \frac{1}{h\sqrt{2\delta\lambda\alpha_т} \operatorname{th}\left(l\sqrt{\dfrac{2\alpha_т}{\lambda\delta}}\right)}\,. \tag{2.81}$$

Expression for thermal resistance of straight circular cross-section strip (spine) (see Figure 2.5*c*) of diameter d will be as follows:

$$R_{th\,p} = \frac{1}{\pi d^{1,5}\sqrt{\lambda\alpha_т} \operatorname{th}\left(2l\sqrt{\dfrac{\alpha_т}{\lambda d}}\right)}\,. \tag{2.82}$$

To estimate performance of extended (finned) surface cooling devices, the notion of fin (spine) efficiency is used in engineering practice. It is usually determined as relation of thermal energy dissipated by the fin (spine) P_0 to its ideal value P_{id} defined as the energy dissipated by the fin (spine) at even distribution of temperature-rise τ_0 over its surface

$$\eta = \frac{P_0}{P_{id}} . \tag{2.83}$$

According to definition the value of ideal thermal energy is defined by the following expression:

$$P_{id} = \alpha_{\text{т}} p l \tau_0 . \tag{2.84}$$

Thus, using expressions (2.79) and (2.84) efficiency of the fin or spine can be represented as follows

$$\eta = \frac{\operatorname{th} bl}{bl} . \tag{2.85}$$

Efficiency of the fin (spine) shows what fraction of the extended surface operates at the temperature drop corresponding to the fin (spine) root τ_0.

For straight fin of rectangular profile the optimal values of its height l and thickness δ can be found provided that predetermined heat exchange surface of the fin dissipates maximal amount of thermal energy or has minimal value of thermal resistance. For this purpose expression for thermal flow through the fin root (2.79) is represented as follows:

$$P_0 = \sqrt{2}\tau_0 \sqrt{\alpha_{\text{т}} \lambda \delta} \cdot \operatorname{th}\left(l \sqrt{\frac{2\alpha_{\text{т}}}{\lambda \delta}} \right). \tag{2.86}$$

To find optimal values expression (2.86) is to be differentiated in respect to δ and is taken equal to zero. Then, expressions defined optimal values will be as follows [13]:

$$\delta_{\text{opt}} = 0.791 \left(\frac{2\alpha_{\text{т}} q_l^2}{\lambda} \right)^{1/3} ; \quad l_{\text{opt}} = \frac{q_l}{\delta_{\text{opt}}} = 1.262 \left(\frac{\lambda q_l}{2\alpha_{\text{т}}} \right)^{1/3} , \tag{2.87}$$

where q_l is the fin profile area, which, in finding the optimal fin parameters, is predetermined by the constant value, that is $q_l = const$.

Let us consider now more complex cases of analysis: fins with variable cross-section. One such case is the fin of triangle profile extensively enough applied in heat-exchange facilities for PSCD (see Figure 2.5*b*).

To find the character of temperature distribution along the fin and the heat flow dissipated by it let us outline element *dx* at distance *x* from the fin tip as shown in Figure 2.7. The fin cross-sectional area perpendicular to heat flux direction will be in this case expressed as follows:

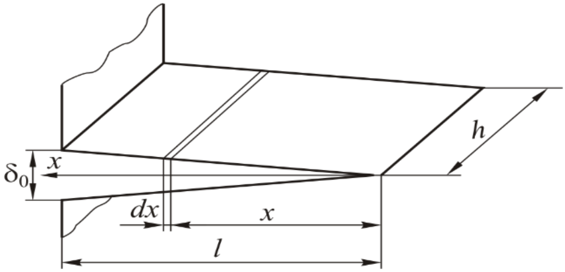


Figure 2.7 – Straight fin of triangle profile

$$q_x = \frac{\delta_0 x}{l} . \qquad (2.88)$$

Cooling surface area of the element *dx* will be defined by the following expression:

$$dF_x = 2hdx . \qquad (2.89)$$

Heat balance equation for the element *dx* in this case will be the same as equation (2.73). However, the expressions for its terms will be somewhat different. Taking expressions (2.88) and (2.89) into consideration, they will be as follows:

$$P_1 = -\lambda\delta_0 h \frac{x + dx}{l} \frac{d}{dx}(\tau + d\tau); \quad P_2 = -\lambda\delta_0 h \frac{x}{l} \frac{d\tau}{dx}; \quad P_3 = 2h\alpha_T \tau dx . \qquad (2.90)$$

Substituting expressions (2.90) in heat balance equation (2.73) and reducing it to normal form, differential equation describing temperature-rise distribution along the fin is obtained. It is as follows:

$$x\frac{d^2\tau}{dx^2} + \frac{d\tau}{dx} - b^2 l\tau = 0 , \qquad (2.91)$$

where *b* is the coefficient including the fin parameters defined in accordance with following expression:

$$b = \sqrt{\frac{2\alpha_T}{\lambda\delta_0}} , \qquad (2.92)$$

Solving equation (2.91) is performed at the following boundary conditions:

$$\tau(l) = \tau_0; \quad \left.\frac{d\tau}{dx}\right|_{x=0} = 0. \tag{2.93}$$

Hence, the expression defined temperature distribution along the fin will be as follows:

$$\tau = \tau_0 \frac{I_0\left(2b\sqrt{lx}\right)}{I_0(2bl)}, \tag{2.94}$$

where I_0 is modified Bessel function of first kind.

Тепловой поток через основание стержня определяется как

$$P_0 = -\lambda\delta_0 h \left.\frac{d\tau}{dx}\right|_{x=l}. \tag{2.95}$$

Differentiating equation (2.95) can be performed by expansion of Bessel function into a series that gives the following equation:

$$\frac{d}{dx} I_0\left(2b\sqrt{lx}\right) = b\sqrt{\frac{l}{x}} I_1\left(2b\sqrt{lx}\right), \tag{2.96}$$

where I_1 is modified Bessel function of first kind of first order.

Then thermal flow through the fin root will be expressed as

$$P_0 = \frac{2\tau_0\alpha_т h I_1(2bl)}{b I_0(2bl)}, \tag{2.97}$$

and thermal resistance of the fin will be

$$R_{th} = \frac{\tau_0}{P_0} = \frac{b I_0(2bl)}{2\alpha_т h I_1(2bl)}. \tag{2.98}$$

The ideal thermal flow for straight fin of triangle profile is expressed as

$$P_{id} = 2\tau_0\alpha_т lh. \tag{2.99}$$

Consequently, efficiency of straight fin of triangle profile will be defined by the following expression:

$$\eta = \frac{P_0}{P_{id}} = \frac{I_1(2bl)}{bl I_0(2bl)}. \tag{2.100}$$

The optimal dimensions of straight triangle profile fin, as well as for square one, are found for predetermined its profile area, which in this case is defined by the following expression:

$$q_l = \frac{l\delta_0}{2} = const \qquad (2.101)$$

To derive optimal values in this case expression (2.97) is to be differentiated in respect to δ_0 and taken equal to zero. The expression defined optimal values of straight triangle profile fin will be as follows [13]:

$$\delta_{0opt} = 1{,}328\left(\frac{2\alpha_T q_l^2}{\lambda}\right)^{1/3}; \quad l_{opt} = \frac{q_l}{\delta_{opt}} = 1{,}506\left(\frac{\lambda q_l}{2\alpha_T}\right)^{1/3}. \qquad (2.102)$$

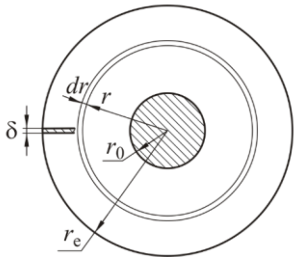


Figure 2.8 – Circular fin of rectangular profile

Let us consider yet another variant of the finned surface, wherein radial type fins of rectangular profile fastened on cylindrical rod are used (see Figure 2.5*d*). In this case the fin is a ring of invariable thickness δ with internal radius r_0 corresponding to radius of cylindrical surface, wherein it is fastened, and external radius r_e as shown in Figure 2.8.

The ring center or the axis of the element that fasts the fin is accepted as the origin of the coordinate. Heat balance equation for the elemental ring *dr*, as well as for another fins, will correspond to equation (2.73). Its terms will be expressed as follows

$$P_1 = -2\pi\lambda\delta r\frac{d\tau}{dr}; \; P_2 = -2\pi\lambda\delta(r + dr)\frac{d}{dr}(\tau - d\tau) \; P_3 = 4\pi\alpha_T\tau r dr. \qquad (2.101)$$

Substituting expressions (2.103) into heat balance equation (2.73) and reducing it to normal form, we will have obtained differential equation describing temperature distribution in respect to coordinate *r*, which is in this case Bessel's equation:

$$r^2\frac{d^2\tau}{dr^2} + r\frac{d\tau}{dr} - b^2 r^2 = 0, \qquad (2.104)$$

where *b* is the coefficient including the fin parameters defined according to the following expression:

$$b = \sqrt{\frac{2\alpha_T}{\lambda\delta}}, \qquad (2.105)$$

If heat transfer from external cylindrical surface of the fin is neglected, then boundary conditions for solving equation (2.104) will be as follows:

$$\tau(r_0) = \tau_0; \quad \left.\frac{d\tau}{dx}\right|_{x=r_e} = 0. \tag{2.106}$$

Its solution defining temperature distribution in respect to coordinate r will be as follows:

$$\tau = \tau_0 \frac{K_1(br_e)I_0(br) + I_1(br_e)K_0(br)}{K_1(br_e)I_0(br_0) + I_1(br_e)K_0(br_0)}, \tag{2.107}$$

where K_1 is modified Bessel function of second kind of first order.

Heat flow through the fin root is

$$P_0 = -2\pi\lambda r_0\delta \left.\frac{d\tau}{dr}\right|_{r=r_0}. \tag{2.108}$$

Differentiating expression (2.107) and substitution $r = r_0$ gives:

$$P_0 = 2\pi\lambda r_0\delta b\tau_0 \frac{I_1(br_e)K_1(br_0) + K_1(br_e)I_1(br_0)}{K_1(br_e)I_0(br_0) + I_1(br_e)K_0(br_0)}. \tag{2.109}$$

Ideal heat flow for ring type fin of rectangular profile will be expressed as

$$P_{id} = 2\pi\alpha_{\text{T}}\tau_0\left(r_e^2 - r_0^2\right). \tag{2.110}$$

Thus, the efficiency of ring type rectangular profile fin will be defined by the following expression:

$$\eta = \frac{P_0}{P_{id}} = \frac{\lambda r_0\delta b}{\alpha_{\text{T}}\left(r_e^2 - r_0^2\right)} \cdot \frac{I_1(br_e)K_1(br_0) + K_1(br_e)I_1(br_0)}{K_1(br_e)I_0(br_0) + I_1(br_e)K_0(br_0)}. \tag{2.111}$$

The analysis performed enables to compare different fin types and to choose most optimal variant for given specific case. For example let us consider comparative analysis of straight fins of square and triangle profiles. Let us define what of that has less area of the profile at equal amount of dissipated thermal energy. For this purpose the expressions of thermal flows through the fin root are to be expressed through the fin profile area at optimal dimensions of the fins.

For square fin the expression for thermal flow through its root will be as follows [13]:

$$P_0 = 1.26\tau_0\left(\alpha_{\text{T}}\lambda q_p\right)^{1/3}. \tag{2.112}$$

From this expression, the square of the fin area is determined

$$q_p = \frac{0.5}{\alpha_T^2 \lambda}\left(\frac{P_0}{\tau_0}\right)^3 \qquad (2.113)$$

For triangle fin, the expression defining its profile area at the predetermined value of thermal flow P_0 will be as follows [13]:

$$q_p = \frac{0.347}{\alpha_T^2 \lambda}\left(\frac{P_0}{\tau_0}\right)^3. \qquad (2.114)$$

The analysis of expressions (2.113) and (2.114) shows that the fin profile area at the predetermined heat flow varies as the third power of the relationship between thermal flow and temperature head taken in the fin root. It is also seen that required profile area is inversely proportional to the fin material heat conductivity and htc squared.

In considering equations (2.113) and (2.114) three important deductions can be drawn. The first one implies that for the same materials at identical heat transfer conditions and the equal relationship between heat flow and temperature head triangle fin requires approximately 69% of the material in comparison with square one.

Second deduction refers to selection of the material for any fin type. It is seen from the expressions that the fin mass, proportional to the profile area q_p, will be direct with the material density and inversely proportional to its heat conductivity.

And, finally, the profile area q_p and consequently the fin volume increase by third power of thermal flow. Hence, to increase thermal flow by two times, it may be used either two fins or one fin with more profile area by 8 times. It is quit evidence that application of more number of the fins will be significantly more effective. However, there is the limit fins number that can be arranged on the given surface [13, 30, 31].

3. THERMAL BEHAVIOR ANALYSIS METHODS

3.1 Theoretic analysis methods of thermal behavior

Any unit of power electric or electronic equipment can be considered as a system of the elements including lumped and distributed sources of thermal energy. And, its temperature field is dependent upon many factors, particularly: thermal energy source capacity and distribution; construction and overall dimensions of the equipment; type of cooling system; physical properties of the material used to make the equipment components; service conditions and others. Dependence of the equipment temperature field from the mentioned factors features its thermal behavior.

Thermal behavior analysis of such systems is very complex task, which is frequently performed with approximate methods. In the process, analysis of various systems enables to formulate general regularities in behavior theirs temperature fields. Application of approximate (schematizing) notions on behavior of the process to particular objects enables in some cases to significantly simplify the task to analyze thermal behavior of power equipment.

System thermal behavior investigation including energy sources usually lies in establishing quantitative relationship between the temperature of limited number of the most responsible equipment points and essential factors that influence the heat exchange process. Then, it is primarily required to establish relation between stationary temperature field of the system and capacities of separate energy sources; as well as it is needed to define influence of shape and dimensions of the regions occupied by discontinuous energy sources on behavior of the system temperature field.

Let's consider several general heat exchange regularities of the complex systems with energy sources that enable to establish the mentioned dependences. One of the most important ones among them is the *temperature field superposition principle*. Its mathematical expression relatively to stationary temperature of any *j*-th point of the system is the following equation [8]:

$$\vartheta_j = \vartheta_0 + \sum_{i=1}^{n} P_i F_{ij} \tag{3.1}$$

where ϑ_0 is ambient temperature; P_i is capacity of *i*-th part of the system; *n* is the number of typical regions system consists of; F_{ij} are thermal coefficients that are independent either upon ambient temperature or source capacities; F_{ij} are thermal coefficients independent upon ambient temperature and sources capacities.

It is important to note that equation (3.1) is valid only in the case when energy source capacities, heat conductivities of separate parts of the system and its heat transfer coefficients are independent upon the temperature. The given equation is also still valid under such conditions with energy absorbers available in the system. Its capacities in equation (3.1) are represented by negative values.

Thus, investigation of stationary temperature field of the systems with energy sources comes down to determine thermal coefficients F_{ij}, which can be found experimentally, theoretically or with the help of analogue methods.

Experimental method of finding thermal coefficient F_{ij} consists in the following. In certain point *j* (point of observation) of the system the temperature probe (e.g. thermocouple junction) is placed, however, capacities of all energy sources, except *i*-th, must be equal zero. By varying capacity P_i, dependence $\vartheta_j - \vartheta_0 = f(P_i)$ is found from experiment. On the other hand, from equation (3.1) it is follows:

$$\vartheta_j - \vartheta_0 = P_i F_{ij} \tag{3.2}$$

Thus, having values P_i, ϑ_j и ϑ_0, from equation (3.2) coefficients F_{ij} are found. If these coefficients are not dependent on temperatures ϑ_j, ϑ_0 (or finally upon capacity P_i), then dependence (3.2) is linear, and one measurement at certain value of capacity P_i will be sufficient for their determination.

Analytical method of finding thermal coefficients comes down to solution of set of heat conduction equations that have been formulated for particular case considered.

It should be noted that during the analyzing thermal behavior of power equipment, some conditions, upon which temperature field superposition principle (3.1) is stated can be trespassed. At most heat transfer coefficient dependence on temperature cannot be neglected. In such cases calculating the temperature field, dependence (3.1) can be used in conjunction with the method of successive approximations. The simplest and most convenient calculation sequence is as follows.

At first initial approximation of dependence of temperature of j-th element surface part ϑ_{jS} on total capacity of energy sources $P = \sum_{i=1}^{n} P_i$ is predetermined as follows

$$\vartheta_{js}^{0} = \vartheta_0 + PF_{js}^{0} \qquad (3.3)$$

It is supposed that this dependence is found at very rough approximations and allows judge about the temperature of certain section of the system surfaces, which are involved in heat exchange with fluid surroundings, only tentatively. The upper index 0 in equation (3.3) emphasizes that it yields result only as initial approximation.

The approximate values of surface temperatures for the predetermined capacity are computed according to the equation (3.3). Considering the surface temperatures, heat transfer coefficients in first approximation are found. They are used to determine first approximation of thermal coefficients F_{js}^{I}. Then according to equation (3.1) the temperatures at certain points and sections of the system $\vartheta_{js}^{\mathrm{I}}$ are evaluated.

Using new values $\vartheta_{js}^{\mathrm{I}}$ heat transfer coefficients are computed again and second approximation of thermal coefficients F_{js}^{II} and temperatures $\vartheta_{j}^{\mathrm{II}}$ and $\vartheta_{js}^{\mathrm{II}}$ are determined. The calculations are repeated until subsequent and preceding approximations result in almost identical.

Since temperature ϑ_j is inversely related to heat transfer coefficients, calculation outcomes converge, and sequential approximation method makes possible to obtain numerical solution of the task with the help of superposition principle.

Another not less important regularity of heat exchange of complex systems with energy sources is so-called *principle of locality* or *non-locality*. Investigating temperature fields of the body system, heat exchange conditions are frequently to be considered at its boundaries. The more complex system is and the more external and internal boundaries it includes, the more difficult study of the temperature field become. For example, switchgear panel or thyristor-based electro-drive panel consists usually of a few compartments with partitions between them, which are the boundaries separating characteristic parts of the system. In its turn, many apparatuses, power semiconductor devices, current-carrying components of various shapes are arranged in the compartments that can be conventionally considered as bounded regions occupied by energy sources. Character and value of the temperature field may be affected by configuration and dimensions of these regions. Consideration of heat exchange conditions at all system boundaries can make the task so complex and time consuming that investigation of the system temperature field can be found as inexpedient.

However, in calculation practice there is not always necessity to know temperature at all points of the system. Sometimes it is enough to find temperature field in most characteristic regions. In the case analyzing temperature field, it is suitable to use locality principle (property of heat flux stability), which can be formulated as follows: any vicinal disturbance of temperature fielding is local and does not propagate on distant sections of the field [7, 8]. However, disturbances of the temperature field may be caused by either geometrical or physical reasons. For example, there can be a group of elements of various configurations dissipating equal capacity. In this case those elements cause the same temperature at certain distance from them. Another example, a group of various configuration elements dissipating certain capacity is arranged in the compartment. This group of the elements causes the same temperature-rise at some distance from energy sources parts of the compartment as evenly distributed over this compartment energy source of the same capacity will. Meanwhile, in the vicinity of the elements both in first and second examples the temperature field may be strongly dependent upon dimensions and configurations of the elements.

In certain cases disturbance of the temperature field can caused by inhomogeneity of the materials included by the system: compartment containing multitude of elements of various materials whose heat conductivities can be several orders different. At the boundary of different materials the temperature gradient can drastically change, but at a sufficient distance from the inhomogeneities, the latter have practically no influence on the character of the temperature field.

However, it should be pointed out that when analyzing a temperature field, there is need to know, at what distance from the considered region its configuration practically does not influence the temperature field behavior in the system. Available research findings testify that those distances from a region center have the same order as its biggest [8]. Such property, named the property of heat flux stability, makes possible to calculate temperature fields of very complex systems with energy sources.

3.2 Electro-thermal analogy

The principle of the analogy. As thermal behaviors of power electric and electronic equipment are investigated, it is necessary to determine the temperature field at various values and distribution of energy sources and absorbers and varying in space or time the temperature of surrounding apparatus medium (temperature of surrounding environment). Thermal sources (absorbers) or external temperature may be considered as disturbance generating in the system or its boundaries. Relationships between disturbance and response of the system can be found mathematically using analytical methods of heat conductance theory in some cases. It was pointed out above that most possible solutions are performed only for simple shaped bodies or for the systems of bodies and are represented so sophistically that it is sometimes difficult not only to find numerical result, but to give physical interpretation of the phenomenon. The solution may be found either using approximate analysis methods (e.g. temperature field regularization theory, principle of locality etc), or involving analogous methods of investigation.

In the investigation of stationary and non-stationary temperature fields the analogy between heat conductance and electric conductance is common. The analogy is established basing upon identity of differential equations of heat conductance and electric conductance

$$\frac{\partial \vartheta}{\partial t} = a\nabla^2\vartheta\,; \qquad \frac{\partial \varphi}{\partial t} = \frac{1}{\rho c}\nabla^2\varphi\,. \tag{3.4}$$

where ϑ is the temperature at point (x, y, z) at time moment t; φ is electrical potential at point (x, y, z) at moment t; a is thermal diffusivity; ρ and c are electrical resistivity and specific capacitance.

If equality $1/r_e c_e = a$ exists and time scales of electrical and thermal phenomena are adequate, then both equations (3.36) for ϑ and φ become identical to each other. When analyzing stationary fields with no available energy sources, the analogy between temperature and electrical potential always takes place because both values comply with structurally adequate equations.

Let's point out however that basic cause of analogy existence is not similitude of differential equations; they are just good means to recognize this analogy. The source of the analogy is the fact that initial regularities are stated in the same manner. Transformation of conversation laws as well as Ohm's and Fourier's laws to mathematical terms results in differential equations

(3.36). Consequently, it is fully possible to establish the analogy (without resort to final differential equations) by means of direct using physical reasoning and initial mathematical expressions. Relationship of the quantities by the scheme of electro-thermal analogy is represented in table 3.1.

Table 3.1 – Electro-thermal analogy

Electrical quantities			**Thermal quantities**		
Quantity	**Unit of measurement**		**Quantity**	**Unit of measurement**	
	Contraction	Dimensionality		Contraction	Dimensionality
Electric charge	C	A·s	Quantity of heat	J	N·m
Difference of electrical potentials; electric voltage	V	$\frac{N \cdot m}{A}$	Difference of temperatures	K	K
Intensity of electric field	V/m	$\frac{V}{m}$	Temperature gradient	K/m	$\frac{K}{m}$
Electric conductivity	S/m	$\frac{A}{V \cdot m}$	Thermal conductivity	Wt/(m·K)	$\frac{N}{s \cdot K}$
Electric resistance	Ohm	$\frac{V}{A}$	Thermal resistance	K/Wt	$\frac{K \cdot s}{N \cdot m}$
Current	A	A	Thermal flux	Wt	$\frac{N \cdot m}{s}$
Electric capacitance	F	$\frac{C}{V}$	Heat capacity of a system	J/K	$\frac{N \cdot m}{K}$

There are two groups of methods to use electro-thermal analogy. In the first group (geometrical analogies) thermal system is simulated by the passage of electric current in models that are geometrically similar to investigated system and are made of materials of continuous conductivity (plating bath, electrical conductive paper etc). The second group of methods (simulating circuits) is based on substitution of the investigated object with distributed parameters by the electric circuit with lumped parameters. Such practice is mathematically equivalent to substitution of partial differential equations describing the process by the finite difference equations. Simulating circuits give rough index about the field because measurements of potential are not possible to make for all points of the field, but for its finite number. Let's point out that between investigated thermal system and its analog (electric simulating circuit) there is no geometrical analogy.

Simulating circuits are combined of resistances in series and capacitors in parallel concentrated at nodal point of the circuit. When analyzing stationary temperature field it is enough to consider simulating circuit of resistances only. Thermal sources are simulated by connection of power sources at certain points of simulating circuit.

A notion about thermal resistance was introduced in first chapter. Taking into consideration foregoing analogy between the processes of transfer of heat and electricity, let's consider again this notion in application of more wide range of the tasks.

Let's assume example shown in Figure 3.1*a* wherein certain volume *1* is round by enclosure *2*; within volume *1* energy sources dissipate capacity *P*. Thermal resistance between volume *1* and enclosure *2* equals R_{th12}; thermal resistance between enclosure *2* and environment equals R_{th2e}; temperature of the surrounding medium ϑ_0. It is necessary to express total thermal resistance of the system R_{th} by resistances of its elements R_{th12} and R_{th2e} and to find the temperatures of the volume surface ϑ_1 and the enclosure ϑ_2. Figure 3.1*b* shows the connection circuit for the thermal resistances.

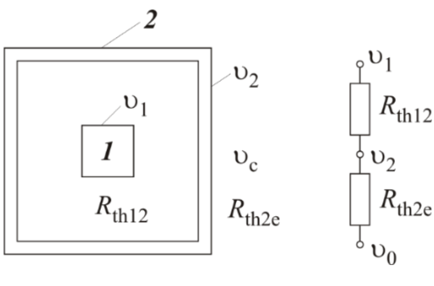


Figure 3.1 – System of the bodies of the type core–gap–enclosure:
a – schematic image of the system; *b* – connection of thermal resistances

Application of Kirchhoff's laws gives:

$$\vartheta_1 - \vartheta_2 = R_{th12} P\ ;\ \vartheta_2 - \vartheta_0 = R_{th2e} P\ ;\ \vartheta_1 - \vartheta_0 = R_{th} P\ ;\ R_{th} = R_{th12} + R_{th2e}\ . \quad (3.5)$$

Figure 3.2*a* shows rod 1 with adiabatic surface, which is connected with plate 2; temperature ϑ_1 is sustained on free butt-end of the rod, temperature of the plate at the place of contact with the rod is ϑ_2. Heat flux P passing through the rod and is dissipated by the plate can be evaluated by formulas:

$$P = \frac{\vartheta_1 - \vartheta_2}{R_{th1}}; \quad P = \frac{\vartheta_1 - \vartheta_2}{F_2} \tag{3.6}$$

where R_{th1} is thermal resistance of the rod; F_2 is thermal coefficient of the plate for surface S_2, through which the flux comes from the rod.

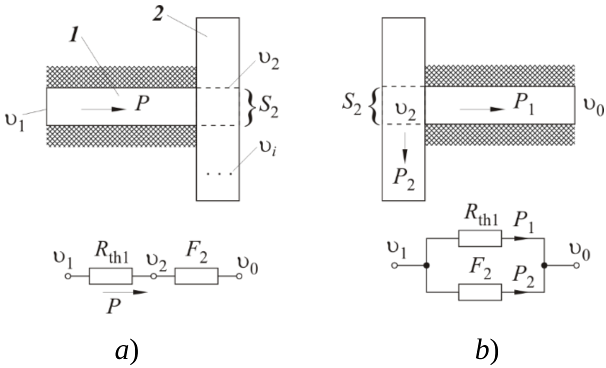


Figure 3.2 – Connection of the wall with the rod: *a* – connection in series; *b* – in parallel

From the equations mentioned it follows that

$$\vartheta_1 - \vartheta_0 = RP; \quad R = R_1 + F_2. \tag{3.7}$$

Figure 3.2*b* shows a plate and a rod with adiabatic boundary; at the place of joint on a surface S_2 a temperature ϑ_2 is maintained. Heat fluxes P_1 and P_2 can be computed by the following formulas:

$$\vartheta_2 - \vartheta_0 = R_1 P_1 \quad \vartheta_2 - \vartheta_0 = F_2 P_2; \quad R = R_1 + F_2. \tag{3.8}$$

Since $P = P_1 + P_2$, then from equations (3.7), (3.8) it follows:

$$P = \frac{\vartheta_2 - \vartheta_0}{R}; \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{F_2} \tag{3.9}$$

As it is seen from these equations Kirchhoff's laws can be applicable for parameters R and F under following condition: investigated isothermal surface coincides with surface S_2, through which thermal flux incomes to the rod or the plate.

4. THERMAL BEHAVIOR ANALYSIS OF CURRENT-CARRYING COMPONENTS

4.1 Basic problems of thermal behavior analysis

When designing power electric and electronic equipment the most important task is to determine (in) computational and experimental way the most advantageous constructive shapes and dimensions of the current-carrying components, which result in most effective use of active materials provided that required features in respect to heating under all possible operating conditions would remain invariant. Successful solution of such task is to a greater extent defined by qualitative thermal behavior analysis basing upon solving heat exchange problem discussed above, as applied to particularly considered current-carrying parts under assumed conditions of theirs heating and cooling.

The main task of thermal behavior analysis (thermal design or thermal calculations) of power equipment current-carrying (and other) components is generally to determine capacity of thermal sources and their temperature field. Thermal behavior analysis is usually performed in two sequential steps:

I. Scoping (short-cut) calculations or **qualitative thermal behavior analysis.** At this stage, first and foremost, thermal sources are revealed. Then, total capacity of thermal sources is calculated and possibility of their reduction is revealed under conditions of least material consumption and optimal thermal behavior. The possible ways to reduce power of thermal sources are application of:

- (gaps) as well as short-circuited turns to reduce magnetic flux.
- the conductive materials with small resistivity;
- the tubular wires, when skin-effect is strongly marked;
- the combined buses that results in leveling off current density over cross-section of the wire;
- the non-ferromagnetic materials in constructions of non-current carrying parts instead of ferromagnetic ones; including non-magnetic inserts (gaps) as well as short-circuited turns to reduce magnetic flux.

At the stage of short-cut calculations, the possibility to increase in heat transfer ability of main power equipment components is outlined. It can be provided by increase in htc or cooling area. Increase in htc can be reached by different ways, such as:

- increase in emissivity factor of the surface cooled (e.g. by means of coating with high emissive material);

- application of liquid cooling;

- application of forced convection, that is, to provide a flow of the cooling medium;

- application of vapor-phase cooling etc.

Increase in cooling area is usually reached either by fining the cooled object surface or application of radiators, that is, standard or special components arranged on the object cooled.

II. Confirmative analysis or **précised thermal calculations.** Performing précised thermal calculations of current-carrying components implies to solve the following problems:

- the **first (implicative) primal problem** is to determine permissible current load at known geometrical dimensions and cooling technology;

- the **first (implicative) reversal problem** is to determine required cooling area at known current load and cooling technology;

- the **second primal problem** is to decide the cooling technology and to determine the parameters of the cooling system at known current load and geometrical dimensions;

- the **second reversal problem** is to find temperature field at known current load and geometrical dimensions as well as cooling technology.

It should be noted that permissible temperature values are always known independently of the type of problem solved.

Current-carrying system of power electric and electronic equipment can generally consist of assemblage of different parts in configuration, dimensions and construction, such as rods, buses, contact blades, arms, pieces of contact units, flexible connections etc. connected between them by means of different types of contacts and mechanical junctions. It is very important here that most current-carrying systems have relatively long length. With, as it has been revealed above, due to high thermal conductivity of electro-conductive materials the temperature distribution over cross-sectional area of current-carrying parts is practically equable. It also takes place in current-carrying components with complexly shaped cross-section, e.g. angle, T-section, I-section etc. This circumstance enables most problems of thermal behavior analysis to reduce to one-dimensional heat conduction problems, which establish the temperature distribution along axis of the current-carrying component and in the time $\vartheta(x,t)$. Boundary conditions will be in this case one-dimensional as well. Internal source strength at even temperature distribution over cross-section can be accepted as being equally distributed and having some midvalue for given cross-section

$$q_{\text{cp}}(x,t) = \delta^2(x,t)\rho_0(1+\alpha\vartheta).$$

Even temperature distribution over cross-section defines conditions of heat transfer from outer surface so that it can be accepted condition of even temperature distribution throughout perimeter of any cross-section. It allows when thermal calculating the current-carrying component having not same heat transfer conditions over the cross-sectional perimeter to accept some averaged thermal resistance between surface of the component and its surroundings.

The heat transfer process is complex as current-carrying components are heated by alternating current. In this case current density distribution, and accordingly, internal cross-sectional heat generation is non-uniform due to skin and proximity effects. Exact solution of such problems at the present day is practically impossible. However, in most cases for current carrying components made of non-ferromagnetic materials the conditions of heating can be accepted the same as at direct current, that is, even temperature distribution over the cross-section and the perimeter with taking into account coefficients of skin and proximity effects (see subsection 1.3).

It should be pointed out that most practical problems of thermal calculations are the particular cases that refer to typical processes of heating the current-carrying components of power electric and electronic equipment. Most typical of them are as follows.

4.2 Analysis of thermal behavior of a homogeneous current-carrying component

The homogeneous current-carrying system will be in given case accepted the one having identical conditions of releasing heat and its driving out to surrounding. In a simplest case it is a conductor with non-varying cross-sectional area, perimeter; it is made of common material; it has lengthwisely equal thermal resistance between the conductor and its surrounding. When analyzing, certain assumptions are made, such as:

- the conductor has infinitely large length resulting in even distribution of the temperature along its length and heat transfer is only radially directed (accordingly, heat transfer in axial direction is not available);

- specific heat of the conductor material, additional losses factor and total htc (and thermal resistance in whole) are **not dependent upon the**

temperature.

Hence, for length unit of the conductor for the time *dt* heat balance equation can be written in the following form:

$$dQ_1 = dQ_2 + dQ_3, \tag{4.1}$$

where dQ_1 is amount of heat released in length unit of the conductor; dQ_2 amount of thermal energy expended on its heating; dQ_3 is amount of heat driven away from outer surface of the conductor.

The terms of this equation are expressed as follows:

$$dQ_1 = \frac{I^2 \rho_0 (1 + \alpha \vartheta) \kappa_д}{q} dt; \quad dQ_2 = c\gamma q d\vartheta; \quad dQ_3 = \frac{\vartheta - \vartheta_0}{r_{т\Sigma}} dt, \tag{4.2}$$

where I is the current load of the conductor; $\kappa_д$ is the additional losses factor; ϑ is the temperature of the conductor; ϑ_0 is ambient temperature; q is the cross-sectional area of the conductor; physical parameters of the conductor material: ρ_0 is resistivity at 0°C, α is thermal coefficient of resistance, c is specific heat, γ is density; $r_{т\Sigma}$ is total thermal resistance between the conductor surface and environment per length unit.

In simplest case when heat is immediately driven out from the conductor surface, thermal resistance is

$$r_{т\Sigma} = \frac{1}{\alpha_т p}, \tag{4.3}$$

where p is the conductor perimeter or cooling surface per its length unit; $\alpha_т$ is total htc from the conductor surface.

In other (more complex) cases of heat transfer between the conductor and its surrounding equivalent thermal circuit is to be set and total thermal resistance is to be calculated.

In such a way, heat balance equation will be as follows:

$$\frac{I^2 \rho_0 (1 + \alpha \vartheta) \kappa_д}{q} dt = c\gamma q d\vartheta + \frac{\vartheta - \vartheta_0}{r_{т\Sigma}} dt$$

or

$$c\gamma q \frac{d\vartheta}{dt} + \left(\frac{1}{r_{т\Sigma}} - \frac{I^2 \rho_0 \kappa_д \alpha}{q} \right) \vartheta = \frac{I^2 \rho_0 \kappa_д}{q} + \frac{\vartheta_0}{r_{т\Sigma}}. \tag{4.4}$$

Solution of this equation with initial conditions: $\vartheta(0) = \vartheta_{нач}$ will be:

$$\vartheta = \vartheta_{\text{уст}} \left(1 - e^{-\frac{t}{T}} \right) + \vartheta_{\text{нач}} e^{-\frac{t}{T}}, \qquad (4.5)$$

where $\vartheta_{\text{уст}}$ is the *steady state temperature* of the conductor, that is, the temperature, which steadies on the surface of the conductor continuously loaded by the current *I*; *T* is the so-called *thermal time constant* (*time constant of heating*) of the wire.

Expression for the steady state temperature can be derived from the heat balance equation at $\frac{d\vartheta}{dt} = 0$:

$$\vartheta_{\text{уст}} = \frac{\dfrac{I^2 \rho_0 \kappa_{\text{д}}}{q} + \dfrac{\vartheta_0}{r_{\text{т}\Sigma}}}{\dfrac{1}{r_{\text{т}\Sigma}} - \dfrac{I^2 \rho_0 \kappa_{\text{д}} \alpha}{q}}. \qquad (4.6)$$

The thermal time constant will be expressed as follows:

$$T = \frac{c\gamma q}{\dfrac{1}{r_{\text{т}\Sigma}} - \dfrac{I^2 \rho_0 \kappa_{\text{д}} \alpha}{q}}. \qquad (4.7)$$

When normal servicing circumstances, then in most cases the condition is valid:

$$\frac{1}{r_{\text{т}\Sigma}} >> \frac{I^2 \rho_0 \kappa_{\text{д}} \alpha}{q}, \qquad (4.8)$$

and the time constant will be

$$T = \frac{c\gamma q}{1/r_{\text{т}\Sigma}}. \qquad (4.9)$$

Hence, the time constant of heating is relation of ability to absorb the heat by the conductor to ability to dissipate the one.

If at zero-time moment the conductor temperature equals ambient temperature $\vartheta(0) = \vartheta_0$, then:

$$\vartheta = \vartheta_{уст}\left(1 - e^{-\frac{t}{T}}\right) + \vartheta_0 e^{-\frac{t}{T}} \qquad (4.10)$$

and temperature-rise of the conductor relative to ambient temperature $\tau = \vartheta - \vartheta_0$ will vary exponentially:

$$\tau = \tau_{уст}\left(1 - e^{-\frac{t}{T}}\right). \qquad (4.11)$$

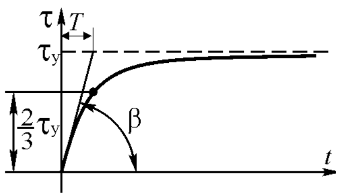


Figure 4.1 – Heating curve

The time constant of heating can be determined from the time plot shown in Figure 4.1. For this purpose the equation (4.11) is differentiated by the time that results in following expression:

$$\frac{d\tau}{dt} = \frac{\tau_{уст}}{T} e^{-\frac{t}{T}}. \qquad (4.12)$$

At $t = 0$

$$\frac{d\tau}{dt} = \frac{\tau_{уст}}{T} = tg\beta. \qquad (4.13)$$

Hence,

$$T = \frac{\tau_{уст}}{tg\beta}. \qquad (4.14)$$

That is, thermal time constant is in this case the section cut off by the line tangent to heating curve on the line $\tau = \tau_{уст}$. It is geometrical sense of thermal time constant. If the heating time equals the time constant $t = T$, then

$$\tau = \tau_{уст}\left(1 - e^{-1}\right) = 0.632\tau_{уст} \approx \frac{2}{3}\tau_{уст}. \qquad (4.15)$$

In other words, after laps of the time constant the conductor temperature-rise becomes equal approximately by 2/3 of steady state value. Note that steady (stationary) state comes practically after the time span equal (3…4)*T*, when the conductor temperature has reached $(0.95\ldots0.98)\tau_{уст}$.

Adiabatic heating a conductor: such conditions of heating occur in the case when thermal energy is not driven out from a conductor to its surrounding ($r_{т\Sigma} = \infty$). Under such conditions initial heat balance equation will be as follows:

$$\frac{I^2 \rho_0 (1 + \alpha \vartheta) \kappa_Д}{q} dt = c\gamma q d\vartheta . \tag{4.16}$$

If in given equation temperature dependence of resistivity is not taken into account and $d\vartheta$ will be substituted by $d\tau$, the equation comes to the form:

$$\frac{I^2 \rho \kappa_Д}{q} dt = c\gamma q d\tau . \tag{4.17}$$

Solution of this equation at $\tau(0) = 0$ gives:

$$\tau = \frac{I^2 \rho \kappa_Д}{c\gamma q^2} t . \tag{4.18}$$

Substitution of the time *t* by the time constant of heating expressed by formula (4.9) gives

$$\tau = \frac{I^2 \rho \kappa_Д}{q} \cdot \frac{1}{1/r_{т\Sigma}} . \tag{4.19}$$

Equation (4.19) corresponds to the expression for steady state temperature-rise. Thus, thermal time constant have one further sense: it is the time required to heat the conductor up to steady state temperature with no driving out heat from its surface to environment.

Cooling process of the conductor is described by the same equation; however, the term dQ_1 (heat release) is not available, therefore, the equation is:

$$0 = c\gamma q d\vartheta + \frac{\vartheta - \vartheta_0}{r_{т\Sigma}} dt . \tag{4.20}$$

If at zero-time the conductor temperature equals the steady state one, that is $\vartheta(0) = \vartheta_{уст}$, then the solution of the equation will be expressed as:

$$\vartheta = \vartheta_{уст} e^{-\frac{t}{T}} + \vartheta_0 \left(1 - e^{-\frac{t}{T}} \right) . \tag{4.21}$$

For the temperature-rises: $\tau = \vartheta - \vartheta_0$; $\tau_{уст} = \vartheta_{уст} - \vartheta_0$, it is

$$\tau = \tau_{уст} e^{-\frac{t}{T}} . \qquad (4.22)$$

Thus, behavior of the conductor's cooling curve will be exponential as shown in Figure 4.2.

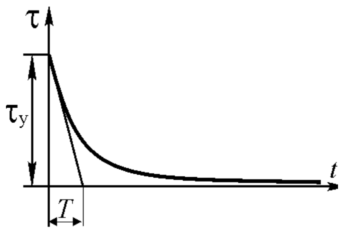


Figure 4.2 – Cooling curve

In the equations of heating and cooling discussed above the quantity $r_{т\Sigma}$ is most important. It is total thermal resistance featuring heat transfer behavior from the conductor surface to environment. Let's consider typical practical example: the wire that comprises conductor of circular cross-section coated by solid insulation layer as shown in Figure 1.2.

The wire is in bulk medium with temperature ϑ_0.

To determine the total thermal resistance, the ways of heat transfer are to be assigned and thermal circuit is to be constructed. In given wire the heat released in the conductor passes through the thin solid insulation and then dissipates from outer surface of the insulation layer to environment. At first, the heat transfer by conduction takes place; at second, it is by convection and/or radiation depending on medium. Hence, equivalent thermal circuit for given wire will contain two thermal resistances in series as shown in Figure 4.3.

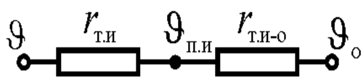


Figure 4.3 – Thermal equivalent circuit

Here $r_{т.и}$ is thermal resistance of the solid insulation layer; $r_{т.и\text{-}о}$ is thermal resistance between outer surface of the insulation layer and ambient medium. Noted thermal resistances are expressed as follows:

$$r_{т.и} = \frac{1}{2\pi\lambda_и} \ln\frac{d_1}{d_2} . \qquad (4.23)$$

where $\lambda_и$ is the insulation material conductivity;

$$r_{т.и\text{-}о} = \frac{1}{\alpha_т p} . \qquad (4.24)$$

where p is outer perimeter of the insulation layer; in given case $p = \pi d_1$;

$\alpha_т$ is total htc from outer surface of insulation layer defined generally as the sum of convective and radiative heat transfer coefficients

$$\alpha_т = \alpha_к + \alpha_и . \tag{4.25}$$

It should be noted that $\alpha_к$ and $\alpha_и$ are composite functions of the temperatures of outer surface of insulation and ambient medium. Therefore, initial differential equation will be non-linear. Its solution can be performed either by successive approximations or numerical methods using PC with relevant software.

4.3 Operational duties of current-carrying components

In terms of time-changing current load behavior for current-carrying components of power electric and electronic equipment the following operational duties are typical.

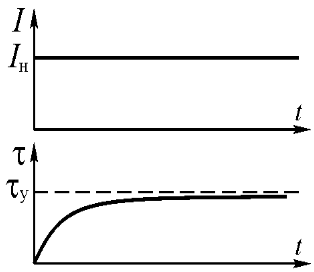


Figure4.4 – Time diagrams for current and temperature-rise at long-term duty

Continuous (long-term) duty, when during under-load operation of the equipment the temperature of its components reaches steady state value as shown in Figure 4.4. In this case under-load operation time may be any much, but not less than the time required to attain steady state temperature. As already noted, this time is approximately of (3…4)T.

Short-time (temporary) duty, when during under-load operation the temperature of the current carrying components does not reach steady state value, but during no-load operation it reaches the ambient temperature as shown in Figure 4.5.

At the current load $I_{кр}$ and the operating time $t_{кр}$ the temperature-rise reach the maximal value derived from heating curve:

$$\tau_{макс} = \tau_{уст}\left(1 - e^{-\frac{t_{кр}}{T}}\right), \tag{4.26}$$

where $\tau_{уст}$ is the steady state temperature-rise for the case continuous flowing the current $I_{кр}$ through the conductor.

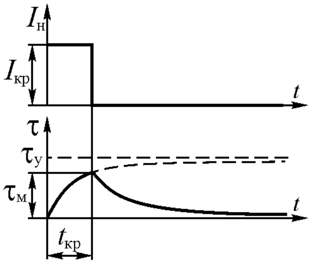


Figure 4.5 – Time diagrams for current and temperature-rise at short-term duty

When calculating current-carrying components for short-time operation so-called *current (power) overload factor* as well as *equivalent value of current* (*power*) are used. Equivalent current (power) is the value that in continuous leads to the same heating as short-time current during the time $t_{кр}$. In this case relationship is valid:

$$\frac{\tau_{уст}}{\tau'_{уст}} = \frac{\tau_{уст}}{\tau_{макс}} = \frac{I_{кр}^2}{I_{экв}^2}. \qquad (4.27)$$

Then:

$$1 = \frac{I_{кр}^2}{I_{экв}^2}\left(1 - e^{-\frac{t_{кр}}{T}}\right) \quad \text{or} \quad I_{экв} = I_{кр}\sqrt{1 - e^{-\frac{t_{кр}}{T}}} = \frac{I_{кр}}{\kappa_I}, \qquad (4.28)$$

where κ_I is the current overload factor expressed as

$$\kappa_I = \frac{1}{\sqrt{1 - e^{-\frac{t_{кр}}{T}}}}. \qquad (4.29)$$

Correspondingly, the power overload factor is

$$\kappa_P = \frac{1}{1 - e^{-\frac{t_{кр}}{T}}}. \qquad (4.30)$$

From the expressions (4.29) and (4.30) it follows that current (power) overload factor is influenced by thermal time constant. With, increase in the time constant enables increase in the overload factors; it is very important for the current-carrying components operating in short-time duty. Increase in thermal constant is usually reached by increase in heat capacity of the current-carrying components.

Intermittent cycle duty is alternative under-load and no-load operation in cycle. Here, during under-load operation the electrical apparatus component has no time to heat up to steady state temperature and during no-load operation it has no time to cool down to ambient temperature as shown in Figure 4.6.

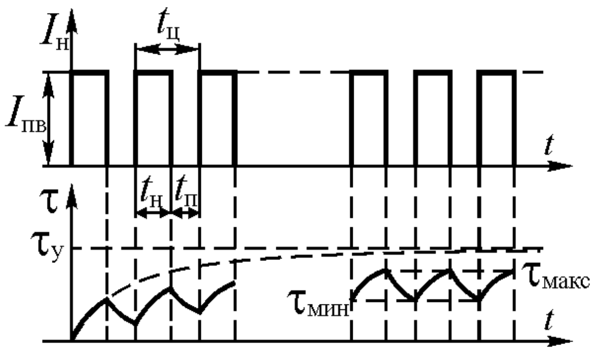


Figure 4.6 – Time diagrams for current and temperature-rise at intermittent cycle duty

The main parameter for this operation type is so-called *relative duty*:

$$ПВ\% = \frac{t_{вкл}}{t_{вкл} + t_{откл}} \cdot 100\% = \frac{t_{вкл}}{t_{ц}} \cdot 100\% \qquad (4.31)$$

As seen from Figure 4.6, after some number of cycles the state steadies when the temperature-rise at the end of under-load state and at the end of no-load state will not change in time, that is: $\tau_{макс} = const1$ and $\tau_{мин} = const2$. This regime is named *quasi-steady state*, when during under-load operation the temperature-rise builds from $\tau_{мин}$ up to $\tau_{макс}$ and the following expression is valid:

$$\tau_{макс} = \tau_{уст}\left(1 - e^{-\frac{t_{вкл}}{T}}\right) + \tau_{мин} e^{-\frac{t_{вкл}}{T}}, \qquad (4.32)$$

whereas during no-load operation the temperature-rise goes from $\tau_{макс}$ down to $\tau_{мин}$, respectively:

$$\tau_{мин} = \tau_{макс} e^{-\frac{t_{откл}}{T}}. \qquad (4.33)$$

Substitution of expression (4.33) into (4.32) gives:

$$\frac{\tau_{\text{макс}}}{\tau_{\text{уст}}} = \frac{1 - e^{-\frac{t_{\text{вкл}}}{T}}}{1 - e^{-\frac{t_{\text{ц}}}{T}}} . \qquad (4.34)$$

In the same way as for short-time duty:

$$\frac{\tau_{\text{уст}}}{\tau_{\text{макс}}} = \frac{I^2_{\text{ПВ}}}{I^2_{\text{экв}}} . \qquad (4.\ 35)$$

Subsequently, the current and power overload factors will be expressed as:

$$\kappa_{\text{I}} = \sqrt{\frac{1 - e^{-\frac{t_{\text{ц}}}{T}}}{1 - e^{-\frac{t_{\text{вкл}}}{T}}}} ; \; \kappa_{\text{P}} = \frac{1 - e^{-\frac{t_{\text{ц}}}{T}}}{1 - e^{-\frac{t_{\text{вкл}}}{T}}} . \qquad (4.\ 36)$$

For this regime, design condition is: $\tau_{\text{макс}} \le \tau_{\text{доп}}$.

4.4 Stationary thermal behavior of current-carrying components with considering equalizing heat fluxes

As already noted, actual current-carrying systems of power equipment consist of different in configuration, dimensions and construction parts and are mostly inhomogeneous in terms of thermal conditions. Inhomogeneous current-carrying system may contain:

- additional thermal sources concentrated on the very small sections: for example, contacts, semiconductor devices etc.
- sections with different cross-sectional areas and materials as well as distinctive heat transfer conditions;

In such cases heat flux is directed not only radially, but axially along to the current carrying system. In other words, in inhomogeneous current carrying systems so- called *equalizing heat fluxes* take place. To determine thermal behavior with taking into account equalizing heat fluxes the inhomogeneous current carrying system is subdivided into a number of homogeneous elements (sections) so that a separate section would have the same heat transfer conditions and other parameters.

Let us consider steady state thermal behavior of any section with taking into account equalizing heat fluxes. This section has cross-sectional area q and total thermal resistance $r_{\text{т}\Sigma}$ as well as is made of one material as shown in Figure 4.7.

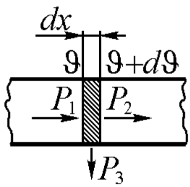


Figure 4.7

For the section element of length *dx*, heat balance equation for stationary conditions can be written:

$$P_1 + P_4 = P_2 + P_3, \tag{4.37}$$

Terms P_1–P_3 of this equation are analogous to respective those of equation (2.73). Term P_4 is thermal power released in element *dx*. Thus, expressions defining the terms of equation (4.37) will be as follows:

$$\begin{cases} P_1 = -\lambda q \dfrac{d\vartheta}{dx}; \\ P_2 = -\lambda q \dfrac{d}{dx}\left(\vartheta - d\vartheta\right) = -\lambda q \dfrac{d\vartheta}{dx} + \lambda q \dfrac{d^2\vartheta}{dx^2}dx; \\ P_3 = \dfrac{\vartheta - \vartheta_0}{r_{т\Sigma}}dx; \\ P_4 = \dfrac{I^2\rho_0\left(1+\alpha\vartheta\right)}{q}dx \end{cases}. \tag{4. 38}$$

Substituting these expressions into the heat balance equation (4.37) and reducing it to normal form, we will have derived differential equation that describes temperature distribution along the section of current carrying system:

$$\frac{d^2\vartheta}{dx^2} + b^2\left(\vartheta - \vartheta_{уст}\right) = 0\,. \tag{4. 39}$$

where the coefficients of the expression are expressed as

$$b = \sqrt{\frac{1}{\lambda q}\left(\frac{1}{r_{т\Sigma}} - \frac{I^2\rho_0\kappa_д\alpha}{q}\right)};\ \vartheta_{уст} = \frac{\dfrac{I^2\rho_0\kappa_д}{q} + \dfrac{\vartheta_0}{r_{т\Sigma}}}{\dfrac{1}{r_{т\Sigma}} - \dfrac{I^2\rho_0\kappa_д\alpha}{q}}. \tag{4. 40}$$

Derived differential equation of second order is in strict sense non-linear because it includes the parameters $\vartheta_{уст}$ and *a* that are temperature dependent. If not to take this dependence into account for the first approximation, its solution can be derived by the classic method. It depends on the manner to predetermine the boundary conditions. Let’s consider two types of sections: section of finite length and half-infinite one.

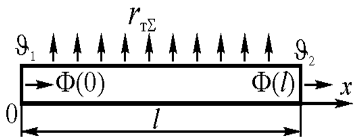


Figure 4.8

For **the section of perfect (finite) length** shown in Figure 4.8 the boundary conditions are predetermined by the temperatures at the section boundaries:

$$\vartheta(0) = \vartheta_1; \; \vartheta(l) = \vartheta_2 , \qquad (4.41)$$

where *l* is length of the section.

Solution of this equation is expressed as:

$$\vartheta = \frac{(\vartheta_1 - \vartheta_{\text{уст}})\text{sh}[b(l-x)] + (\vartheta_2 - \vartheta_{\text{уст}})\text{sh}bx}{\text{sh}bl} + \vartheta_{\text{уст}} . \qquad (4.42)$$

Using Fourier's law the equalizing heat fluxes at the section boundaries can be written in following form:

$$\Phi(0) = u\vartheta_1 - v\vartheta_2 + s ; \quad \Phi(l) = v\vartheta_1 + u\vartheta_2 - s , \qquad (4.43)$$

where $u = b\lambda q \dfrac{\text{ch}\,bl}{\text{sh}bl}; \; v = \dfrac{b\lambda q}{\text{sh}\,bl}; \; s = \vartheta_{\text{уст}}(v-u).$

For the half-infinite section shown in Figure 4.9 the boundary conditions are predetermined as follows

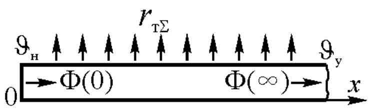


Figure 4.9

$$\vartheta(0) = \vartheta_{\text{н}}; \quad \vartheta(\infty) = \vartheta_{\text{уст}}; \quad \left.\frac{d\vartheta}{dx}\right|_{x=\infty} = 0. \qquad (4.44)$$

The temperature distribution along the section will be in this case expressed as:

$$\vartheta = (\vartheta_{\text{н}} - \vartheta_{\text{уст}})e^{-bx} + \vartheta_{\text{уст}} . \qquad (4.\ 45)$$

The equalizing heat at the section boundary (that is at $x = 0$) will be expressed as

$$\Phi(0) = b\lambda q(\vartheta_{\text{н}} - \vartheta_{\text{уст}}). \qquad (4.\ 46)$$

4.5 Heating current-carrying components by short-circuit currents. Thermal stability of equipment

The **short circuit (sc)** is faulty conditions for current-carrying components of power electrical and electronic equipment. The magnitudes of sc currents significantly exceed the currents of normal circumstances. However, duration of their flowing is very limited since in the event of sc relay protections actuate and a faulty subsystem is rapidly de-energized. With taking into account selective action of protection it not excess of a few seconds, because of that adiabatic heating conductors happen. For example, the increase in current tenfold results in increase in power released a hundred times while heat dissipation is practically fixed since increase in temperatures of conductors is insignificant.

So, the heat balance equation will not contain the term of withdrawing heat:

$$\frac{I_{кз}^2(t)\rho_0(1+\alpha\vartheta)}{q}dt = c_0(1+\beta\vartheta)\gamma q d\vartheta$$

or
$$j_{кз}^2(t)dt = \frac{c_0(1+\beta\vartheta)\gamma}{\rho_0(1+\alpha\vartheta)}d\vartheta\,. \tag{4. 47}$$

Integration of this equation gives:

$$\int_0^{t_{кз}} j_{кз}^2(t)dt = \frac{c_0\gamma}{\rho_0}\int_{\vartheta_н}^{\vartheta_к}\frac{(1+\beta\vartheta)}{(1+\alpha\vartheta)}d\vartheta = A(\vartheta_к) - A(\vartheta_н), \tag{4. 48}$$

where the dependence $A(\vartheta)$ is named *adiabatic heating curve*.

Such curves for different conductive materials (without taking into account additional losses) are represented in reference sources [6].

If the value of sc current is not time-varying, then expression (4.48) can be written in the form:

$$j_{кз}^2 t_{кз} = A(\vartheta_к) - A(\vartheta_н). \tag{4.49}$$

where $\vartheta_н$ is the conductor temperature prior to sc; $\vartheta_к$ is the conductor temperature at the instant of clearing sc, which must not exceed allowable value.

Hence, permissible sc current density is determined from the following equation:

$$j_{кз\,доп} = \sqrt{\frac{A(\vartheta_{доп}) - A(\vartheta_н)}{t_{кз}}}\,, \tag{4.50}$$

and minimal required cross-sectional area of the conductor is:

$$q_{мин} = \frac{I_{кз}}{j_{кз\,доп}}\,. \tag{4.51}$$

If the cross-sectional area is known, then the temperature is determined as:

$$A(\vartheta_к) = A(\vartheta_н) + j_{кз}^2 t_{кз}\,; \tag{4.52}$$

and $A(\vartheta_к) \Rightarrow \vartheta_к$.

Note that the expressions represented above can be used only in the case when rms sc current in time is not variable. Such situation takes place when the component considered is installed for the long distance from power generators. If the sc happens in the neighborhood of power generator, then rms sc current varies in time. If automatic excitation controller is not available in the subsystem, sc current squared decreases from initial value $I_{кз0}$ down to steady state one $I_{кз\infty}$ as shown in Figure 4.10*a*. When automatic excitation controller operates in the subsystem, then sc current at first decreases as well, and then under action of automatic excitation controller and increase in generator EMF the sc current squared builds up as shown in Figure 4.10*b*. In such cases thermal calculations are somewhat more complicated.

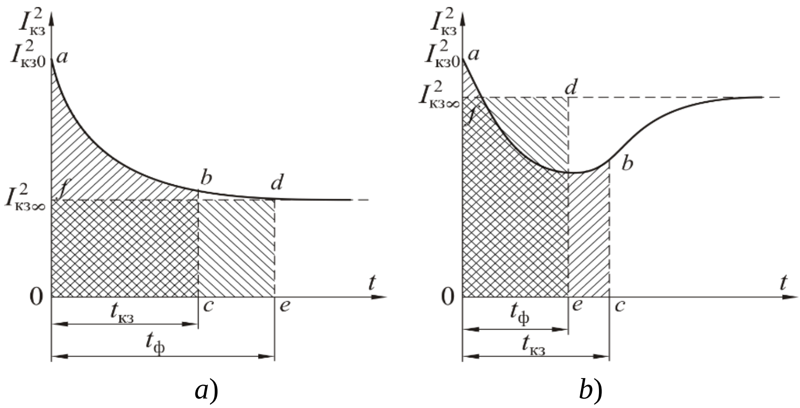


Figure 4.10 – Behavior of sc current: *a*) automatic excitation controller is not available; *b*) automatic excitation controller is operating

The value of thermal pulse for time-varying sc current is determined through integrating:

$$Q_{кз} = \int_0^{t_{кз}} I_{кз}^2(t)dt\,. \tag{4.53}$$

If the law of time-varying sc current is known, then $Q_{кз}$ can be graphically quantified from the plot $I_{кз}^2(t)$ through the area of the figure *0abc* (see Figure 4.10).

There exists another method involved to exchange the area *0abc* by equivalent rectangular-shaped one *0fde* bounded by the steady state sc current and so-called **virtual fault duration (time)**. The virtual fault duration is the time span that it takes for the steady state sc current results in the same heat released as the time-varying sc current, that is

$$\int_0^{t_{кз}} I_{кз}^2(t)dt = I_{кз\infty}^2 t_{кз\,ф}\,. \tag{4.54}$$

The procedure to find virtual fault time is as follows: since sc current includes two (symmetrical and direct) components, thermal pulse and virtual fault time are respectively subdivided into two components:

$$Q_{кз} = Q_{кз.п} + Q_{кз.а} = I_{кз\infty}^2\left(t_{ф.п} + t_{ф.а}\right). \tag{4.55}$$

The virtual time of symmetrical component is found from specific curves

$$t_{ф.п} = f\left(\beta'', t_{кз}\right). \tag{4.56}$$

where β'' is relation of initial sc current to its steady state value:

$$\beta'' = \frac{I_0}{I_\infty}\,. \tag{4.57}$$

Such curves are plotted at $t_{кз} \le 5\text{ s}$. If $t_{кз} > 5\text{ s}$, then:

$$t_{ф.п} = t_{ф.п}(5\text{ s}) + \left(t_{кз} - 5\right). \tag{4.58}$$

Thermal action of the dc sc current is significant, when $t_{кз} \le 1\text{ s}$, then:

$$t_{ф.а} = (\beta'')^2 T_а \left(1 - e^{-\frac{2t_{кз}}{T_а}}\right), \tag{4.59}$$

where $T_а$ is the time constant of decaying dc sc current depending on the type of power generator and comprises usually 0.2–0.3 s.

Further calculations for the sc conditions are performed in the same manner.

Thermal stability is ability of the equipment and its components to withstand (without failures inhibiting its further normal operation) thermal action of the sc current during predetermined time interval.

The main quantity that features thermal stability of the apparatus is the *short-time withstand current* (*current of thermal stability*). It is maximal rms symmetrical sc current whose thermal action does not result in loss of operating ability of the apparatus. The equipment and its components are also featured by accepted fault time. As per valid standards the fault time is usually 1, 3, 5 or 10 s depending on the type of equipment, its functional application etc. The values calculated are usually permissible current density and required cross-sectional area of the current-carrying parts. They are determined by the procedure discussed above.

4.6 Thermal behavior of current-limiting resistors of on-load taps changers

Current-limiting resistors are the key components of high speed on-load tap changers. Theirs functions are to limit the current when changing over the current load from one tap of power transformer to another that happen in the process of the power network voltage control. Changing over the load current is carried out in two stages. At first stage unloaded branch of selector is changed over and high-speed spring mechanism is simultaneously wound up. At second stage with the help of contactor (especial contact group) actuated by high-speed spring mechanism very rapidly current load is changed over to unloaded branch of selector. Just in this time interval current-limiting resistor is loaded by current and is accordingly subjected by heating [18].

Duration of changing over the current load is usually hundredth fractions of second, which is significantly less than its thermal time constant. Hence, the process of heating a current-limiting resistor can be considered as adiabatic. So, an increment in temperature of the resistor per one change over is determined by the following formula:

$$\Delta\vartheta = \frac{W}{C} \tag{4.60}$$

or for the resistor of round wire:

$$\Delta\vartheta = \frac{16 I^2 \rho t_{н}}{\pi^2 c \gamma d^2}, \tag{4.61}$$

where W and C are thermal energy generated by the resistor per one change over and its full heat capacity, respectively; I is the current to be changed over; ρ, c, γ are, respectively, resistivity, specific heat and density of the resistor material; d is diameter of the resistor wire; $t_{н}$ is equivalent time of heating the resistor with taking into account overload and arcing time.

Most heavy duty in thermal respect for current-limiting resistor takes place in the case when the tap changer executes automatic voltage control and changes over follow uninterruptedly one after another. In such duty after changing over the load and heating the resistor at once without whatever pause follows another changing over. However, at the first stage of new changing over, when high-speed mechanism is wound up and selector contacts are changed over, the current-limiting resistors are not flowed by current and are cooled down in transformer oil medium. Duration of cooling is dependent on operating time of the electro-motor drive $t_п$. Behavior in time of resistor temperature-rise over transformer oil temperature in the process of cooling a resistor is described by the following differential equation:

$$cm\frac{d\tau}{dt} + \alpha S_{охл}\tau = 0\,, \qquad (4.62)$$

where m is the resistor mass; $S_{охл}$ is cooling surface of the resistor; α is htc from the resistor surface.

Mass of the resistor is:

$$m = \frac{\pi d^2}{4} l\gamma\,. \qquad (4.63)$$

Cooling surface of the resistor is: $S_{охл} = \pi dl$.

Htc from the resistor surface is in complex manner dependent on temperature of the resistor surface, as well as on temperature of oil in transformer tank $\vartheta_М$. To determine htc using well-known expressions for convective heat exchange is in this case not impossible since oil temperature in power transformers is high enough and frequently close to its boiling point. Hence, when cooling a resistor so-called regime of nucleate boiling takes place, which is little investigated for transformer oil. For given particular case htc can be quantified by empirical formula verified experimentally [18]:

$$\alpha = \left(10.1 + 0.193\vartheta_М + 0.097\tau\right)\sqrt[4]{\tau/d}\,. \qquad (4.64)$$

Substituting expression for $S_{охл}$ and mass of resistor as well as taking into account $\tau = \vartheta - \vartheta_М$, we will have:

$$\frac{d\vartheta}{dt} = \frac{4\alpha\left(\vartheta - \vartheta_М\right)}{c\gamma d}\,. \qquad (4.65)$$

Complex dependence of htc from the resistor temperature introduces significant non-linearity into equation derived. Nevertheless, solution of this equation can be easily realized with the help of ***MathCAD***. Analysis of thermal behavior of current-limiting resistor is executed in following sequence:

1) The resistor temperature after first change over of current load is evaluated:

$$\vartheta_1 = \vartheta_{0-1} + \Delta\vartheta , \tag{4.66}$$

where ϑ_{0-1} is the resistor temperature prior to first change over; it is here accepted corresponding to the temperature of oil in the transformer tank, which in turn must correspond to maximal one.

2) The temperature prior to second change over of current load ϑ_{0-2} is determined. It is performed by solving differential equation of cooling the resistor (4.65) for the time span $t_п$ with following initial condition $\vartheta(0) = \vartheta_1$.

3) The resistor temperature after second change over of current load is evaluated:

$$\vartheta_2 = \vartheta_{0-2} + \Delta\vartheta . \tag{4.57}$$

4) The temperature prior to third change over of current load ϑ_{0-3} by solving differential equation of cooling the resistor (4.65) for the time span $t_п$ with following initial condition: $\vartheta(0) = \vartheta_2$ is determined, and so depending on required number of changes over of current load.

As per ГОСТ 24126-80 the number of following one after other changes over in conditions of automatic voltage control must correspond to half the range of the transformer voltage control. The value of temperature obtained after last changeover is compared with allowable value.

5. THERMAL BEHAVIOR ANALYSIS OF CONTACTS AND CONTACT SYSTEMS

5.1 General information. Classification of contacts

Electrical contact is vital component of any electric installation and power equipment. It is well known that current-carrying systems of power electrical and electronic equipment are not unitized constructions. They are assembled of separate elements (conductors) connected by means of *electrical contact*. It can be pointed out here that in most switching apparatuses electrical contacts play key role and is main component. In spite of intensive development of semiconductor-based switches, which supersede partially the contact-based switches, the problems bounded up with operational ability and reliability of contacts have not lost their significance and are still very topical. Operating experience of electrical installations shows that major part of failures is resulted from disturbances just in contact functioning.

On the other hand, it should be noted that physical phenomena occurred with functioning electrical contacts are very complex and have not in full measure studied to present time. By this why at the present time major number of scientific-and-research works is devoted to electrical contacts and now it is large scientific segment including a few separate scientific directions.

The *electrical contact* is the touch point of two conductors serving to pass electrical current from one conductor to another. The conductors producing electrical contact may be in different aggregative states. Therefore, from the qualitative standpoint there are the contacts formed with:

- solid conductors (metal – metal);
- solid and liquid conductors (metal – liquid metal);
- liquid conductors (liquid metal – liquid metal);
- solid conductor and plasma (metal – arc discharge);
- liquid conductor and plasma (liquid metal – arc discharge).

In this handbook the contacts formed with solid conductors (metal–metal) are considered.

The surfaces of solid conductors contacting directly to one another and producing electrical contact are named *contact faces*. The conductors forming immediately electrical contact are named *contact-pieces* or simply *contacts*. To ensure operational ability of contacts it is not sufficient that the contact faces would be only touched to one another, they must be pressed with predetermined force between them called *contact pressure* or *contact* *load*. Assemblage of the contact-pieces with the elements providing contact pressure forms *contact unit*. The contact units in electrical installations and separate electrical apparatuses perform different functions and are variously designed, respectively. Depending on the functions performed the contact units may be shared into two fundamental groups:

1. *Contact connections (joints)* perform function to connect the conductors, which in operation do not rupture an electrical circuit and any movement of contact-pieces relative to one another is not available. There exist such variations of contact joints as:

- *releasable (make-and-break) contact connections*, which can be opened and closed without disassembling and assembling (without tool operations); plug-and-socket can exemplify releasable contact joint;

- *disassembled contact joints***:** here the contact-pieces are jointed and are pressed by means of hardware (bolts, screws etc.); the contact joints of such type are extensively used in electricity distribution centers, electrical apparatuses to connect electrically buses, cables as well as electrical machines and apparatuses to other equipment of electrical power system;

- *permanent contact joints***:** in this case electrical connections are performed by means of weld, soldering, rivet and finally cemented contact connections have been come used recently.

2. *Circuital contacts* perform switching (make-break, change-over) operations in electrical circuits as well as current collection by movable conductors from fixed ones. Respectively, they are shared into two subgroups:

- *switching contacts* carrying out physical (mechanical) rupture (break) of electrical circuit or its restoring under current or without one;

- *current-collecting contacts***:** here one contact-piece travels relative to other one without disturbance of electrical contact between them.

Both switching and current-collecting contacts contain moving contact-piece termed as *movable contact* and contact-piece free from movement termed as *fixed contact*. The contact pressure in both contact types is mainly carried out with springs of various types. In order to ensure predetermined contact pressure, predetermined deformation of the spring must be constructively provided.

As the switching contacts operate they can be in two static positions: opened (off) and closed (on). In opened position the contacts are featured by the *contact gap*. It is shortest distance between contact faces of movable and fixed contacts.

5.2 Contact resistance

Contact resistance is usually the resistance resulted from available electrical contact. The contact resistance is in turn produced by two main causes:

I. curvature (diversion from shortest path) of current lines in the neighborhood of the contact face;

II. available pollutions on the contact faces as well as surface films resulted from chemical reactions with surrounding; it should be noted that surface films offer frequently high enough electrical resistance.

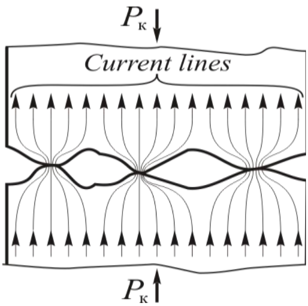


Figure 5.1

Let's consider in more detail influence each of noted factors on contact resistance. It is well known that direct metal contact occurs not throughout whole contact face, but only at certain points (areas) according to microgeometry of the contact interface as shown in Figure 5.1. These areas are usually named the *contact spots* or α-*spots*. Because of this, in the neighborhood of contact interface, the current lines bend and the current is constricted into contact spots.

Region in the vicinity of contact interface, wherein the current lines bend is named the *constriction region*. Bending the current lines and increase in their path result in appearance of additional resistance in the contact. Resistance resulted from bending current lines in the constriction region is named *constriction resistance*.

To determine the constriction resistance let's consider the idealized contact formed by two isotropic conductors with infinite dimensions. The contact has one circular-shaped contact spot with radius *a*.

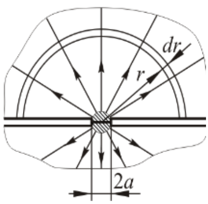


Figure 5.2 – Spherical model of contact

Let's consider at first so-called spherical model of contact which assumes that the current lines equally radiate outward from the contact spot in every which way and equipotential surfaces are half-spheres. Such contact is in longitudinal section shown in Figure 5.2. Electrical resistance of the elemental layer of constriction region with width *dr* on the distance *r* from the center of contact spot will be expressed as:

$$dR_c = \rho \frac{dr}{2\pi r^2} . \tag{5.1}$$

Integration of this expression in limits from a to ∞ gives the expression of constriction resistance of one of the contacts:

$$R_{c1/2} = \frac{\rho}{2\pi} \int_a^{\infty} \frac{dr}{r^2} = \frac{\rho}{2\pi}\left(-\frac{1}{r}\right)\Bigg|_a^{\infty} = \frac{\rho}{2\pi}\left(-\frac{1}{\infty} + \frac{1}{a}\right) = \frac{\rho}{2\pi a} . \tag{5.2}$$

Total constriction resistance of contacts of the same materials will be twice:

$$R_c = \frac{\rho}{\pi a} . \tag{5.3}$$

In theory of electrical contacts so-called *elliptical model* is extensively used. It assumes that for circle contact area the equipotential surfaces in the constriction region are rotational half-ellipsoid shaped with focuses located on its bound. As this model is used the constriction resistance is determined by usage of the analogy between pattern of current lines radiation outward from the contact spot with radius a and electrical field pattern of infinitely thin charged disc with radius a. For such analogy there is well-known relationship between capacitance of the disc C and electrical resistance of the medium R:

$$RC = \rho\varepsilon\varepsilon_0 . \tag{5.4}$$

where ρ, ε, ε_0 are the parameters of the medium, that is, the contact material.

Capacitance of the disc relatively to half-space considered is defined by well-known expression:

$$C = 4\varepsilon\varepsilon_0 a . \tag{5.5}$$

If it is substituted to previous expression, the formula to determine constriction resistance will be as follows:

$$R_{c1/2} = \frac{\rho}{4a} . \tag{5.6}$$

Total constriction resistance of the contacts produced by the same materials will be twice:

$$R_c = \frac{\rho}{2a} . \tag{5.7}$$

Derived expression is named the *Holm's formula*.

If the contact interface is plane, then actual contact will occur by a few spots. If the contact spots are located far enough from each other, so mutual effect on the pattern of current lines radiation will be absent and then constriction resistance will be proportional to the number of contact spots *n*:

$$R_{\text{c}} = \frac{\rho}{2an}. \tag{5.8}$$

The radius of contact spot *a* is at first defined by the behavior of the contact interface deformation. For example, in the case of *elastic deformation*, when the plane contact face and spherical one with radius *r* are pressed, then the radius of contact spot can be evaluated by the following expression [5, 17]:

$$a = 1.11\sqrt[3]{\frac{r \cdot P_{\text{к}}}{E}}, \tag{5.9}$$

where $P_{\text{к}}$ is the value of the contact pressure; *r* is the radius of contact face sphere; *E* is the modulus of elasticity.

As two spherical surfaces with the same radiuses are pressed the radius of the contact spot will be determined by the following expression [5, 17]:

$$a = 0.86\sqrt[3]{\frac{r \cdot P_{\text{к}}}{E}}. \tag{5.10}$$

It should be noted that elastic deformation occurs only at small forces pressing the contact interface (order of a few Newtons). By this why it takes place in contacts of low current apparatuses (relay contacts etc). When the contact pressure per unit of contact spot area in excess of ultimate bearing strength of the contact material, then *plastic deformation* will take place in the contacts. In this case the radius of the contact spot will be expressed as

$$a = \sqrt{\frac{P_{\text{к}}}{\pi[\sigma_{\text{см}}]}}, \tag{5.11}$$

where $[\sigma_{\text{см}}]$ is the ultimate bearing strength of the contact material.

Substituting this expression to the Holm's formula, we will have derived the expression to determine the constriction resistance for the case of plastic deformation:

$$R_{\text{c}} = \frac{\rho}{2}\sqrt{\frac{\pi[\sigma_{\text{см}}]}{nP_{\text{к}}}}. \tag{5.12}$$

From the derived expression it is seen that constriction resistance depends first on contact material properties: resistivity, plasticity as well as the value of contact pressure and the number of contact spots. It may be pointed out here that expression (5.12) is appropriate only for qualitative estimation of contact resistance. For engineering calculations the formulas based on experience findings are mainly used. Most extensively used formula for the contact resistance calculations is:

$$R_{\text{к}} = R_0 + \frac{\kappa}{(0.102P_{\text{к}})^{\text{m}}}, \qquad (5.13)$$

where R_0, κ and m are the contacts depending on materials, shape and condition of the contact interface; $P_{\text{к}}$ is the value of the contact pressure.

As it is seen from expression (5.13) the contact resistance for given contact interface is mainly dependent on contact pressure. Typical experimental dependence between contact resistance and contact pressure is shown in Figure 5.3.

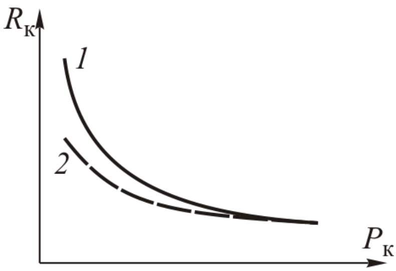


Figure 5.3 – Typical dependence contact resistance from contact pressure

In the region of small contact pressures the contact resistance is highly dependent on contact pressure. It is explained by the fact that conditions of bearing here are more favorable and small increase in contact pressure leads to substantial increase in total actual contact area. At high contact pressures large enough actual contact area undergoes the force and the contact resistance increases insignificantly (see curve 1). With decrease in contact pressure the contact resistance increases by the curve 2 since the microridges are partially broken under plastic deformation.

The temperature in constriction region has substantial effect on the contact resistance since it effects resistivity of contact material. To determine more accurately temperature dependence of resistivity the temperature distribution in constriction region must be considered. In engineering practice simplified dependence is used as follows:

$$\rho = \rho_0 \left(1 + \frac{2}{3} \alpha \vartheta_{\text{к}} \right). \tag{5.14}$$

Then, contact resistance with taking into account the temperature dependence of resistivity can be evaluated by the following formula:

$$R_{\text{к}} = R_{\text{к}0} \left(1 + \frac{2}{3} \alpha \vartheta_{\text{к}} \right). \tag{5.15}$$

The factors considered effecting on contact resistance are valid only for case the contact interface is *brightly scraped*, when constriction of the current is practically once cause of available contact resistance. However, in actual conditions of production, transportation, storage and servicing the contact interface is exposed by its surrounding. It results in formation of surface layer on the contact face (surface film). Its properties may be drastically distinctive on the properties of the metals producing the contact interface. It is first and foremost concerned such property as electrical conductivity. In this case constriction of current lines is important, but not once cause of available contact resistance. Availability of surface films on the contact face in some cases increases the contact resistance in tens of thousands times. It in turn results in additional release of heat in the contacts, reduction of reliability and service life, and in some cases full failure of the contacts.

Formation of films on the metal surface is most frequently caused by the corrosion in metal. *Corrosion* is usually defined as the disintegration of a material into its constituent atoms due to chemical reactions with its surroundings. In the most common use of the word, this means a loss of electrons of metals reacting with water and oxygen. The main factors defining the rate of corrosion processes in metals are humidity of atmosphere, its pollution by dust and gases as well as the temperature on the contact face. Certain metals, when contacting with oxygen of air, are coated by thin oxide film (thickness of order 10^{-6} sm, resistivity of order 10^5 Ohm·sm), which inhibits further disintegration of the metal. Noble metals are little or full not exposed by corrosion and, therefore, are frequently used as contact material. The hydrogen-sulfide medium produces sulfide films on the surface of metals.

In parallel with corrosion processes, formation of films may be resulted from *absorption* and *adhesion*, that is, taking up and deposition on contact faces of substances having frequently high enough resistivity. For example:

- deposition of evaporation products: solid insulation of coil wire, coil forms etc;

- deposition of arc thermal action products: fly ash, thermal black, products of contact electrical erosion etc;

- pollution by the products of mechanical wear resulted from friction of apparatuses pieces, factory dust etc.

Wide variety of processes resulting in formation of surface films does not enable to determine accurately enough contact resistance with considering resistance of surface films. This is why in engineering practice it is taken into account basing upon experimental investigation and mathematical statistic methods.

In the contacts designed to operate at high currents the surface films are usually mechanically failed. If the contact pressure is low enough, the film may be failed under action of voltage drop as result of electrical disruption with formation of thin metal bridge. This phenomenon derived name *fritting*. Hence, for the contacts designed for operation under high currents production of surface films does not substantially effect on the contact operation and is not a problem of great concern.

For the contacts designed to operate at small currents and having small contact loads the surface films is grave problem since they do not fail when the contacts close. Therefore, in such cases engineering solutions of the contact units are usually lined to restrict production of surface films. One of effective technique to solve this problem is application of noble metals as contact materials such as silver, gold, platinum and others. Another method to reduce formation of surface films is encapsulation of contacts. Hermetically sealed and magnetically operated contacts of reed switches may exemplify it. This technology enables to stabilize contact resistance since environment exposure (including active medium) is precluded. Encapsulation of contacts enables eliminate exposure of environment (including active one) on the contact interface and stabilize the value of the contact resistance. There are also other constructive techniques to reduce influence of surface films on the contacts operation. It is first and foremost application of so-called *self-scraped bright (wiping) contacts*.

5.3 Heating contacts under continuously flowing current

As already noted the power electric and electronic equipment contains electrical contacts, which are additional thermal sources and are correspondingly most thermally loaded components. Hence, decision and optimization theirs thermal behavior is vital problem in the process of power equipment development. Available contact resistance results in, at first, releasing Joule's heat. Along with it, in closed contacts additional thermal losses resulted from thermo-electrical effects (Thomson, Peltier and Kohler effects) may take place. However, it should be noted that they significantly influence on thermal behavior of the contacts passed through by very small currents.

Thermal behavior analysis of contacts, evaluation of theirs temperature is generally formidable task since contacts are frequently the element of complex current-carrying system, wherein the equalizing heat fluxes occur. Let's discuss simplest example of the contact system that is two half-infinite rods pressed to each other by the end faces as shown in Figure 5.4.

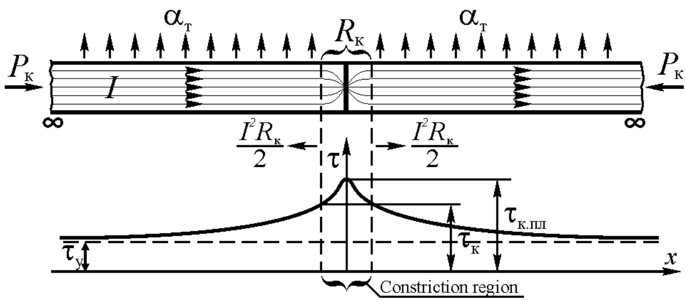


Figure 5.4 – Temperature distribution at the contacts formed by half-infinite rods

Within the constriction region the heat transfer to environment does not occur. Thermal power released here $I^2R_к$ is driven out to the rods by conduction. The contact system is in this case symmetrical; hence, half of the heat is driven out to one rod while other half to another. From the side surfaces of the rods the heat is evenly dissipated to its surroundings. The task of thermal calculations is here to determine the temperature-rises at the boundary of constriction region $\tau_к$ and at the contact spot $\tau_{к.пл}$.

At first let's discuss thermal behavior of half-infinite rod. As already noted, distribution of temperature with length of half-infinite conductor is describe by differential equation of second order as follows:

$$\frac{d^2\tau}{dx^2} - \frac{p\alpha_т}{\lambda q}\left(\tau - \tau_y\right) = 0, \quad (5.16)$$

where τ_y is the temperature-rise of the rod surface at infinitely far point; it is expressed as:

$$\tau_y = \frac{I^2\rho}{\alpha_т pq}, \quad (5.17)$$

where p, q are the heat dissipative perimeter and cross-sectional area of the rod, respectively; λ is the heat conductivity of the rod's material; $\alpha_т$ is heat transfer coefficient from the rod surface.

Solution of this equation at the following initial conditions:

$\tau(0) = \tau_к$; $\frac{I^2 R_к}{2} = -\lambda q \frac{d\tau(0)}{dx}$ will be:

$$\tau = \tau_y + \frac{I^2 R_к}{2\sqrt{\lambda q \alpha_т p}} e^{-bx}, \quad (5.18)$$

where $b = \sqrt{\frac{p\alpha_т}{\lambda q}}$.

This solution at $x = 0$ give expression for the temperature-rise at the boundary of constriction region:

$$\tau_к = \tau_y + \frac{I^2 R_к}{2\sqrt{\lambda q \alpha_т p}}. \quad (5.19)$$

To find the temperature at the contact spot it is necessary to determine temperature distribution within constriction region. For this purpose spherical model of contact is used. For outlined elemental spherical layer *dr* at the distance *r* from the contact spot (see Figure 5.2) the following heat balance equation will be valid:

$$\Phi_1 + \Phi_2 = \Phi_3, \quad (5.20)$$

where Φ_1 is the heat flux coming into the spherical layer; Φ_2 is thermal energy released within the layer; Φ_3 is the heat flux going out from the layer.

The terms of the heat balance equation will be expressed as:

$$\Phi_1 = -2\pi r^2 \frac{d\tau}{dr}; \; \Phi_2 = \frac{I^2\rho}{2\pi r} dr; \; \Phi_3 = -2\pi(r + dr)^2 \frac{d}{dr}\left(\tau + d\tau\right). \quad (5.21)$$

Substituting these expressions into the heat balance equation (5.20) and neglecting by infinitesimal quantities of second order, the differential equation describing the temperature distribution in constriction region will be derived. It is as follows:

$$\frac{d^2\tau}{dr^2}+\frac{2}{r}\frac{d\tau}{dr}+\frac{I^2\rho}{4\pi^2\lambda r^2}=0. \qquad (5.22)$$

Integration of this equation with boundary conditions:

$$\tau(\infty)=\tau_{\text{к}};\ \Phi(\infty)=-2\pi\lambda r^2\left.\frac{d\tau}{dr}\right|_{r=\infty}=\frac{I^2R_{\text{к}}}{2}, \qquad (5.23)$$

gives expression for the temperature distribution in constriction region:

$$\tau=-\frac{I^2\rho}{8\pi^2\lambda r^2}+\frac{I^2R_{\text{к}}}{4\pi\lambda r}+\tau_{\text{к}}. \qquad (5.24)$$

The temperature-rise of the contact spot is determined by substitution of the radius of the contact spot $r = a$ expressed by the constriction resistance for the spherical model: $R_{\text{к}}=\frac{\rho}{\pi a}$. This substitution gives:

$$\tau_{\text{к.пл}}=\frac{I^2R_{\text{к}}^2}{8\rho\lambda}+\tau_{\text{к}}=\frac{U_{\text{к}}^2}{8\rho\lambda}+\tau_{\text{к}}. \qquad (5.25)$$

Derived expression is the Holm-Kohlrausch formula. Thus, the temperature-rise of the contact spot is expressed as:

$$\tau_{\text{к.пл}}=\tau_{\text{у}}+\frac{I^2R_{\text{к}}}{2\sqrt{\lambda q\alpha_{\text{т}}p}}+\frac{I^2R_{\text{к}}^2}{8\rho\lambda}. \qquad (5.26)$$

Thermal behavior of contacts is required of such that their temperature would not result in recrystallization of contact material that in turn lead to softening and cold welding of the contacts. By that why the temperature in the neighborhood of contacts is specified by standards and confirmed by the temperature-rise tests.

As it was already noted, contacts of power electric and electronic equipment are usually the component of complicated current carrying system. In such cases, as differentiated from discussed above one, the contact system does not considered as symmetrical one since the contact is generally may consist of two dissimilar components with different cross-sectional area and distinctive materials. Moreover, heating the contacts will be also defined by equalizing heat fluxes resulted from non-homogeneity of the current-carrying system, namely, configuration its components and conditions their heat exchange with environment. As the result the temperatures at the boundaries of such contact constriction region will be in general case different. To analyse thermal behavior of inhomogeneous current-carrying system with available contacts it is important to determine expressions for equalizing heat fluxes at the boundaries of the contact constriction region. To make it, set of differential equations (5.55) describing temperature distribution within constriction region is used. The boundaries in this case will be as follows:

$$\begin{cases} \tau_1(\infty)=\tau_{\text{к}1}; \quad \tau_2(\infty)=\tau_{\text{к}2}; \quad \tau_1(a)=\tau_2(a)=\tau_{\text{к.пл}}; \\ -2\pi\lambda_1 r^2 \left.\dfrac{d\tau_1}{dr}\right|_{r=\infty} -2\pi\lambda_2 r^2 \left.\dfrac{d\tau_2}{dr}\right|_{r=\infty} = I^2 R_{\text{к}}. \end{cases} \qquad (5.27)$$

where τ_1 and τ_2 are the temperature-rises within constriction regions of the components forming given contact; $\tau_{\text{к}1}$ and $\tau_{\text{к}2}$ are the temperatures at the boundaries of the contact constriction region; a is radius of the contact spot; $R_{\text{к}}$ is the contact resistance expressed in accordance with spherical model for the contacts made of dissimilar materials as follows

$$R_{\text{к}} = \frac{\rho_1+\rho_2}{2\pi a}, \qquad (5.28)$$

where ρ, λ are resistivity and thermal conductivity of the contact materials.

Solution of such set of differential equations with boundary conditions (5.27) give following expressions for equalizing heat fluxes at the boundaries of the contact constriction region:

$$\begin{cases} \Phi_1(\infty)=s_1-s_{12}(\tau_1-\tau_2); \\ \Phi_2(\infty)=s_2+s_{12}(\tau_1-\tau_2). \end{cases} \qquad (5.29)$$

where

$$\begin{cases} s_1=\dfrac{I^2R_{\text{к}}}{\lambda_1+\lambda_2}\lambda_1 \; 1-\lambda_2 G_{12} \; ; s_2=\dfrac{I^2R_{\text{к}}}{\lambda_1+\lambda_2}\lambda_2 \; 1-\lambda_1 G_{12} \; ; \\ s_{12}=\dfrac{\lambda_1\lambda_2 \;\; \rho_1+\rho_2}{\lambda_1+\lambda_2 \;\; R_{\text{к}}}; G_{12}=\dfrac{1}{2 \;\; \rho_1+\rho_2}\left(\dfrac{\rho_2}{\lambda_2}-\dfrac{\rho_1}{\lambda_1}\right). \end{cases} \qquad (5.30)$$

Temperature-rise of the contact spot in this case will be determined according to the following expression:

$$\tau_{к.пл} = \frac{I^2 R_к^2}{2(\lambda_1 + \lambda_2)(\rho_1 + \rho_2)} + \frac{\lambda_1 \tau_{к1} + \lambda_2 \tau_{к2}}{\lambda_1 + \lambda_2}. \qquad (5.31)$$

5.4 Stationary thermal behavior of inhomogeneous current-carrying systems

As already noted, actual current-carrying systems of power electric and electronic equipment are in most cases inhomogeneous in thermal respect. They may contain complex in shape current-carrying components, including contact-pieces, with significantly distinctive and frequently unfavorable cooling conditions as well as switching contacts, contact joints, power semiconductor devices etc. responsible for additional heat evolution. It should be noted here that each current-carrying system has own structure depending on constructive features of equipment. Therefore, thermal behavior analysis of inhomogeneous current-carrying system can be letter discussed by particular example. Such example can provide stripping contact unit of on-load tap changer selector shown in Figure 5.5.

It contains such main components: strips 1; fixed contact 2 fastened on insulating pillar 3 and connected with corresponding transformer winding tap via busduct 4; current-collecting ring 5 fastened on insulating cylinder 6 and connected with contactor via busduct 7.

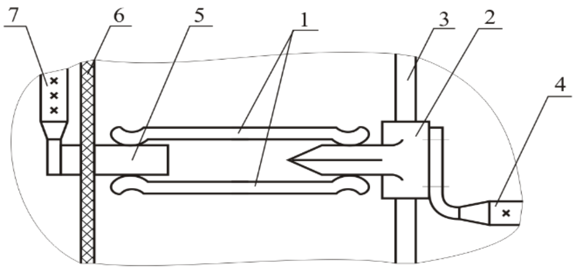


Figure 5.5 – Current-carrying system of on-load tap changer selector

To solve the task of thermal behavior analysis the current-carrying system is subdivided into series sections so that each has equable geometrical configuration and material, invariable current density and homogeneous conditions of heat exchange with its surrounding. This subdivision results in thermal equivalent scheme shown in Figure 5.6.

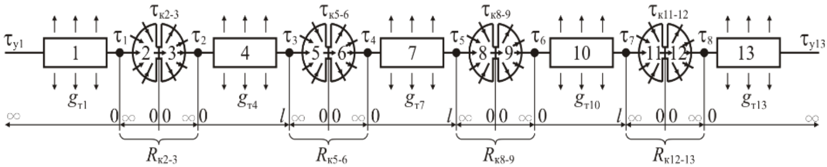


Figure 5.6 – Thermal equivalent scheme of the selector current-carrying system

Just represented equivalent scheme includes a few sections that are homogeneous elements:

1 is the busduct connecting selector with contactor 7 (see Figure 5.5); it is considered as half-infinite section;

4 is the current-collecting ring 5 (see Figure 5.5);

7 is the strips 1 (see Figure 5.5);

10 is fixed contact 2 (see Figure 5.5).

Sections 7, 10 and 13 are considered as the ones with certain (finite) length.

13 is the busduct connecting selector with transformer 4 (see Figure 5.5) is accepted by half-infinite section.

Equivalent scheme (see Figure 5.6) also contains switching contacts and contact joints represented by constriction regions described by spherical model:

2–3 is the contact joint between sections 1 and 4;

5–6 is sliding contact between sections 4 and 7;

8–9 is switching contact between sections 7 and 10;

11–12 is the contact joint between sections 10 and 13;

$\tau_1 - \tau_8$ are the temperature-rises at the nodal points of current-carrying system (at the boundaries of the sections and the constriction regions).

Solution of the task is based upon conditions of temperature field continuity. To realize this principle the expressions for heat fluxes at the boundaries of the sections and constriction regions are required. They are gained from expressions (4.43), (4.46), (5.29).

$$\left.\begin{aligned}
&\Phi_1(0)=a_1\lambda_1 q_1(\tau_1-\tau_{y1});\\
&\Phi_2(\infty)=s_2-s_{23}(\tau_1-\tau_2);\quad \Phi_3(\infty)=s_3+s_{23}(\tau_1-\tau_2);\\
&\Phi_4(0)=u_4\tau_2-v_4\tau_3+s_4;\quad \Phi_4(l_4)=u_4\tau_2+v_4\tau_3-s_4;\\
&\Phi_5(\infty)=s_5-s_{56}(\tau_3-\tau_4);\quad \Phi_6(\infty)=s_6+s_{56}(\tau_3-\tau_4);\\
&\Phi_7(0)=u_7\tau_4-v_7\tau_5+s_7;\quad \Phi_7(l_7)=u_7\tau_4+v_7\tau_5-s_7;\\
&\Phi_8(\infty)=s_8-s_{89}(\tau_5-\tau_6);\quad \Phi_9(\infty)=s_9+s_{89}(\tau_5-\tau_6);\\
&\Phi_{10}(0)=u_{10}\tau_6-v_{10}\tau_7+s_{10};\quad \Phi_{10}(l_{10})=u_{10}\tau_6+v_{10}\tau_7-s_{10};\\
&\Phi_{11}(\infty)=s_{11}-s_{11-12}(\tau_7-\tau_8);\quad \Phi_{12}(\infty)=s_{12}+s_{11-12}(\tau_7-\tau_8);\\
&\Phi_{13}(0)=a_{13}\lambda_{13}q_{13}(\tau_8-\tau_{y13}).
\end{aligned}\right\}\qquad (5.32)$$

In the expressions represented, the indexes denote the number of the section or constriction region according to equivalent scheme, Figure 5.6.

The expressions defining parameters for the constriction regions are as follows:

$$\left.\begin{aligned}
&s_i=\frac{I^2R_{\text{к}\,i,i+1}}{\lambda_i+\lambda_{i+1}}\lambda_i(1-\lambda_{i+1}G_i)\\
&s_{i+1}=\frac{I^2R_{\text{к}\,i,i+1}}{\lambda_i+\lambda_{i+1}}\lambda_{i+1}(1+\lambda_i G_i)\\
&s_{i,i+1}=\frac{\lambda_i\lambda_{i+1}(\rho_i+\rho_{i+1})}{R_{\text{к}\,i,i+1}(\lambda_i+\lambda_{i+1})}\\
&G_i=\frac{1}{2(\rho_i+\rho_{i+1})}\left(\frac{\rho_{i+1}}{\lambda_{i+1}}-\frac{\rho_i}{\lambda_i}\right)
\end{aligned}\right\} i=2,5,8,11;\qquad (5.33)$$

The expressions defining parameters for the current-carrying components are as follows:

$$\left.\begin{aligned}
&a_i=\sqrt{\frac{1}{\lambda_i q_i}\left(g_{\text{т}i}-\frac{I^2\rho_{0i}\alpha_i}{q_i}\right)}\\
&\tau_{yi}=\frac{I^2\rho_{0i}(1+\alpha_i\vartheta_0)}{g_{\text{т}i}q_i-I^2\rho_{0i}\alpha_i}
\end{aligned}\right\} i=1,4,7,10,13,\qquad (5.34)$$

where I is the load current continuously flowing through the current-carrying system; ϑ_0 is the cooling medium temperature; $R_{\text{к}.i,i+1}$ is the contact resistance of the constriction regions according to equivalent scheme shown in Figure 5.6; λ_i, ρ_{0i}, α_i are material heat conductivity, resistivity and temperature coefficient of resistivity of *i*-th section of the current-carrying system including the contact constriction regions, respectively; g_{Ti} is specific thermal conductance from the current-carrying component surface (i = 1, 4, 7, 10, 13); q_i is cross-sectional area of the current-carrying component (i = 1, 4, 7, 10, 13); l_i is the current-carrying component length (i = 4, 7, 10).

Using condition of the temperature field continuity, that is, the equality of the heat fluxes at boundaries of the sections and constriction regions:

$$\left.\begin{aligned} &\Phi_1(0)=\Phi_2(\infty); \Phi_3(\infty)=\Phi_4(0);\\ &\Phi_4(l_4)=-\Phi_5(\infty); \Phi_6(\infty)=\Phi_7(0);\\ &\Phi_7(l_7)=-\Phi_8(\infty); \Phi_9(\infty)=\Phi_{10}(0);\\ &\Phi_{10}(l_{10})=-\Phi_{11}(\infty); \Phi_{12}(\infty)=\Phi_{13}(0) \end{aligned}\right\} \quad (5.35)$$

gives set of linear algebraic equations of three-diagonal type:

$$\left.\begin{aligned} &a_{11}\tau_1+a_{12}\tau_2 = b_1\\ &a_{21}\tau_1+a_{22}\tau_2+a_{23}\tau_3=b_2\\ &\dots\dots\dots\dots\dots\dots\\ &a_{76}\tau_6+a_{77}\tau_7+a_{78}\tau_8=b_7\\ &a_{87}\tau_7+a_{88}\tau_8=b_8 \end{aligned}\right\} \quad (5.36)$$

where
$$\left.\begin{aligned} &a_{11}=a_1\lambda_1 q_1+s_{23};\quad a_{12}=-s_{23};\quad b_1=s_2+a_1\lambda_1 q_1;\\ &a_{21}=s_{23};\quad a_{22}=-s_{23}-u_4;\quad a_{23}=u_4\quad b_2=s_4-s_3;\\ &\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ &a_{87}=s_{11-12};\quad a_{88}=-s_{11-12}-a_{13}\lambda_{13}q_{31};\quad b_8=s_{12}-a_{13}\lambda_{13}q_{31}. \end{aligned}\right\}. \quad (5.37)$$

From solution of the set derived, the temperature-rises at the nodal points (at the boundaries of the sections and the constriction regions) are found.

Basing upon the described procedure, relevant computer software realizing solution of the task to determine steady thermal conditions of current-carrying system can be developed. To improve accuracy of calculations, solution of this task is to be performed by sequential approximations with following sequence:

1) initial temperature-rises at the nodal points τ_i are predetermined;

2) specific thermal conductance between the conductor surface and its

surrounding $g_{\text{т}i}$ for each section (except constriction regions) is calculated;

3) set of equations (5.36) is solved and new approximations of temperature-rises at the nodal points τ_i are determined and are compared with former approximations;

4) if the difference between new and former approximations in excess of the value predetermined, then calculations according items 2 and 3 are to be repeated using new approximations of temperature-rises at the nodal points; when this difference is less than this value, then calculations are over; their outcomes are the last approximations of temperature-rises at the nodal points;

5) using calculated temperature-rises at boundaries of the constriction regions, temperature-rises at the contact spots according to expression (5.31) are calculated;

6) basing upon resulting temperature-rises at the nodal points according to expressions (4.42) and (4.45) distribution of temperature throughout the current-carrying system length can be obtained

Average temperature-rise within a constriction region is determined according to following relationship [5, 17]:

$$\tau_{i\text{ср}} = \tau_j + \frac{2}{3}\left[\tau_{\text{к.пл}\,i,i+1} - \tau_j\right], \tag{5.38}$$

where τ_j is the temperature-rise at boundary of given constriction region; $\tau_{\text{к.пл}i,i+1}$ is the temperature-rise at the contact spot between *i*-th and (*i*+1)-th constriction region.

5.5 Operation of contacts under short-circuit conditions

Passing sc current (a few or even tens kiloamperes) through closed contacts may lead to their failure and failure of the equipment. Therefore, the requirement of stability (survival) under passage of sc currents is quite reasonable. The sc currents have on the contacts at first *thermal action* resulting in elevated heating compared with normal circumstances. It leads frequently to melting the contact spot and welding the contacts. The sc currents have on the contacts *electrodynamic action* as well occurred with forces that tend to part the contacts. It usually leads to falling off contact pressure, and in certain cases to momentary opening the contacts resulting in arcing and welding the contacts after closing.

Thermal action of sc current: The passage of sc current through the closed contacts involves in runaway increase in voltage drop across the contacts as well as heat evolution in the constriction region. It is resulted not only from high value of sc current, but due to increase in contact resistance because of significant elevation of the temperature in the neighborhood of the contact. Increase in heat evolution may lead to the temperature reaches melting point of contact material at the contact spot. Because of metal melting the surface films disappear (are burned through) in the neighborhood of the contact zone. Dimensions of the contact spot are increased that involve in runaway decrease in contact resistance. With this, heat evolution in the contacts somewhat decrease, because of this cooling down and welding the contacts occur.

For contacts certain degree of welding under sc current is permissible. It is featured by the force or mechanical work required to break away the contacts of one another. The more breaking away force, the more degree of welding is permissible. The current resulting in welding the contact spot when force required to open the contacts equals zero is named *initial melting current* $I_{пл}$. There exists more definite concept: *initial welding current* $I_{св}$. It is the current that leads to a minimal degree of welding when comparatively small force to break away the contacts is required (usually in the range 1 to 2 N).

It should be noted that value of initial welding current is dependent on time of its passing. The more duration of initial welding current, the less its value is. By this why there is more definite concept used in engineering practice; it is *minimal welding current* $I_{св\infty}$, that is, the current, continuous passing of which, results in the same effect (minimal degree of welding).

To determine the minimal welding current the equation as per Ohm’s law can be used:

$$I_{св.\infty} = \frac{U_{к.св}}{R_к}, \tag{5.39}$$

where $U_{к.св}$ is voltage drop across the contacts according to the minimal welding current; $R_к$ is transition contact resistance, which in this case may be expressed by Holm’s formula

$$R_к = \frac{\rho}{2a_{пл}}, \tag{5.40}$$

where $a_{пл}$ is radius of the contact spot.

Basing upon the Holm-Kohlrausch formula derived above, it can be written as follows:

$$\vartheta_{пл} - \vartheta_{к} = \frac{U_{к.св}^2}{8\rho\lambda}, \qquad (5.41)$$

Since $\vartheta_{пл} >> \vartheta_{к}$, then:

$$U_{к.св} = \sqrt{8\rho\lambda\vartheta_{пл}}. \qquad (5.42)$$

Substitution of expressions (5.84) and (5.86) into Ohm's law equation (5.83) gives expression for minimal welding current:

$$I_{св.\infty} = a_{пл}\sqrt{\frac{32\vartheta_{пл}\lambda}{\rho}}. \qquad (5.43)$$

It may be pointed out here that dimensions of the contact spot due to melting the metal increases approximately by 1.5–2 times in comparison with normal circumstances.

The value of minimal welding current is indicator of stability of the contacts to thermal action of sc current. The more minimal welding current is, the higher will be stability of the contacts. Gained expression shows that to increase in minimal welding current, and hence, stability of the contacts to thermal action of sc current the materials with high as possible melting point, heat and electrical conductivity is to be at first used. Furthermore, stability of the contacts is dependent on the radius of the contact spot, and hence, on the contact pressure. It may be pointed out here that gained expression may be used only for rough approximations since deals with idealized models of contacts. For practical engineering calculations in respect to welding the empirical formulas are applicable like as follows:

$$I_{св} = \kappa\sqrt{P_{к}}, \qquad (5.44)$$

where κ is the coefficient taking into account physical properties of the contacts material as well as constructional features of the contact unit.

The contacts welding under sc current will be avoided, if the value of minimal welding current gained by calculations (or experimentally) (is) more than or equal to initial rmc sc current.

6. THERMAL BEHAVIOUR ANALISYS OF ELECTRICAL EQUIPMENT COILS

6.1 General

A coil is one of main components of electrical equipment. In particular, it is an integral part of equipment's electromagnetic systems, such as, power and instrument transformers, electrical reactors, electromagnetic actuators for switching apparatus etc. Its functional purpose is to produce magneto-motive force (mmf) of prescribed value determined from analysis of the magnetic circuit.

The task of the coil electromagnetic analysis is to determine basic winding data: the number of turns and cross-sectional area of the wire (type from the wire gage) at given values of mmf and power voltage (or current through the coil).

Depending on the manner of connection into electric circuit the coils are shared into two variations: shunt (parallel) coils connected across whole power voltage and series (current) coils inserted into circuit on predetermined value of current. Depending on the type of current ac and dc coils are distinguished.

Circular enameled-isolated wire is usually applied for the magnet series coils. The current and power equipment coils are usually made of bus with rectangular cross-section. As conductor material for coils cooper is used more often than aluminum. Depending on constructional version formed coils wound on rigid insulating form and unformed coils wound directly on a magnet core are distinguished. In order to enhance mechanical rigidity the magnet coils are impregnated with compound. The coils of bus may be wound by net spiral or flat spiral. Let us consider the basic relations used for calculations of winding data.

For dc windings of parallel connection the initial data for electromagnetic calculations are mmf (*Iw*) and supply voltage *U*. The basic relation for calculations is the Ohm's law equation:

$$I = \frac{U}{R}, \tag{6.1}$$

where *R* is pure resistance of the winding wire:

$$R = \rho_{\vartheta} \frac{l_{\text{в.ср}}}{q} w, \tag{6.2}$$

where ρ_ϑ is resistivity of the winding wire material at operational temperature ϑ; $l_{\text{в.ср}}$ is average length of the coil turn; *w* is number of the winding turns.

Substituting expression (6.2) into Ohm's law equation (6.1) and multiplying both sides by *w* gives:

$$(Iw) = \frac{Uqw}{\rho_\vartheta l_{\text{в.ср}} w}. \qquad (6.3)$$

From this expression required cross-sectional area of the winding wire can be determined

$$q = \frac{(Iw)\rho_\vartheta l_{\text{в.ср}}}{U}. \qquad (6.4)$$

For the coils wound of round-sectional wire required diameter of the wire (without insulation) will be expressed as:

$$d = \sqrt{\frac{4(Iw)\rho_\vartheta l_{\text{в.ср}}}{\pi U}}, \qquad (6.5)$$

Using the calculated value cross-sectional area or diameter by the formula (6.4) or (6.5), respectively, the winding wire is selected and values *d* and *q* are to be refined.

The number of turns of the winding is determined basing upon a total cross-sectional area of the winding space Q_0

$$w = \frac{f_o Q_o}{q}, \qquad (6.6)$$

where f_o is space factor of winding; it is a ratio of the total cross-sectional area of the winding conducting material to the total cross-sectional area of the winding.

Space factor of a winding is one of important constructional indexes defining efficiency of winding space usage. Its value is expressed by multiplication of layering coefficient $\kappa_{\text{у}}$ and coefficient of insulation $\kappa_{\text{из}}$.

$$f_o = \kappa_{\text{у}} \cdot \kappa_{\text{из}}. \qquad (6.7)$$

The layering coefficient is defined by the type of winding, which, in turn, depends on diameter of the winding wire. Manufacturing electromagnet coils occurs with application of following winding types [6]:

- *layer winding*, as the turns are arranged by layers; it is applied in the case when diameter of the winding wire in the range of 0.3 to 0.8 mm; then $\kappa_{\text{у}} = 0.9\ldots0.95$;

- *stagger winding*, as the turns are staggered; it is applied in the case when diameter of the wire is in excess of 0.8 mm; then $\kappa_{\text{у}} = 0.95\ldots1.05$;

- *random winding*, as the turns are not arranged in strict order; it is applied at small (less than 0.3 mm) diameter of the wire; then $\kappa_{\text{у}} = 0.7\ldots0.9$.

Insulation coefficient is dependent on thickness of the insulation and diameter of the winding wire. Increase in thickness of the insulation results in decrease of insulation coefficient. Enameled wires have most small thickness of insulation. At small wire diameter, that is, great number of turns per one unit of cross-sectional area of winding space, insulation of the wire occupies more area.

As already noted, to calculate the winding data, geometrical dimensions of the coil, such as, average length of the coil turn $l_{\text{в.ср}}$ and cross-sectional area of the winding space Q_0 are required to be predetermined. These values can be determined as result of short-cut thermal calculation with the help of heat balance equation. Such approach in some degree complicates calculations of winding data. To perform calculations using permissible current density $j_{\text{доп}}$ is considerably simpler. It is known for the number of practical cases. For example, for the current flowing continuously through the coil with natural air cooling, permissible value current density can be presumed in the range of 2 to 4 A/mm^2.

Number of the coil turns is determined from Ohm's law equation expressed as

$$U = I \cdot R = j_{\text{доп}} q \frac{\rho_{\vartheta} l_{\text{в.ср}} w}{q}. \qquad (6.8)$$

Accordingly:

$$w = \frac{U}{j_{\text{доп}} \rho_{\vartheta} l_{\text{в.ср}}}. \qquad (6.9)$$

Then, required cross-sectional area of the winding space and, respectively, dimensions of "magnet circuit window" will be determined as follows

$$Q_{\text{о}} = \frac{qw}{f_{\text{о}}}. \qquad (6.10)$$

For ac parallel coils the number of turns is determined from well-known equation of ac magnetic circuit:

$$w = \frac{U}{4.44 \cdot f \cdot \Phi_m}, \qquad (6.11)$$

where U is rms supply voltage of the coil; Φ_m – is amplitude value of magnetic flux that linkages with the coil; f – is frequency of the power network.

Active cross-sectional area of the wire can be in this case determined from permissible value of current density. However, it should be pointed out here that current value (and hence current density) in a coil is dependent on total reluctance of the magnetic circuit. If the magnetic circuit contains air gaps varying when the apparatus operates, then calculations are to be performed for those values, at which the coil is continuously flowed by current. For example, for electromagnet devices current and cross-sectional area of the wire are calculated for attracted position of the armature because at the initial its position (when the gap is maximal), the coil is momentary flowed by starting current when the electromagnet is actuated. In this case the coil is almost net inductive resistance, and, therefore, its pure resistance can be in first approximation neglected and then current through the coil is

$$I = \frac{U}{\omega L}, \qquad (6.12)$$

where L is inductance of the coil defined by relation of its linkage flux to the current

$$L = \frac{\Psi_m}{I_m}, \qquad (6.13)$$

At small values of air gaps leakage flux can be neglected and linkage flux can be determined as the product of magnetic flux to number of the coil turns

$$\Psi_m = w\Phi_m, \qquad (6.14)$$

Substitution of expression (6.14) in (6.13) and multiplication its numerator and denominator by number of coil turns w gives

$$L = \frac{w\Phi_m}{I_m} \cdot \frac{w}{w} = \Lambda_\Sigma w^2, \qquad (6.15)$$

where Λ_Σ is the total permeance of the magnetic circuit including permeances of air gaps.

Substituting expression (6.15 into (6.12), we obtain:

$$j_{доп} q = \frac{U}{\omega \Lambda_{\Sigma} w^2} . \tag{6.16}$$

Then, the expression for calculating required cross-sectional area of the winding wire will be as follows:

$$q = \frac{U}{j_{доп} \omega \Lambda_{\Sigma} w^2} , \tag{6.17}$$

Cross-sectional area of the winding space for ac coil will be calculated in the same manner.

Magneto-motive force (*Iw*) and current through the winding *I* are predetermined for calculations of the winding data for dc and ac series windings. In this case the number of the coil turns is calculated as follows:

$$w = \frac{(Iw)}{I} , \tag{6.18}$$

Active cross-sectional area of the winding wire can be here determined from permissible current density $j_{доп}$ for given heat exchange conditions of the coil and its surrounding as well as its operational duty.

$$q = \frac{I}{j_{доп}} , \tag{6.19}$$

Cross-sectional area of the winding space and respective dimensions of the “magnetic circuit window” are determined by the same way.

It should be noted that the winding data calculated are preliminary ones and to be refined by confirmative thermal analysis of the winding and electromagnetic system as a whole.

6.2 Thermal calculations of coils: main task and presumptions

The task of heat behavior analysis (thermal calculations) of the coil is usually to determine the temperatures on its surface and within the thickness. Its central problem is the fact that body of coil is inhomogeneous: metal wire covered by solid insulation layer, gaps between separate turns and layers of coil are available. Figure 6.1 shows cross-section of multilayer coil wound by round wire.

Basic presumption used to perform thermal calculations is substitution of actual coil body by so-called *equivalent body*. Complex real heat transfer process within the body of a coil is here substituted by simplified one, in which heat flux passes through homogeneous body with so-called *equivalent coil body heat conductivity* $\lambda_з$. It is usually found basing upon experimental data since its value substantially depends on several factors, such as, conductor material, its cross-sectional area, insulation material thickness and its material, impregnation within the coil thickness as well as quality of the winding (density, layering).

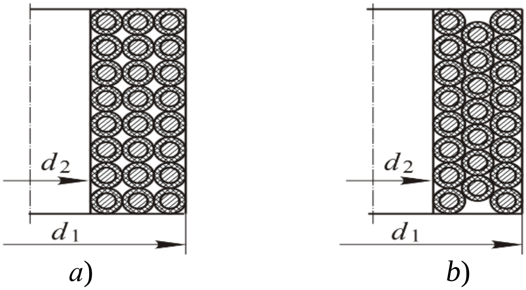


Figure 6.1 – Cross-sectional view of winding window of coil:
a) layer winding; *b*) stagger winding

For example, for non-impregnated layer coils wound of cooper round cross-sectional wire (Figure 6.1 *a*), heat conductivity of equivalent coil body can be determined by a following empirical formula [5]:

$$\lambda_з = 1.45\sqrt{\lambda_и \lambda_0 \left(\frac{d}{\delta} + 1\right)} - 1.57\lambda_0, \tag{6.20}$$

where $\lambda_и$ is heat conductivity of the winding wire insulation; λ_0 is heat conductivity of air; d is conductive diameter of the wire; δ is insulation thickness of the wire.

For impregnated coils [5]:

$$\lambda_з = \lambda_и \left(\frac{d}{\delta}\right)^{2/3}, \tag{6.21}$$

For stagger non-impregnated coils shown in Figure 6.1*b* heat conductivity of equivalent coil body is [5]

$$\lambda_з = 2.18\sqrt{\lambda_и \lambda_0 \left(\frac{d}{\delta} + 1\right)} - 1.33\lambda_0. \tag{6.22}$$

For impregnated coils [5]:

$$\lambda_{з} = 1.45\lambda_{и}\left(\frac{d}{\delta}\right)^{3/4}. \tag{6.23}$$

Rigorous theoretical consideration of steady-state temperature distribution in coil is generally formidable task. Therefore, to solve this problem, there is a number of simplifying presumptions as follows:

1) heat is equally released within all cross-section of the coil;

2) heat removal from coil end faces is not available or it is stated that heat released within the coil thickness is driven out to its surrounding from side surfaces only;

3) heat conductivity within the coil thickness is heat conductivity of its equivalent body $\lambda_{з}$;

4) influence of the coil temperature on its pure resistance and heat conductivity are neglected.

Thus, if the coil is wound on cylindrical form, as shown in Figure 6.1, it can be considered as homogeneous cylindrical wall (hollow cylinder) of infinite length with a uniform internal heat generation. In this case distribution of temperature within the coil thickness will be governed by heat conduction equation that in cylindrical coordinates will be as follows:

$$\frac{\partial^2\tau}{\partial r^2} + \frac{1}{r}\frac{\partial\tau}{\partial r} + \frac{q_V}{\lambda_{з}} = 0, \tag{6.24}$$

where τ is temperature-rise within the coil thickness over the ambient temperature; q_V is volumetric internal heat generation expressed as

$$q_V = j^2\rho f_o, \tag{6.25}$$

where j is current density in the coil wire; ρ is resistivity of the winding wire conductive material.

Put the general case that heat is driven out from both inner and outer surfaces of the coil. Then boundary conditions of third kind can be written as follows:

$$\frac{\tau_1}{r_{т1}} = -\lambda_{з}\left.\frac{\partial\tau}{\partial r}\right|_{r=R_1} \quad \frac{\tau_2}{r_{т2}} = -\lambda_{з}\left.\frac{\partial\tau}{\partial r}\right|_{r=R_2}, \tag{6.26}$$

where τ_1, τ_2 are temperature-rises on outer and inner surfaces of the coil, respectively; $r_{т1}$, $r_{т2}$ are total thermal resistances per length unit of the coil from its inner and outer surfaces, respectively; R_1, R_2 are outer and inner radius of the coil, respectively.

General solution of equation (6.24) will be as follows:

$$\tau = A \cdot \ln r - \frac{q_V}{4\lambda_з} r^2 + B, \qquad (6.27)$$

where A and B are the integration constants.

From general solution (6.27) expressions for temperature-rises on outer and inner surfaces of the coil can be written as follows:

$$\tau_1 = A \cdot \ln R_1 - \frac{q_V}{4\lambda_з} R_1^2 + B \quad \tau_2 = A \cdot \ln R_2 - \frac{q_V}{4\lambda_з} R_2^2 + B. \qquad (6.28)$$

Derived system of equations (6.28) makes it possible to find the integration constants:

$$A = \frac{1}{\lambda_з}\left[S(\tau_1 - \tau_2) + P\right]; \quad B = \tau_1 - \frac{\ln R_1}{\lambda_з}\left[S(\tau_1 - \tau_2) + P\right] + \frac{q_V}{4\lambda_з} R_1^2, \qquad (6.29)$$

where $S = \tau_1 - \dfrac{\lambda_з}{\ln \dfrac{R_1}{R_2}}$; $P = \dfrac{q_V}{4} \cdot \dfrac{R_1^2 - R_2^2}{\ln \dfrac{R_1}{R_2}}$

Using boundary conditions (6.26) gives:

$$\begin{aligned} \frac{\tau_1}{r_{T1}} &= -\lambda_з\left(\frac{A}{R_1} - \frac{q_V}{2\lambda_з} R_1\right) \\ \frac{\tau_2}{r_{T2}} &= -\lambda_з\left(\frac{A}{R_2} - \frac{q_V}{2\lambda_з} R_2\right) \end{aligned}. \qquad (6.30)$$

Substituting expression for integration constant A (6.29) into derived system of equations (6.30) and solving it relatively to temperature-rises on outer and inner surfaces of the coil gives:

$$\tau_1 = \frac{C_1 D_2 + D_1}{1 - C_1 C_2}; \quad \tau_2 = \frac{C_2 D_1 + D_2}{1 - C_1 C_2}, \qquad (6.31)$$

where $C_1 = \dfrac{S}{S + \dfrac{R_1}{r_{T1}}}$; $C_1 = \dfrac{S}{S + \dfrac{R_1}{r_{T1}}}$; $D_1 = \dfrac{R_1^2 \dfrac{q_V}{2} - P}{\dfrac{R_1}{r_{T1}} + S}$; $D_2 = \dfrac{R_2^2 \dfrac{q_V}{2} - P}{\dfrac{R_2}{r_{T2}} - S}$.

Basing upon general solution (6.28), one can define in general case a radius in the coil thickness body $R_{\text{макс}}$ corresponding to maximal temperature-rise, when the following condition is fulfilled

$$\left.\frac{\partial\tau}{\partial r}\right|_{r=R_{\text{макс}}} = 0\,, \qquad (6.32)$$

which gives:

$$R_{\text{макс}} = \sqrt{\frac{2}{q_{\text{V}}}\left[S(\tau_1 - \tau_2) + P\right]}\,, \qquad (6.33)$$

Maximal temperature-rise within a coil thickness body is determined from general solution (6.28)

$$\tau_{\text{макс}} = A \cdot \ln R_{\text{макс}} - \frac{q_{\text{V}}}{4\lambda_{\text{з}}} R_{\text{макс}}^2 + B\,. \qquad (6.34)$$

It may be pointed out that calculations of temperature-rises τ_1 and τ_2 are to be performed by sequential approximations as thermal resistances $r_{\text{т1}}$ and $r_{\text{т2}}$ are their complex functions.

6.3 Thermal behavior calculations of dc coils

Thermal behavior of coils generally depends on conditions of heat exchange with its surroundings. In case of direct current heat is usually driven out both from outer and inner surfaces of the coil.

Outer surface of the coil is as a rule in direct contact with environment; heat is generally transferred by convection and radiation. Hence total thermal resistance from outer surface of the coil will be expressed as

$$r_{\text{т1}} = \frac{1}{\alpha_{\text{т1}} p_1}\,, \qquad (6.35)$$

where $\alpha_{\text{т1}}$ is total heat transfer coefficient from outer surface of the coil; p_1 is outer perimeter of the coil from which heat is driven out to environment.

In electric equipment a coil is in most cases placed on ferromagnetic core. Therefore, heat from its inner surface is driven out to environment in more complex manner. In dc electromagnetic systems a core in steady state is not thermal source (internal heat generation is not available). In such systems heat flux from inner surface of the coil is transferred into the core, and then is dissipated to environment from its uncovered surface. It should be noted that contact between inner surface of the coil and the core is not perfect. Usually an air-gap, wall of the coil form and other factors setting up considerable resistance to heat flux are between them, which in most cases are to be taken into account.

Thus, total thermal resistance between inner surface of the coil and environment will be added of two components:

$$r_{т2} = r_{т.к-с} + r_{т.с-о}\,, \tag{6.36}$$

where $r_{т.к-с}$ is thermal resistance between inner surface of the coil and the core; $r_{т.с-о}$ is thermal resistance between the core and its surroundings.

Thermal resistance between the inner surface of cylindrical coil and the core is mostly determined as follows:

$$r_{т.к-с} = \frac{2\pi}{\lambda_{сл} \ln \dfrac{R_2 + \delta}{R_2}}\,, \tag{6.37}$$

where $\lambda_{сл}$ is heat conductivity of the gap material between inner surface of the coil and the core; if an air gap is available convective component is to be taken into account in this value (see section 1); δ is the gap thickness.

Thermal resistance between the core and its surroundings is most often determined with the help of a simplified model, where the coil is placed on straight core with thermally isolated ends as shown in Figure 6.2 [5, 17].

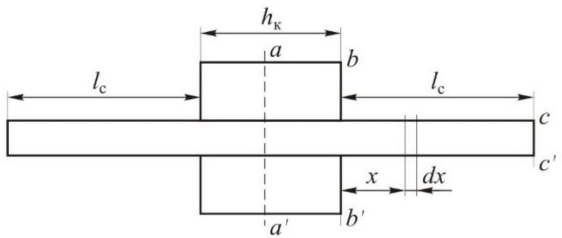


Figure 6.2 – Computational scheme for the coil with the core

Given model is applicable in the case the core is closed and is symmetric relatively to the plane normal to coil axis. Thereby one can consider that heat fluxes through cross-sections $a - a'$ and $c - c'$ are zero (see Figure 6.1).

Let us presume that within covered section of the core temperature is evenly distributed due to high thermal conductivity of the core material. To determine thermal resistance between the core and its surroundings, let us consider heat balance of element *dx* located on distance *x* from cross-section

$b-b'$ (see Figure 6.2). It will be the same as equation (2.73):

$$P_1 + P_2 = P_3 . \tag{6.38}$$

Hence, equation describing distribution of temperature along uncovered section of the core will be the same as equation (2.74):

$$\frac{d^2\tau}{dx^2} + b^2\tau = 0 , \tag{6.39}$$

where b is the coefficient including parameters of the core expressed as

$$b = \sqrt{\frac{p_c \alpha_{\text{т.с}}}{\lambda_c q_c}} \tag{6.40}$$

where $\alpha_{\text{т.с}}$ total heat transfer coefficient from the core surface; p_c is perimeter of the core from which heat is driven out to its surroundings; q_c cross-sectional area of the core; λ_c is heat conductivity of the core material.

Solution of equation (6.39) with regard to boundary conditions:

$$\tau(0) = \tau_0; \quad \left.\frac{d\tau}{dx}\right|_{x=l} = 0 \tag{6.41}$$

will be as follows:

$$\tau = \tau_0 \frac{\operatorname{ch} b(l_c - x)}{\operatorname{ch} b l_c} . \tag{6.42}$$

Derived solution enables to determine expression for heat flux through cross-section $b-b'$

$$\Phi = -\lambda_c q_c \left.\frac{d\tau}{dx}\right|_{x=0} = \frac{\lambda_c q_c \tau_0 b}{\operatorname{ch} b l_c} \operatorname{sh} b l_c . \tag{6.43}$$

Hence, thermal resistance between the core and environment per its length unit will be expressed as

$$r_{\text{т.с-о}} = \frac{h_{\text{к}}}{2\sqrt{\lambda_c q_c p_c \alpha_{\text{т.с}}}} \frac{\operatorname{ch} b l_c}{\operatorname{sh} b l_c} . \tag{6.44}$$

6.4 Thermal behavior calculations of ac coils

Passage of alternating current through the coil occurred with active losses in the core due to eddy currents and remagnetization. Accordingly, thermal behavior of the coil will be dependent on internal heat generation not only in the coil, and in the core. Since volumetric heat generation in the coil and in the core are generally distinctive, heat exchange happens frequently between them. It should be noted that analysis of the coil thermal behavior with taking into account its heat exchange with the core is formidable mathematical task and it is not considered in given textbook. In practical engineering calculations the task is simplified presuming that heat exchange between the coil and the core can be at least in first approximation neglected. In this case heating the coil and heating the core can be considered independently from each other.

Distinctive feature of ac coil thermal calculations is that the heat released within the coil thickness is driven out only from its outer surface as absence of heat exchange between the coil and the core is in given case presumed. Power losses in ac coils within frequency range taking place actually in electrical equipment is determined in the same way as at direct current, that is, basing upon its ohmic resistance. Otherwise thermal calculations for ac coils are not different from ones for dc coils (see subsection 6.2).

Analysis of heating ferromagnetic core has some distinctive features. Let us consider simplest structure of electromagnetic system containing one coil with series magnetic circuit homogeneous over its entire length. Internal volumetric heat generation $p_{\text{уд}}$ in the core is available, which is most suitably determined from experimental curves [6]. Thermal power released in the core is driven out to environment from its surfaces uncovered by the coil. In this case the core is as well as at direct current straightened as shown in Figure 6.3.

It is evident that highest temperature will be at its middle cross-section $a - a'$. However, to find it, we should know distribution of temperature along the core length. In terms of thermal behavior the core has two characteristic zones:

- **zone I:** it is covered by the coil; heat is driven out from it to zone II only by conduction;
- **zone II:** it is uncovered by the coil; heat is driven out from its side surface directly to environment.

It is evident that distribution of temperature in both zones will be described with different equations. Let us consider, at first, thermal balance of element *dx* located in zone I distance *x* from cross-section $a - a'$ (see Figure 6.3). It will be as follows:

$$P_1 + P_4 = P_2 . \quad (6.45)$$

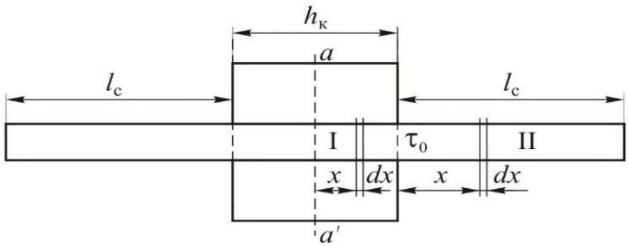


Figure 6.3 – Computational scheme of ac coil with the core

Given equation contains the same terms as equation (2.37), except P_3, as in zone I of the core thermal power is not driven out to environment. Heat released in element dx in this case is

$$P_4 = p_{уд} q_c dx , \quad (6.46)$$

where q_c is cross-sectional area of the core;

Hence, distribution of temperature in zone I will be described with following equation:

$$\frac{d^2\tau_I}{dx^2} + \frac{p_{уд}}{\lambda_c} = 0 , \quad (6.47)$$

where λ_c is thermal conductivity of the core material.

Solution of equation (6.47) with boundary conditions:

$$\left.\frac{d\tau}{dx}\right|_{x=0} = 0; \quad \tau_I\left(\frac{h_к}{2}\right) = \tau_0 \quad (6.48)$$

will be as follows:

$$\tau_I(x) = \frac{p_{уд}}{2\lambda_c}\left(\frac{h_к^2}{4} - x^2\right) + \tau_0 . \quad (6.49)$$

This solution enables to derive expression for maximal temperature-rise at middle cross-section of the core $a - a'$

$$\tau_{с.м.ма} = \tau_0 + \frac{p_{уд} h_к^2}{8\lambda_c} . \quad (6.50)$$

In order to determine temperature-rise at the boundary of I and II zones

τ_0, let us consider the task about distribution of temperature in II zone. Describing equation for this case will be the same as equation (4.39)

$$\frac{d^2\tau_{II}}{dx^2} + b^2\left(\tau_{II} - \tau_{уст}\right) = 0 , \qquad (6.51)$$

where
$$\tau_у = \frac{p_{уд} q_с}{p_с \alpha_{т.с}} ; \quad b = \sqrt{\frac{p_с \alpha_{т.с}}{\lambda_с q_с}} , \qquad (6.52)$$

where $p_с$ is the perimeter of the core cross-section dissipating thermal power to its surrounding; $\alpha_{т.с}$ is total heat transfer coefficient from the core surface.

Boundary conditions for solving equation (6.51) will be heat fluxes at boundaries of zone II:

$$-\lambda_с q_с \left.\frac{d\tau}{dx}\right|_{x=0} = p_{уд} q_с \frac{h_к}{2}; \quad -\lambda_с q_с \left.\frac{d\tau}{dx}\right|_{x=l_с} = 0 . \qquad (6.53)$$

Solution of equation (6.51) with boundary conditions (6.53) will be:

$$\tau_{II}(x) = \frac{p_{уд} h_к}{2b\lambda_с} \cdot \frac{\operatorname{ch} b(l_с - x)}{\operatorname{ch} bl_с} + \tau_у . \qquad (6.54)$$

Derived solution enables to find expression for temperature-rises on border between I and II zones:

$$\tau_0 = \tau_{II}(0) = \frac{p_{уд} h_к}{2b\lambda_с} \cdot \frac{\operatorname{ch} bl_с}{\operatorname{sh} bl_с} + \tau_у . \qquad (6.55)$$

Hence, maximal temperature-rise in the core with taking expressions (6.52) and (6.55) into account will be defined with following expression:

$$\tau_{с.м.ма} = \frac{p_{уд} h_к}{2b\lambda_с} \cdot \frac{\operatorname{ch} bl_с}{\operatorname{sh} bl_с} + \frac{p_{уд} h_к^2}{8\lambda_с} + \frac{p_{уд} q_с}{p_с \alpha_{т.с}} . \qquad (6.56)$$

Specific calculations and operational experience show that temperature of the core may considerably exceed temperature of the coil. When good contact between them is available, the core will heat the coil that lead to increase in temperature of the coil. To avoid this, highest thermal isolation of the coil from the core must be in such cases provided. For example, it can be reached by insertion of air gap between the core and coil. In the absence or insufficiently good thermal isolation thermal calculations of the coil is to be performed with taking into account heat exchange between the core and the coil.

7. THERMAL BEHAVIOR OF POWER SEMICONDUCTOR DEVICES

7.1 General

Power semiconductor devices (PSCD) are elemental base of advanced power electronic equipment, such as, power converter devices (rectifiers, invertors, frequency converters) as well as solid-state (net semiconductor-based) and combined (contact-semiconductor-based) switching apparatuses (contactors, magnetic starters, circuit-breakers etc.). Designing of PSCD-based switching equipment is one of developing direction of electrical apparatus engineering. The most important features of net PSCD-based switching apparatuses are arcing-less switching electrical circuit as well as no movable components. Those properties eliminate many undesirable phenomena occurring in contact-based apparatuses, such as, mechanical wear, enhanced noise, electro-erosion wear and welding of contacts and others. There are also important features of PSCD-based switching apparatuses, such as, enhanced response speed, switching frequency, switching life; ability to survive under unfavorable internal conditions; ability to regulate voltage and current at phase control in ac circuits; low cost to maintenance.

In power converter devices and net semiconductor-based switches PSCDs operate in different duties: continuous – under nominal load, short-term – under overload conditions and also impulse – under short-circuit current. It should be noted that in those devices serviceability (optimal thermal behavior) of PSCDs in continuous and short-term duties can not be provided with any use of different type coolers, such as, finned radiators (including forced cooling), coolers with use of liquid and evaporative cooling. This circumstance leads to substantial increase in overall dimensions and complicates the construction of electrical apparatuses that is grave theirs shortcoming. Another significant shortcoming of net PSCD-based switching apparatuses is high sensitivity to overcurrents and excess of voltages. That is why high-speed responsive protections against overcurrents and overvoltages must be incorporated in the apparatus circuits.

Especial group of switching apparatuses are combined (contact-and-semiconductor-based) switching devices, wherein contacts are paralleled with PSCD-based component. In those apparatuses the positive qualities of the contacts and PSCDs are good combination. The performance of combined switching apparatuses is accomplished by separation of the functions such that the contacts conduct continuously flowing current load, whereas PSCD-based component performs the function of electrical current switching. In this case PSCDs undergo action of current load only from the moment of parting the contacts to crossing the current through zero value, when PSCDs are locked.

Thus, basic operational conditions of PSCDs in combined apparatuses is action of single and series of impulses at electrical circuit switching by the duration no more than one half-cycle of power frequency. Operational ability of PSCDs in this case will be provided by only theirs overload capability. Therefore, in combined apparatuses PSCDs are not completed with some kind of coolers. It should be noted further essential quality of combined switching apparatuses. It is implies in that PSCD-based component in the process of functioning is not acted by let through short circuit currents since it is in open position shunted by the contacts.

7.2 Thermal parameters and thermal behavior analysis methods of PSCDs

The task posed by the designers of power electronic equipment is usually to select the type of PSCD and then to check its thermal behavior. It is usually implies in that to quantify the temperature of its *p-n* junction in the operational duties mentioned above and to compare it with permissible one.

As already noted, PSCD is concentrative source of thermal losses that are generated in very small volume of its *p-n* junction. From here thermal flux passes through series of layers of different materials, such as, thermo-compensators (tungsten, molybdenum), spacers (silver, tin) base, heat sink cooler (aluminum, copper), and is driven out to surroundings. Each of those layers sets up resistance to propagation of the heat flux, which results in temperature difference between the junction and each layer. Considering the path of heat flux through constructively separate components, it can be said about thermal resistances of the following sections:

- junction – device case R_{thjc};
- device case – contact face of heat sink R_{thch};
- contact face of heat sink – cooling medium R_{thha};
- junction – cooling medium R_{thja};

Accepting conventionally thermal resistance as temperature drop per unit of the heat flux, following expressions can be derived:

$$R_{thjc} = \frac{\vartheta_j - \vartheta_c}{P}\ ;\ R_{thjh} = \frac{\vartheta_c - \vartheta_h}{P}\ ;\ R_{thha} = \frac{\vartheta_h - \vartheta_a}{P}\ ;$$

$$R_{thja} = R_{thjc} + R_{thch} + R_{thha} = \frac{\vartheta_j - \vartheta_a}{P}\ , \qquad (7.1)$$

where ϑ_j is the junction temperature; ϑ_c is the device case temperature; ϑ_j is the heat sink contact face temperature; ϑ_a is the cooling medium temperature.

Thermal resistance junction–case is mainly defined by the area of semiconductor structure, the quality of contact junctions and construction of the case. The expression of thermal resistance junction–case R_{thjc} represented above can be valid only for PSCDs of finger-type construction. But the devices of pill-type construction are characterized with thermal resistances junction – anode terminal of case R_{thjcA} and junction – cathode terminal of case R_{thjcK}. They are expressed as follows

$$R_{thjcA} = \frac{\vartheta_j - \vartheta_{cA}}{P_A}\ ;\quad R_{thjcK} = \frac{\vartheta_j - \vartheta_{cK}}{P_K}\ , \qquad (7.2)$$

where P_A and P_K is thermal power driven out from the side of anode and cathode, respectively.

Represented expressions refer to steady state thermal behavior. In transient (unsteady state) conditions thermal behavior of PSCD is characterized by transient thermal resistances junction – case Z_{thjc} and junction – cooling medium Z_{thja} determined as functions of time from the start of current flowing.

Dependences $Z_{thjc}(t)$ are represented in nameplate data for certain types of PSCDs. Dependences $Z_{thja}(t)$ are also represented in nameplate data, but they refer to certain type of heat sink. In the case the device operates in assemblage with another heat sink, then transient thermal resistance junction – medium is quantified in accordance with the following expression

$$Z_{thja} = Z_{thjc} + R_{thch} + Z_{thha}\ . \qquad (7.3)$$

In represented expression the value of the contact thermal resistance is not in strict sense constant in time. However, because of that it is insignificant in total thermal resistance and has small thermal time constant, it can be neglected in engineering calculations provided that compliance with requirements of assemblage and quality of the contact faces.

Transient thermal resistance of a cooler $Z_{thha}(t)$ for specific time intervals t is represented in reference data on the coolers of each particular type [26]. When using this parameter as the component of total thermal resistance $Z_{thja}(t)$ for calculated values of the time $t > 1\,\text{s}$ it is to be added to the contact thermal resistance R_{thch}. It is cased by the fact that when thermal flux reaches the surface of cooling device, thermal resistance of the contact between PSCD case and contact surface of the cooling device is inserted into thermal equivalent circuit.

In steady state conditions

$$Z_{thja}(t) = R_{thja}\,. \tag{7.4}$$

Having values $Z_{thja}(t)$ for specific time intervals t, the temperature of the junction at the time instant t can be evaluated when the value of released power P is known

$$\vartheta_j(t) = \vartheta_a + P \cdot Z_{thja}(t)\,. \tag{7.5}$$

As it was mentioned, current loading capability of PSCDs is defined by the maximal permissible temperature of its *p-n* junction, which cannot be exceeded in any its operational duty.

Calculation of *p-n* junction temperature is performed basing upon power of thermal losses and thermal resistance. Let's suppose that it is necessary to calculate *p-n* junction temperature-rise τ_{j2} at the instant t_2, when during the time span t_0–t_1 the constant power P was released in it, and within the span t_1–t_2 the power was not released ($P = 0$), as shown in Figure 7.1.

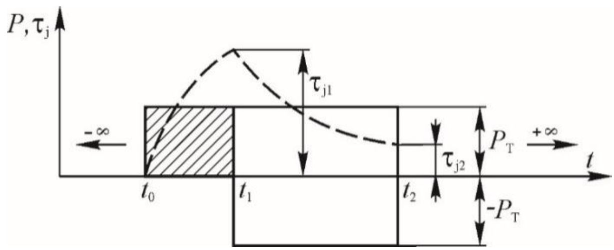


Figure 7.1 – Time diagrams of power losses and junction temperature-rise as single power impulse passes

In the process, over the time interval from t_0 to t_1 the junction temperature-rise builds up τ_{j1}, and then on the time span t_1–t_2 falls down to

τ_{j2}. To calculate the temperature-rise at the instant t_2 the power impulse duration is conventionally increased to this moment supposing that within the span t_1–t_2 the negative power with identical absolute value P is dissipated. Thus, expression of the junction temperature-rise at the moment t_2 can be written as follows

$$\tau_{j2} = \tau'_{j2} - \tau''_{j2} = PZ_{thja}(t_2 - t_0) + (-P)Z_{thja}(t_2 - t_1), \tag{7.6}$$

where $Z_{thja}(t_2 - t_0)$ and $Z_{thja}(t_2 - t_1)$ are the values of transient thermal resistance junction – medium corresponding to intervals t_0–t_2 and t_1–t_2, respectively, determined by dependences $Z_{thja}(t)$ at given cooling.

Rearrangement of expression (7.6) gives

$$\tau_{j2} = P\left[Z_{thja}(t_2 - t_0) - Z_{thja}(t_2 - t_1)\right]. \tag{7.7}$$

It should be noted that in actual practice more complex types of load with numerous values of released power P and thermal resistances Z in computational expressions may take place. Therefore, to simplify the computational expressions the following more short designations are accepted in further analysis:

$$\begin{aligned} &Z_{thja}(t) \Rightarrow Z; \\ &R_{thja} \Rightarrow R_{\mathrm{T}}; \\ &Z_{thja}(t_2 - t_0) \Rightarrow Z_{2-0}; \\ &Z_{thja}(t_2 - t_1) \Rightarrow Z_{2-1}; \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ &Z_{thja}(t_\mathrm{m} - t_\mathrm{n}) \Rightarrow Z_{\mathrm{m-n}}. \end{aligned} \quad . \tag{7.8}$$

Then expression (7.7) with designations taken from (7.8) will be as follows

$$\tau_{j2} = P(Z_{2-0} - Z_{2-1}). \tag{7.9}$$

In represented computational expressions action of rectangular power impulse is presumed. However, actual shape of impulses for PSCDs operating in various circuits of electronic equipment is usually different from rectangular one. Thermal behavior analysis method of PSCDs, which is represented, requires transforming power impulses of any shape to rectangular one [26].

7.3 Temperature calculations under different load behavior

Let’s consider thermal behavior of PSCDs in equipment of switching applications. In net semiconductor-based switching apparatuses (contactors, magnetic starters, circuit-breakers) PSCDs operate in the duty, which for given apparatus is design one.

Continuous duty: when continuous invariable current load occurs, the junction temperature in steady state with taking into account designations (7.8) are evaluated according to following expression

$$\vartheta_j = \vartheta_a + PR_{\text{т}} . \tag{7.10}$$

In the process, allowable power of thermal losses will be determined as follows

$$P_m = \frac{\vartheta_{jm} - \vartheta_a}{R_{\text{т}}} . \tag{7.11}$$

Temporary duty: when short-term load with duration $t_{\text{кр}}$ and released power $P_{\text{кр}}$ occurs, maximal temperature of the junction according to expression (7.5) will be

$$\vartheta_{j\text{макс}} = \vartheta_a + P_{\text{кр}} Z(t_{\text{кр}}). \tag{7.12}$$

The junction temperature after a lapse of no-current pause with duration $t_{\text{п}}$ after short-term operation under load will be

$$\vartheta_j(t_{\text{п}}) = \vartheta_a + P_{\text{кр}}\left[Z(t_{\text{кр}} + t_{\text{п}}) - Z(t_{\text{п}})\right]. \tag{7.13}$$

Intermittent cycle duty: maximal quasi-steady state temperature of the junction in intermittent cycle duty with on-load time $t_{\text{р}}$ and cycle duration $t_{\text{ц}}$ at released power $P_{\text{ПВ}}$ will be [26]

$$\vartheta_{j\text{макс}} = \vartheta_a + P_{\text{ПВ}}\left[\frac{t_{\text{р}} R_{\text{т}}}{t_{\text{ц}}} + \left(1 - \frac{t_{\text{р}}}{t_{\text{ц}}}\right) Z(t_{\text{р}} + t_{\text{ц}}) - Z(t_{\text{ц}}) + Z(t_{\text{р}})\right]. \tag{7.14}$$

As already noted, in combined (contact-and-semiconductor-based) switching apparatuses PSCDs can operate under action of either impulse series (in normal switching mode) or single impulses (in infrequent switching mode). Duration of current impulses is usually no more than one half-cycle of power frequency $(t_{\text{и}} \le 0.01 \text{ s})$ for PSCDs in combined switching apparatuses. Hence, the temperature at the end of single current impulse with power $P_{\text{и}}$ can be evaluated according to expression (7.5), which in terms of designations accepted in (7.8), will be as follows

$$\vartheta_{j\text{макс}} = \vartheta_a + P_\text{и} \cdot Z(t_\text{и}). \tag{7.15}$$

When a switching apparatus operates in normal switching mode, PSCD is subjected by infinite series of current impulses with duration of the order of 0.01 s. Maximal quasi-steady state temperature in this case at power released by one impulse $P_\text{и}$ will be quantified with the formula identical to one for intermittent cycle duty [26]

$$\vartheta_{j\text{макс}} = \vartheta_a + P_\text{и}\left[\frac{t_\text{и} R_\text{т}}{T} + \left(1 - \frac{t_\text{и}}{T}\right) Z(T + t_\text{и}) - Z(T) + Z(t_\text{и})\right], \tag{7.16}$$

where T is the period of following the current impulses depending on the frequency of switching operations, which is defined by the purpose of the apparatus and ranges up to 3600 per one hour.

It should be noted that in this case $T >> t_\text{и}$ and, therefore, formula (7.15) with taking into account

$$\frac{t_\text{и}}{T} \approx 0 \quad \text{and} \quad Z(T + t_\text{и}) \approx Z(T),$$

can be converted to the following

$$\vartheta_{j\text{макс}} = \vartheta_a + P_\text{и}\left[\frac{t_\text{и} R_\text{т}}{T} + Z(t_\text{и})\right]. \tag{7.17}$$

In derived formula transient thermal resistance between *p-n* junction and environment is used. It should be noted here that in reference books [26 and others] transient thermal resistances between *p-n* junction and casing are represented. However, considering very small on-load time, when adiabatic heating will be occurred, transient thermal resistances will be dependent on only specific heat of PSCD casing. By this why reference data of thermal resistances junction – casing can be used as thermal resistances junction – cooling medium.

Aggregate steady state thermal resistance junction – medium of most frequently used pill-type construction PSCDs is defined by the equivalent circuit represented in Figure 7.2.

$$R_{thjc} = \frac{(R_{thj-cA} + R_{thc-aA})(R_{thj-cK} + R_{thc-aK})}{R_{thj-cA} + R_{thc-aA} + R_{thj-cK} + R_{thc-aK}}, \tag{7.18}$$

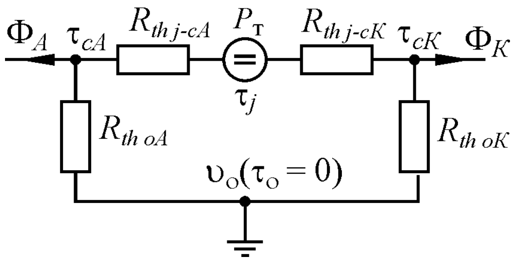


Figure 7.2 – Thermal equivalent circuit of pill-type construction PSD

where $R_{thj\text{-}cA}$, $R_{thj\text{-}cK}$ are steady state thermal resistances junction – anode terminal of casing and junction – cathode terminal of casing, respectively, determined by reference books [26];

$R_{thc\text{-}aA}$, $R_{thj\text{-}aK}$ are steady state thermal resistances anode terminal of casing – cooling medium and cathode terminal of casing – cooling medium, respectively, determined by capability of the buses connected to anode and cathode terminals to drive out heat.

If to neglect transition thermal resistances between casing and buses as well as heat transfer through conduction within buses, then thermal resistances casing–cooling medium can be quantified according to following formula:

$$R_{thc-a} = \frac{1}{\alpha_т S_{охл}}\,, \tag{7.19}$$

where $\alpha_т$ is total heat transfer coefficient from the surface of the buses connected to anode or cathode terminals of the PSCD; $S_{охл}$ is the buses surface contacting with cooling medium.

The *p-n* junction temperature calculated by formula (7.16) must not exceed permissible one for selected type of PSCD.

As it was noted above, thermal behavior of PSCDs operating continuously under load is mainly defined by thermal resistances of their individual heat sinks. There is no doubt that most part of thermal energy released in *p-n* junction of PSCD will be driven out to environment through the heat sinks. However, some part of heat will be transferred into current-carrying parts or vice versa from current-carrying parts to the PSCD through equalizing heat fluxes. Hence, in given case we have to deal with inhomogeneous current-carrying system that apart from current-carrying elements and contact joints includes PSCDs, which can be represented as separate section. In this case it is important to derive expressions for equalizing heat fluxes at its boundaries that will be defined by constructional features of the PSCD.

For press pack PSCDs having two-side cooling the expressions for equalizing heat fluxes can be obtained on the basis of its thermal equivalent circuit shown in Figure 7.2. For represented circuit the following equations are valid:

$$P_т = \frac{\tau_j - \tau_{cA}}{R_{thj-cA}} + \frac{\tau_j - \tau_{cK}}{R_{thj-cK}};$$
$$\frac{\tau_j - \tau_{cA}}{R_{thj-cA}} = \Phi_A + \frac{\tau_{cA}}{R_{th\,oA}}; \quad , \tag{7.20}$$
$$\frac{\tau_j - \tau_{cK}}{R_{thj-cK}} = \Phi_K + \frac{\tau_{cK}}{R_{th\,oK}},$$

where Φ_A, Φ_K are the equalizing heat fluxes incoming to current-carrying parts connected to anode and cathode of the PSCD (or vice versa), respectively.

Solving equations (7.19) respectively equalizing heat fluxes will be as follows:

$$\begin{aligned} \Phi_A &= s_A - \tau_{cA} u_A + \tau_{cK} v_A; \\ \Phi_K &= s_K - \tau_{cK} u_K + \tau_{cA} v_K. \end{aligned} \quad , \tag{7.21}$$

The coefficients in expressions (7.21) will be found by the following expressions:

$$\begin{aligned} s_A &= \frac{R_{thj-cK}}{R_{thj-cK} + R_{thj-cA}} P_т; \quad s_K = \frac{R_{thj-cA}}{R_{thj-cK} + R_{thj-cA}} P_т; \\ u_A &= \frac{1}{R_{th\,oA}} \left(1 - \frac{R_{thj-cK}}{R_{thj-cK} + R_{thj-cA}}\right) + \frac{1}{R_{thj-cA}}; \\ u_K &= \frac{1}{R_{th\,oK}} \left(1 - \frac{R_{thj-cA}}{R_{thj-cK} + R_{thj-cA}}\right) + \frac{1}{R_{thj-cK}}; \\ v_A &= \frac{R_{thj-cK}}{R_{th\,oK}\ R_{thj-cK} + R_{thj-cA}}; \quad v_K = \frac{R_{thj-cA}}{R_{th\,oA} \left(R_{thj-cK} + R_{thj-cA}\right)} \end{aligned} \quad , \tag{7.22}$$

The principal approach to PSCD thermal behavior analysis with taking into consideration of equalizing heat fluxes will be in this case the same as thermal behavior analysis of inhomogeneous current-carrying system with available contacts (see subsection 5.4). In this case the only peculiarity will be the presence of PSCDs with individual heat sinks in current-carrying system. They are to be considered as separate section. Solution of the task in given case will be based on the conditions of the temperature field continuity. To do this apart from expressions for equalizing heat fluxes at the boundaries of the sections (4.43), (4.46) и (5.29) expressions for equalizing heat fluxes of PSCD (7.20) should be used.

7.4 Cooling technologies for PSCD

There is well-established classification of cooling technologies for PSCD. According to that they are shared into air, liquid, evaporative and combined ones. Nevertheless, such subdivision is common enough and is not adequate for the level of modern tasks aimed to intensify driving out a heat from PSCD.

Existing cooling technologies may be characterized by a number of features such as manner of the heat transfer agent movement, construction type (individual or grouped) manner of driving out a heat (single-sided, double-sided, three-dimensional), material of the cooling device etc. However, further specification and refinement of researches in various lines of given field necessitates to classify as a whole PSCD cooling systems in respect to most fundamental features. The most important among them is the availability or freedom from intermediate heat transfer agent, and basic characteristic of the cooling system is the manner of thermal energy transportation from PSCD to cooling agent. Other features are the kind of heat transfer agent as well as the types of applied cooling devices and heat exchangers. As the cooling systems are grouped together in such manner those having no intermediate heat transfer agent are subdivided into those with air (natural and forced) and water single-loop cooling, and those with available intermediate heat transfer agent are subdivided into homogeneous and heterogeneous ones, i.e. those with single-phased and double-phased heat transfer agent.

Air cooling systems for PSCD are referred to those where air is used as once heat transfer agent and thermal energy by convection and radiation is transferred to environment. Such systems are most spread that is resulted from that in most cases the air is finally that medium where thermal energy is transferred.

Natural air cooling has found application in spite of considerable mass, dimensions and low heat transfer intensity. In application of natural air cooling not more than 10 W/(m^2·K) that is explained by great thin of boundary layer at the sink fins. In the process PSCD are loaded by the current that no more than 25% of its ultimate value and capacity of power electronic equipment unit usually does not exceed 15 kW.

In the case when natural cooling does not provide normal thermal behavior, forced air cooling is applied. It is used in most power electronic equipment produced. It is explained considerable growth of heat transfer to air by means of forced convection in blast and due to that higher load capability of PSCD, smaller dimensions and mass of the equipment. Depending on velocity of air movement the value of htc is rated 70 to 150 W/(m^2·K). Cooling devices (heat sinks) for PSCD have as a rule low hydraulic resistance, and therefore for theirs blast low-head axis fans suitably arranged in constructions of power electronic equipment.

Nevertheless, capabilities of forced air cooling are restricted. Increase in velocity of air movement more than 15–25 m/s is not economically efficient since leads to growth of aerodynamic resistance, and consequently capacity of fans. Low efficiency of heat transfer to air even under blast necessitates significant increase in sink surface area that results in growth of dimensions. It should be pointed out here that increase in dimensions and mass of cooling devices happens more quick than growth of current through a device.

Thus, the main direction to solve the problem of cooling PSCD is application of more effective cooling technologies allowing increase htc and reduces mass and dimensions of cooling devices. One of such technologies using at the present time it is water single-loop cooling.

In such cooling the water circulation system is opened, that is, the water, after passing cooling device, goes to exhaust and is not used in further. Denoted technique is very effective, but requires significant consumption of water. It has gained acceptance in power electronic equipment for metallurgical and chemical industry, in ship installations.

The cooling systems with intermediate heat-transfer agent (hta) are less widespread, but more promise. The systems with intermediate single-phase hta operate usually in closed loop with forced or natural circulation of the hta. Forced circulation is produced by the help of pump. Natural circulation in the loop is produced by difference in densities of hot and cold liquid. In those systems various types of heat transfer agents are used, such as, water, transformer oil, synthetic dielectric liquids, liquid metals etc.

Heat transfer process occurs in such manner. Thermal energy is taken by single-phase heat transfer agent and is transferred in recuperative heat-exchanger to air or liquid. In choosing cooling liquid the complex of properties providing reliable and safety operation of power electronic equipment in different climatic conditions are taken into consideration.

From the heat engineering standpoint the most effective liquid is water since it can provide htc in the range of 10^3 to $7\cdot10^3$ Wt/(m$^2\cdot$K). For transformer oil it comprises $3\cdot10^2$–10^3 Wt/(m$^2\cdot$K). Water, used as hta, has two most evident drawbacks – comparatively high freezing point and low electrical strength. Nevertheless, this cooling technology is extensively enough used in low voltage equipment.

In single-phase thermo-siphons heat transfer occurs in natural convection conditions of liquid hta. Because of this they provide less intensive heat driving out than the systems with forced circulation.

At the present time the process of active implementation of cooling systems with two-phase hta into power electronic engineering is evident. They enable to attain high efficiency of heat exchange through change of the hta aggregative state. In this case two-phase stated liquid is used as hta. It is either into the chamber of individual cooling device or into the chamber with power electronic unit as group cooling is applied. When such cooling technology is used the heat, for example in the chamber of individual cooling device, to cooling air is transferred in closed evaporative-condensate cycle. When evaporation (boil) of the liquid occurs in direct vicinity of power electronic unit, then the values of htc may be $(30–50)\cdot10^3$ Wt/(m$^2\cdot$K). Heat transfer by steam occurs practically at constant temperature and can be completed for significant distances. The steam condenses in cooled condenser. High intensity of heat transfer at condensation provides high temperature potential of condenser from the side passed over the air. Most important benefit of such cooling technology is absence of the pump to force circulation of intermediate hta that much simplifies the equipment and do it more reliable.

Returning the condensate from condensing zone to evaporating one in cooling devices with intermediate hta may be produced either due to gravity force (two-phase thermosiphon) or capillary forces (heat pipe). Necessary condition for normal operation of two-phase thermosiphon is such its position in space as unobstructed draining down to vapor zone is provided. Heat pipes, as differentiated from two-phase thrmosiphons, can operate both in zero-g condition and opposite to gravity forces. To cool PSCD it is possible application of combined cooling devices, in which returning the condensate from cooling to heating zone is produced due to both gravity and capillary forces. Considered three types of cooling devices with two-phase hta have usually condenser cooled by air at natural or forced convection conditions and is applied for individual cooling PSCD or in power electronic units.

Close, in operational principle, to two-phase thermosiphons is transpiration system with individual cooling devices and common condenser as well as transpiration system PSCD immersed into dielectric liquid. The condenser is in these cases either air-cooled or water-cooled.

Application of effective cooling systems is especially promise in high voltage (HV) electronic equipment that usually contains much number of PSCD. It is for example rectifiers for dc transmission line substations. The main components of such substations are HV thyristor unit that may include 100–200 and more thyristors. The cooling system of such unit must provide not only small mass and dimensions, but high quality of insulation.

As air cooling is used for HV units, the last requirement defines its position in a room developing indoor dc distribution center. The cooling air is fed to HV thyristor unit by the fan and exhausted from it to the room and then is directed for cooling to indoor heat exchanger. It is air loop of cooling system. To compensate air losses from leakage additional amount of dry unpolluted cooled air passes into the room. Into another loop ethylene glycol is pumped to circulate. This loop serves to cool the air. Heat driven out in this loop is transferred to outdoor air or to the water in outdoor heat exchanger.

At the present time oil cooling power electronic units for rectifying outdoor substations is also used. In this case one or two HV units are positioned within steel tank. All units consist of a few identical tiers wherein some thyristor groups are arranged. The thyristors and other heat-releasing components are cooled by forced oil circulation in the loop. Driving out the heat from oil happens in the heat-exchanger cooled by indoor air or water. In servicing HV power electronic equipment with oil insulation to maintain isolation performances at invariable level is more simple, no necessity periodically to clean the equipment from dust.

The water cooling system includes two closed loops of water circulation with common circulating pumps and heat-exchangers. De-ionized water is fed by pumps into the group of the heat-exchangers and then, passing the filters of mechanical particles it is shared into two flows. The main part passes to HV unit and furthermore into pressure tank, another part directs to ion-exchange filters and then is return into sucking line of pumps. All components contacting with circulating de-ionized water are made of non-corroding materials. The cooling devices for thyristors are made of cooper with cover and are conducted in series to common pipe with the help of separating isolating spacers. The water is fed to HV unit by insulating pipes.

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