

ANALYSIS OF THE DISPLACEMENT FIELD IN A RING DOMAIN UNDER TANGENTIAL LOADING

A system of partial differential equations for a domain in the form of a ring, known [1] as the 1st fundamental boundary problem of the elasticity theory, is solved using the method of Muskhelishvili's complex potentials. The ring models the power element of engine, transmitting rotational motion, therefore the boundary conditions are formulated taking into account the tangential load. Such a problem has already been considered [2]. However, the load in these works was thought to be uniformly distributed along the contour. It is physically realistic to assume that the load is uniformly distributed only within the angles corresponding to the ring bindings and that it is equal to zero outside these limits. The aim of the paper [3] is to find and analyze the solution of the corresponding boundary value problem.

In this paper, a solution of the boundary value problem of elasticity theory for a domain in the form of a ring with piecewise constant boundary conditions on the contour was constructed using the Muskhelishvili's method of complex potentials. It is established that in the absence of a normal component of external loading, the problem can be solved by the method of complex potentials in static formulation. In terms of complex potentials, the boundary condition [1, § 41, (23)] in the form of Kolosov-Muskhelishvili in the absence of a normal component has the form ($j = 1, 2$):

$$\left\{ \Phi + \bar{\Phi} - e^{2i\theta} \left[\bar{z} \cdot \Phi' + \Psi' \right] \right\} \Big|_{|z|=R_j} = -iT.$$

Here $T = T_{0j} \cdot t(\theta)$, $j = 1, 2$, when $t(\theta)$ is a piecewise function of the boundary conditions.

The solution of this problem is obtained in an analytical form (as a Fourier expansion). Further the analytical solution is reduced to a form suitable for numerical simulation. Numerical simulation of a similar problem for specific values of physical parameters was carried out in [3]. A numerical analysis showed that the displacement of points near the load zone has practically no radial components; the tangential external load causes only the angular displacements of the points of the area. A similar situation arose when the external load was uniformly distributed along the contour [2]. In this case, the stress-strain state is pure shift. On the contrary, near the free part of the boundary the angular

displacements of the points noticeably weaken, but there are significant radial movements in the direction of the center. In this case, the stress-strain state is compression in the radial direction.

Thus the results of this paper [3] generalize the result obtained in [2]. The results were also set out in [4].

Conclusion. Therefore analysis of the solution shows that in the neighborhood of the contour there is deformation of the region close to the shift (on the sections of the boundary with a nonzero boundary condition) or to radial contraction (on the sections of the boundary with the zero boundary condition).

The results of this paper can be applied to describe the state of rotating mechanical structures.

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