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SEARCH FOR PHYSICS LAWS

ПОШУК ФІЗИЧНИХ ЗАКОНОМІРНОСТЕЙ

(переклад з української)

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Рекомендовано Вченою радою Національного університету “Запорізька політехніка” як навчальний посібник (протокол №… від ……..2026 р.)

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This publication is a translation of the first part of the textbook “Search for Physics Laws” published by the authors in Ukrainian in 2021. The textbook contains didactical materials for a new type of introductory physics laboratory course. The novelty of the course lies in the fact that its main educational goal is to teach students the basic techniques for processing experimental data. The guide may be useful for physics lecturers at higher educational institutions, students of pedagogical and technical universities, physics teachers, high school students, and for everyone interested in modern methods of teaching physics.

Це видання є перекладом першої частини посібника “Пошук фізичних закономірностей”, виданого авторами українською мовою у 2021 році. Посібник містить методичні матеріали, що забезпечують проведення занять вступного фізичного лабораторного практикуму нового типу, головною навчальною метою якого є навчання студентів базовим приймам обробки експериментальних даних. Посібник може бути корисним для викладачів фізики вищих навчальних закладів, студентів педагогічних та технічних університетів, учителів фізики, старшокласників та для всіх, хто цікавиться сучасними методами викладання фізики.

CONTENTS

INTRODUCTION

How do physicists empirically uncover the laws of physics? What methods do they employ to rigorously extract these principles from experimental data? The first part of this course aims to address these foundational questions, providing students with insight into the systematic approaches used to discover physical laws through carefully selected classical examples. The second part of the course introduces students to the professional techniques and analytical tools that physicists use, so called “professional secrets” of physicists. This includes:

– Dimensional Analysis: Techniques for predicting research outcomes in advance based on dimensional analysis.
– Extrapolation and Interpolation: Methods for deriving reliable information beyond or within the measured data range.
– Similarity Theory: Understanding how measurements on small-scale models can predict behavior of large-scale systems, such as dams, airplanes, and ships.
– Mathematical Modeling: Techniques that allow physicists to infer the behavior of complex systems – like neutron distribution in nuclear reactors – through simpler analogous experiments, such as current distribution in conductive paper.

The course also covers essential experimental procedures, such as recording measurement data, creating graphical representations, and deriving conclusions from graphs. Developing these skills is critical for students and represents the third component of the course.

The development of computer technology has introduced new dimensions to experimental physics. Modern data analysis now includes methods such as: Hypothesis Testing (χ-squared test) (for confirming or refuting theoretical predictions), Data Mining (for identifying hidden patterns or parameters within experimental data), Computer Calculations of Means and Uncertainties (enabling a more nuanced understanding of experimental results).

These computational skills are not merely supplementary but are integral to the practice of modern experimental physics. Therefore, we have incorporated them into our course, alongside a focus on interpreting results with accuracy and depth.

The development of these course components arose naturally during the design process, as each followed logically from the core principles of the course. However, at the heart of this course there are two guiding ideas around which the entire structure of our introductory laboratory physics course is built:

1. The Fundamental Objective of Laboratory Work: The main object of each laboratory work is to explore and quantify a specific physics law. The ultimate purpose of experimental research in this context is to determine the parameters that define these laws.

2. Interval Paradigm in Experimental Answers: In experimental research, the outcome is expressed as a confidence interval. Teaching students how to construct and interpret confidence intervals for various types of physics relationships is central to our curriculum.

This course structure is the result of our efforts to convey both the practical methods and the conceptual frameworks essential to experimental physics. It is our hope that we have succeeded in creating a cohesive and comprehensive educational guide. Ultimately, however, it is for the reader to judge whether we have met our objectives. As the classic phrase goes, *“Feci quod potui, faciant meliora potentes”* – “I have done what I could; let those who can, do better”.

Acknowledgments

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Authors

STRUCTURE OF THE COURSE "SEARCH FOR PHYSICS LAWS"

The structure of a Laboratory Session (Sessions 1-12) is outlined in Table SC.1 below.

Table SC.1 — The Structure of a Laboratory Session

1	Work in a laboratory	Duration 80 minutes
1.1	*Written Test*. Each session, except the first, begins with a brief written test of five simple tasks reviewing the material from the previous session. Students receive examples of these tasks in advance to guide their preparation.	10 minutes
1.2	*Instructor's Lecture.* The instructor's lecture introduces and contextualizes new concepts. This part of the session provides the theoretical foundation necessary for students to understand and engage with the practical work.	30 minutes
1.3	*Laboratory Practice*. Half of the laboratory works is conducted in a group setting under the instructor's guidance, while the other half is performed in small groups independently, fostering collaborative and self-directed learning.	30 minutes
1.4	*General Discussion.* Each session concludes with a group discussion. This interactive exchange allows students to share their findings, raise questions, and engage in reflective dialogue. This discussion enhances their ability to communicate scientific ideas effectively, an essential skill for professional practice.	10 minutes
2	Homework	80 minutes
2.1	*Laboratory Report*. For homework, students compile a laboratory report including data tables, processed data, and an experimental graph built on graph paper. The report must include a conclusion stating the confidence interval for the measured value.	60 minutes
2.2	*Quiz Practice.*	20 minutes

The last two sessions are dedicated to assessing students' cumulative knowledge and their ability to apply skills acquired throughout the course. Each control session is designed with distinct objectives, ensuring that students can demonstrate both their theoretical understanding and practical application of physics laws.

In the first assessment session, students' understanding of core concepts and their proficiency in essential laboratory skills are evaluated. Skills assessed include:

– Data Interpretation: Interpreting graphs and identifying physical relationships.
– Quantitative Analysis: Conducting calculations and rounding results appropriately.
– Mathematical Representation: Expressing experimental data in the form of mathematical equations.

The assessment takes the form of a multiple-choice test with 25 questions, enabling a standardized evaluation of students' competence across key topics.

The second assessment session tests students' ability to derive physics laws from experimental data, simulating a real research scenario. Students are presented with experimental measurement data and are expected to deduce the underlying physics law.

Key features of this session include:

– Focus on Law Derivation: Emphasis is placed on the ability to analyze data and draw conclusions, a critical skill in experimental physics.
– Exclusion of Uncertainty Questions: This session specifically excludes questions on experimental uncertainty, all provided data has the uncertainty of 1% allowing students to concentrate on identifying the physics law rather than error analysis.

TOPIC 1. FOUR AXIOMS OF PHYSICAL MEASUREMENTS

Session 1. Understanding the Concept of a Confidence Interval. Laboratory work 1 “Buffon – de Morgan Experiment”.

In this session, students are introduced to a fundamental concept in measurement theory: the confidence interval as the primary outcome of an experimental study. To illustrate this concept, students, guided by the instructor, conduct an experiment aimed at determining the probability of obtaining “heads” in a coin toss. Following the experiment, students formulate rules for constructing confidence intervals. The result of this session is the establishment of a confidence interval for the probability of obtaining “heads” in repeated trials.

1. Nothing exists!
2. Even if something exists, nothing can be known about it!
3. Even if something can be known about it, knowledge about it cannot be conveyed to others.

Three theorems by the sophist Gorgias

4. Even if it can be conveyed, no one would be interested in hearing about it.

D. Fon-Der-Flaass

1. Theoretical Background: *The First Three Axioms of the Theory of Measurements*

As we begin our exploration of experimental methods for understanding the natural world, it is reasonable to consider the fundamental question: “Is it truly possible to comprehend the World, particularly through experimental means?”

The possibility of gaining knowledge about the world has been a central question in philosophy, eliciting diverse perspectives ranging from optimistic belief in human knowledge to deep skepticism. A particularly pessimistic view is encapsulated in the three theorems attributed to the ancient sophist Gorgias, cited in the epigraph above. These statements may appear paradoxical and even discouraging. However, they have endured for nearly two and a half millennia – a testament to their profound reflection of the complex and often unconscious connections between our knowledge and the reality we seek to understand.

Mathematician D. Fon-Der-Flaass, reflecting on the structure of contemporary mathematical knowledge, observed that much of it aligns with Gorgia's words. Moreover, he expanded these assertions with a fourth, equally disheartening observation.

In physics, issues of understanding arise at the very onset of study, particularly during practical laboratory work. In our initial laboratory exercise, we will examine the nature of these issues and explore how physicists resolve them.

Defining Objectives. Our primary objective is to determine the true value of a certain physical quantity, x through measurement. Suppose we conduct n measurements and obtain a series of values x_1, x_2, …, x_n for the measured quantity. The question then arises: How can we determine the true value x_{true} of the measured quantity based on these results?

This question serves as the focal point of our course, and our exploration begins with three unsettling propositions.

First, can we assume that the result of the i-th measurement (value x_i) coincides with the true value x_{true}?

To this, there is unanimous agreement: "Of course not!" This forms our first unpleasant axiom:

1. None of the measured values of x_i will be equal to x_{true}!

Consequently, any student who reports only a single measurement in a lab report cannot expect a favorable evaluation.

At this point, many students may see an apparent solution: "Just calculate the arithmetic mean!" While finding the arithmetic mean of all measurements is indeed valuable, does it yield the true value? Unfortunately, it does not. This brings us to the second unpleasant axiom:

2. The average value of all measurements $\bar{x}$ will not coincide with x_{true}!

Lastly, the third unpleasant axiom fixes the fact that if by chance some result of measurements aligned with x_{true}, there would be no indication – no thunderclap or triumphant fanfare – to mark the occasion. In practice, such an event would go entirely unnoticed:

3. And if, by any chance, either x_i or $\bar{x}$ coincides with x_{true}, we would remain unaware.

These three fundamental assertions constitute essential principles for the theory of measurements, defining the whole philosophy of working with experimental data. To solidify your understanding of these axioms, we encourage you to verify them through experimentation.

2. Joint Laboratory Work 1: "Buffon – de Morgan experiment". Introduction to Laboratory Work Stages and Rules of Confidence Interval Construction.

1. Count Georges-Louis de Buffon, a member of the Paris Academy of Sciences, and Augustus de Morgan, the first president of the London Mathematical Society, are renowned not only for their expertise in their respective fields but also for their notable experiment involving over 5 000 coin tosses. Why did they undertake such a task? Their goal was to experimentally determine the probability of obtaining "heads" in a coin toss. Today, we will replicate their experiment, though with a significantly reduced sample size of just three tosses.

The aim of this laboratory work is to estimate the probability of obtaining "heads" in a coin toss through direct experimental measurements. We selected this specific probability as our subject of study due to the advantage of knowing its theoretical value beforehand. The probability of obtaining "heads" in a coin toss is ½ ($x_{\text{true}} = 0.5$), providing us a unique opportunity to compare our empirical findings with an established theoretical value.

In this laboratory experiment, we proceed as follows: all students present (we denote their number as N) simultaneously toss a coin on the count of "one, two, three". Those whose coins land on "heads" raise their hands, and the instructor counts the total number of raised hands (we denote this number as N_{head}), recording this result in a table.

Table 1.1 provides an example of a typical set of measurements (the last two columns will be completed later). The probability of obtaining "heads" in the i-th experiment and the arithmetic mean of the measured outcomes are calculated as follows: $x_i = N/N_{\text{head}}$ and $\bar{x} =$ $= (x_1 + x_2 + x_3)/3$. A lifehack: so that the game of chances does not raise doubts in the validity of our first axiom, the number of those who toss a coin should be odd. Therefore, if the total number of students is even, the instructor should consider participating in the coin toss as well.

Table 1.1 — Results of the 2017 Experiment

Number of a test, i	Total number of participants, N	Number of "heads", N_{head}	Probability of getting "heads" in the i-th experiment, x_i			
i = 1	21	9	0.429			
i = 2	21	13	0.619			
i = 3	21	10	0.476			
The mean $\bar{x} = \frac{x_1+x_2+x_3}{3}$			$\bar{x}$ = 0.508			

Our measurement results are both illustrative and instructive. Discussion with the students:

– Tell me please, did the result of any measurements coincide with the true value?

– No, no one matched!

– And what about the mean value?

– That didn't match either!

At this point, the students are ready to reach a key conclusion: "Through measurement alone, we cannot obtain the exact true value of the quantity being measured!"

Indeed, this is a fundamental insight in experimental physics. However, it leads us to a further question: "What should we consider as an answer in a laboratory experiment?"

2. *What is the Answer?* The answer to this question has evolved with the development of human knowledge (see Fig. 1.1). In ancient times, answers to profound questions were often ambiguous, as illustrated by the tale of Croesus, King of Lydia. Croesus once asked the Delphic Oracle, "Should I start a war?" The oracle replied, "If you start a war, a great empire will be destroyed." Confident in this prophecy, Croesus waged war, only to see his own empire fall.

The development of mathematics has led to crystallization of a specific concept of an "answer" as a single, precise number. "The answer is a number!" mathematicians often claim, and we have grown so accustomed to this idea that it seems the only valid approach. In scientific experimentation, particularly in laboratory work, the answer takes on a distinct form. Due to advances in science and technology, it has been established that *the result of an experimental measurement is not a single number but rather an interval, calculated following specific guidelines*.

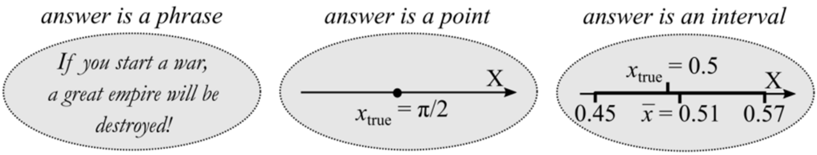

Figure 1.1 — What is the answer?

This interval is called a *confidence interval*. It is implied that this interval contains the true value of the measured quantity. Below are the steps and rules for calculating this interval, ensuring it accurately represents the experimental measurement.

a) b)

a — an interval in mathematics; b — an interval in physics

Figure 1.2 — Setting an interval

3. *Rules for Calculating Parameters of a Confidence Interval*. The representation of a confidence interval can be approached in two distinct forms. The first method specifies the interval by denoting its endpoints, which is commonly expressed as $a \leq x \leq b$ (see Fig. 1.2a). This notation is prevalent among mathematicians.

In contrast, physicists typically adopt a different approach (see Fig. 1.2b). They define the interval by identifying the coordinate of its middle $\bar{x}$ (referred to as the *mean value*), and the half-width, denoted as Δx(which is also known as the absolute uncertainty. Consequently, this interval is represented as $x = \bar{x} \pm \Delta x$.

The procedures for calculating $\bar{x}$ and Δx are outlined below (see Equations (1.1-1.2)), specifically for the scenario involving the measurement of a constant physical quantity:

1. The mean value (midpoint of the confidence interval) is equal to the arithmetic mean of all measurement results:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}. \qquad (1.1)$$

2. The half-width of the confidence interval (absolute uncertainty) is calculated by the formula

$$\Delta x = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2}{n(n-1)}} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n(n-1)}}. \qquad (1.2)$$

The quantities $\Delta x_i = x_i - \bar{x}$, whose squares are summed under the square root, represent the random deviations of the *i*-th measurement.

Calculating both $\bar{x}$ and Δx as described in the above equations is central to processing experimental data when measuring a constant physical quantity. Once these values are obtained, the result can be expressed in the standard form:

$$x = \bar{x} \pm \Delta x. \qquad (1.3)$$

With this foundation, we will proceed to analyze the data from our microlaboratory experiment.

4. *Joint Construction of the Confidence Interval*. Having already calculated the mean value $\bar{x}$ (the arithmetic mean) for our measurements, the remaining task is to calculate the half-width of the confidence interval, or the absolute uncertainty, Δx. Given the extent of these calculations, we will employ two additional columns to streamline the process.

First, we fill in the column of random deviations Δx_i, followed by a column for their squares, Δx_i^2 (see Table 1.2). After completing these columns, we sum and calculate the half-width of the confidence interval, which represents our absolute uncertainty.

At this stage, we begin to introduce to our students “two rules of good form” for working with numbers: necessity to use the exponential form for very large and very small numbers, and rules of rounding results of calculations.

Table 1.2 — Complete Table of Experimental Results

Number of a test, i	N	N_{head}	Probability of getting "head", $x_i = \frac{N_i^{head}}{N}$	Deviation of x_i from the mean, $\Delta x_i = x_i - \bar{x}$	Auxiliary column, Δx_i^2
1	21	9	0.429	– 0.079	$62.4 \cdot 10^{-4}$
2	21	13	0.619	0.111	$123.2 \cdot 10^{-4}$
3	21	10	0.476	– 0.032	$10.2 \cdot 10^{-4}$
The mean $\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$					$\bar{x} = 0.508$
Half-width of the confidence interval (the absolute uncertainty) $\Delta x = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2}$					$\Delta x = 0.0571$

5. *The First "Rule of Good Form": Using the Exponential Form.* Let's consider two calculation tasks as an illustration:

$$\frac{20}{5} = \ \dots, \qquad \frac{0.00002}{500} = \ \dots .$$

Ask yourself which of these tasks any student would solve quickly and accurately, and which one might feel overwhelming. Naturally, everyone will say, "The first task is simple; the answer is obviously 4." But in the second case, many would hesitate before answering, and it may not be clear without further calculation.

This reaction is normal. People are comfortable with whole numbers, especially smaller ones, say within the range of the first ten, or perhaps the first hundred. For instance, we can easily see that $7.9^2 \approx 60$ and $11.1^2 \approx 120$. However, when numbers become very large or very small, we tend to lose our intuitive understanding and often end up relying heavily on calculators. But what if you don't have a calculator on hand?

The solution to this issue is to adopt the exponential form, which simplifies working with both extremely large and small numbers. By converting values to those within a manageable range (1 to 10) and expressing the rest as powers of ten, we "put the zeros in a bag" and write them as an exponent of 10. For example, in the second task, we can represent 500 as $5{\cdot}10^2$, and convert the "inconvenient" number 0.00002 into a number $20{\cdot}10^{-6}$. This makes the calculation simple and straightforward:

$$\frac{0.00002}{500} = \frac{0.00002}{5 \cdot 10^2} = \frac{20 \cdot 10^{-6}}{5 \cdot 10^2} = 4 \cdot 10^{-8}.$$

In our course, we will consistently require that all numbers in laboratory reports be written in this exponential form.

6. *Rounding*. Let's return to the concept of a confidence interval (see Equation (1.3)). Once we have calculated both the mean value, $\bar{x} = 0.508$ and half-width (the absolute uncertainty) $\Delta x = 0.0571$, therefore, we can write down our result for the experiment as follows:

$$x = 0.508 \pm 0.0571.$$

From mathematical perspective, the notations $m = 0.5$ kg and $m = 0.50$ kg may seem equivalent, but in the context of measurement theory, they are not. In the first case, we're indicating accuracy to one decimal place, while in the second, we're expressing accuracy to two decimal places.

Measurement accuracy is a critical parameter. Improving accuracy by an order of magnitude (ten times) often requires fundamentally new equipment and can be thousands of times more costly. Writing all the digits a calculator provides implies a precision level that isn't realistic for most measurements. To maintain clarity and accuracy, we'll follow standard rounding rules for reporting experimental results, reflecting a realistic level of precision and avoiding overstated accuracy.

Rules for rounding the final result.

1. *Begin rounding by addressing the absolute uncertainty, Δx. The value of the absolute uncertainty is rounded to one significant figure if that figure is 2, 3, …, or 9, if the first digit is 1, then it is rounded to two significant figures.*

Examples:

$$\Delta x = 0.00\underline{3}268 \quad \Rightarrow \quad \Delta x = 0.00\underline{3},$$
$$\Delta x = \underline{5}17.634 \quad \Rightarrow \quad \Delta x = 500,$$
$$\Delta x = \underline{17}.372 \quad \Rightarrow \quad \Delta x = 17.$$

Note that this rule specifies which figure should remain after rounding; however, the actual rounding follows standard rounding rules.

2. *Once the absolute uncertainty has been rounded, proceed to round the mean value, $\bar{x}$, so that it aligns with the same order of magnitude as the rounded absolute uncertainty*.

Examples:

$x = 2.7481 \pm 0.03 \quad \Rightarrow \quad x = 2.75 \pm 0.03,$

$x = 43606.7 \pm 500 \quad \Rightarrow \quad x = 43600 \pm 500,$

$x = 542.7 \pm 17 \quad \Rightarrow \quad x = 543 \pm 17.$

By following these guidelines, the final answer for our laboratory measurement is as follows:

$$\underline{x = 0.51 \pm 0.06}.$$

7. *Relative Uncertainty of a Measurement*. In addition to stating the mean and absolute uncertainty, it is often helpful to calculate the relative uncertainty, ε. It can be calculated using the formula:

$$\varepsilon = \frac{\Delta x}{\bar{x}} \tag{1.4}$$

and is usually rounded to one or two significant figures (if the first digit is 1), and often expressed as a percentage.

The relative uncertainty helps clarify the significance of the result. For instance, stating that “Puchnastyk lost 2 kilograms” it is difficult to understand how serious the situation is, because it is not clear who Puchnastyk is: an elephant or a dog.

The relative mass change $\varepsilon = \frac{\Delta m}{m} = 0.005 = 0.5\%$ (see Equation (1.4)) is more informative. We immediately understood that nothing terrible happened!

In our case $\varepsilon = \Delta x / \bar{x} = 0.12 = 12\%$ and now we can formulate the conclusion where we will collect all the information we found.

8. *Example Conclusion for the Laboratory Work.*

Conclusion: In the laboratory work, we measured the probability of getting “head” in a coin toss. The result is $x = 0.51 \pm 0.06$. The fractional uncertainty is $\varepsilon = 12\%$.

3. General Discussion *“The fourth unpleasant axiom of the theory of measurements”*

We obtained an excellent result; our interval “captured” the true value (see Fig. 1.3). This is certainly a positive result. However, an important question remains: will a confidence interval constructed according to our method always capture the true value?

In scientific practice, it’s common to confront the conflict between logic and our psychological desire for precision. While we naturally want our method to be perfect and entirely accurate, logic offers no guarantee. Only a technically correct but ultimately uninformative answer $0 \leq x < \infty$ always captures the true value.

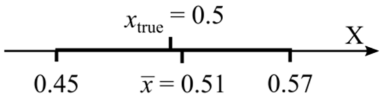

Figure 1.3 — In our laboratory work the interval caught the true value!

We have an inherent limitation: intervals calculated from measurements using this method capture the true value only in a subset of cases. Thus, we introduce our fourth axiom:

An interval, constructed according our rules, captures the true value of a measured quantity only in 2/3 of cases.

In practice, this means that in around one-third of cases, the true value will lie outside the interval we calculated. This is simply a part of the scientific process.

If a higher probability of capturing the true value is required, we can adjust the confidence interval by multiplying its width by a specific factor, known as *“Student’s coefficient”*. This coefficient is the greater, the more probability you are to provide, and its values can be found in special tables.

We always end our conversation about the World and its cognoscibility with the most important and optimistic, in our opinion, statement of A. Einstein: “The most incomprehensible thing about the Universe is that it is comprehensible!”

4. Example of the Quiz

1. A student obtained an interval for coordinates of an object at one instant: from 46.3 cm to 46.9 cm. Express this as $\bar{x} \pm \Delta x$.

2. Round the following experimental result: $h = (1.6432 \pm 0.237)$ m.

3. Find the fractional uncertainty in the following experimental result: (32 ± 8) μC.

4. Calculate the absolute uncertainty in the following experimental result: $\ell = (80 \pm ?)$ Cm, $\varepsilon = 5\%$.

5. A student measured some quantity three times, and obtained the following values: $x_1 = 69$, $x_2 = 69$, $x_3 = 63$. Present the result as $\bar{x} \pm \Delta x$.

[*Answers:* 1) $x = (46.6 \pm 0.3)$ cm; 2) $h = (1.6 \pm 0.2)$ m; 3) $\varepsilon = 25\%$; 4) $\Delta\ell = 4$ cm; 5) $x = 67 \pm 2$.]

5. Additional Tasks

1°[1]. Use reference materials to find the value of the Student's coefficient for our measurements, if the confidence interval must capture the true value with the probability of 99%.

When consulting the table of the Student's coefficients, remember:

– In simpler cases, select the appropriate coefficient from the row corresponding to $(n-1)$.

– In more complex cases, select it from the row with number $(n-2)$.

In order to give uniformity to relevant rules, the additional term *degrees of freedom* ν is introduced. In the first case we should assume that $\nu = (n-1)$, and $\nu = (n-2)$ in the second case. Using this concept, the general form of an answer is $x = \bar{x} \pm t_\nu^\alpha \cdot \Delta x$, where α is the value of confidence level (the probability that the constructed interval contains the true value). In our case number of measurements is $n = 3$, the degree of freedom is $\nu = (n-1) = 2$ (we measured the constant value), the given value of the confidence level is $\alpha = 0.99$.

2. Write a program that calculates the confidence interval (values $\bar{x}$ and Δx) using equations (1.1-1.2). The input data should be the number of measurements n and the column of measurement results x_i.

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[1] For questions and tasks marked with a symbol ° answers and solutions are given at the end of the book.

Session 2. An Introduction to the Graphical Method. Laboratory work 2 "Measurement of Time".

Practical objectives: constructing a confidence interval for real measurements, instrumental uncertainty, and representing experimental results graphically. Theoretical objectives: differentiating between two concepts, the random uncertainty of a measurement and the random uncertainty of the mean; discussing the possibility of experimental cognition of the World.

In the immobility of the mean lies the greatest World's perfection.
Confucius

1. Theoretical Background: *Learning how to Determine the Parameters of a Confidence Interval Using a Graph.*

1. *Types of Uncertainties*. There are four types of experimental uncertainties: random uncertainty, instrumental uncertainty, systematic uncertainty and a mistake.

Mistakes are easy to handle. *Mistakes* are data that clearly stand out from the rest, often due to carelessness, and should be excluded[2].Though mistakes are often easy to identify and correct, they can sometimes lead to unexpected discoveries, sometimes even paving the way toward the Nobel Prize award!

Systematic uncertainty is an uncertainty that is permanent. Example: a voltmeter with miscalibrated scale. It's essential to inspect instruments for systematic issues before measuring[3].

So, mistakes and systematic uncertainties can be eliminated, but two other types of uncertainties, random uncertainty and instrumental uncertainty, will always persist. Why is that? Let's explain.

2. *Separated yet United!* Any laboratory setup consists of two components: the object being measured and the instrument measuring it. Both contribute to the total experimental uncertainty. The instrument

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[2]*For the curious ones*. There is a statistical method (Grubbs's test), which can help to exclude, or on the contrary, to keep suspicious measurements.

[3]*For the curious ones.* There is a statistical method (E. Abbe's method), which allows us to draw a conclusion about the presence or absence of systematic uncertainty in the set of measurement data.

uncertainty Δx_{inst} arises because no instrument is perfectly precise. And the uncertainty associated with the object under the study, Δx_{ran}, arises due to randomness in its behavior.

These uncertainties can differ in significance depending on the experiment. So, in the previous laboratory work the instrumental uncertainty was zero, $\Delta x_{\text{inst}} = 0$ (it is unlikely that we made mistakes calculating the "heads"), thus randomness was born due to "random" movements of tossed coin. During this laboratory work we will deal with a system for which $\Delta x_{\text{ran}} = 0$, therefore the resulting uncertainty will be determined mainly by the instrumental uncertainty. Though, such division takes place only in some cases. In a real experiment both uncertainties appear to an experimenter as the whole, as the resultant (total) uncertainty Δx, which is measured experimentally. That is the uncertainty we will learn to find from the results of measurements.

Though we cannot directly observe the individual contributions of these uncertainties, probability theory allows us to model them as follows:

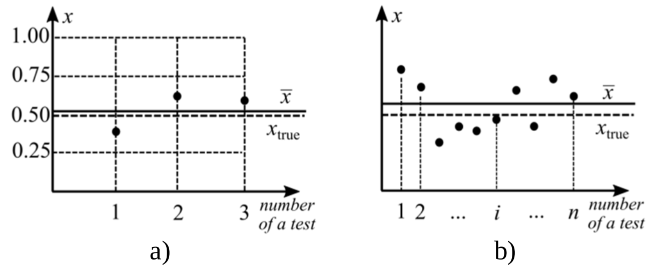
$$\Delta x^2 = \Delta x_{\text{ran}}^2 + \Delta x_{\text{inst}}^2. \tag{1.5}$$

This is the probability theory theorem about the addition of variances of independent random variables. Keep in mind, that applying this equation you should use the correct mathematical terms for the quantities Δx^2, Δx_{ran}^2 and Δx_{inst}^2, they should be interpreted as *variances of the corresponding random variables*.

Instructor's note on cunning words "Random Behavior of the Object Under Study". There is no place for mythical Randomness in modern classical physics. Classical physics wholly stands on the positions of Laplace's determinism: if we accurately formulate the differential equation of motion for all object parameters that determine the behavior of the object, and if we set their initial values with absolute precision, the behavior of the object will be described by solutions of these equations, with no element of randomness involved. Randomness is a visible result of our ignorance, when due to the fact that we did not take into account all important parameters, the actual behavior of the system differs from the expected one. Note that physicists refer to parameters that play a significant role in describing the behavior of a system, yet are not initially suspected, as *hidden parameters*.

Practical consequences of Equation (1.5): if observed uncertainty cannot be explained by the presence of an instrumental uncertainty, then the system under the study does not behave as we expect. Conclusion for the methodologist: the laboratory setup should be checked; conclusion for the physicist-theorist: it is necessary to create a new physics theory of the studied phenomenon.

In the last session we have already learned how to calculate the total uncertainty Δx for a special case of a linear relationship $y = b$. Now, we will get acquainted with the instrumental uncertainty. But first let's take a fresh look at what we discussed last time from a new perspective.



a — results of Session 1; b — when the number of data points is greater

Figure 1.4 — The visual representation of the experiment results, the experimental line (the mean value) is shown by a solid line

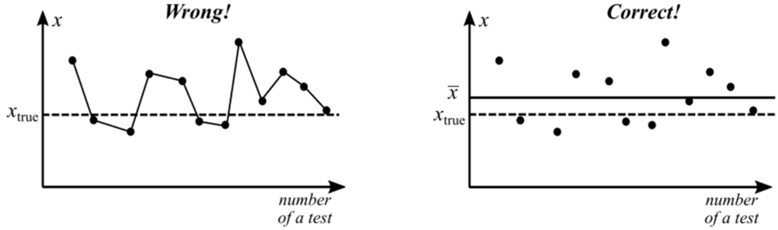
3. *The Mean Value*. In our previous session, we tossed a coin three times and got three different values for the probability of getting "head". These results of our work are displayed in Figure 1.4a. For clarity, we plotted each result on a two-dimensional plane, where the *x*-axis represents the experiment number, and the *y*-axis shows the observed probability. Additionally, we calculated the mean value of these results and confirmed, that three unpleasant statements are really true: neither the experimental values, nor the mean coincide with the true value. As a result, we formulated the main principle of experimental research: *the result of a laboratory work is not a number but rather a confidence interval, which is calculated according to certain rules*.

For many students the rules for calculating a confidence interval $x = \bar{x} \pm \Delta x$, which we formulated previous time, may seem complex, but there's good news: we can achieve a high level of accuracy by constructing it graphically. When the number of data points is relatively high, as in Figure 1.4b, the graphical approach is particularly effective.

So, let's put everything in order.

It is quite simple to draw a straight line on the graph (Fig. 1.5b) that corresponds to the mean value $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$.

The straight line corresponding to the mean value is a line for which the number of experimental points above and under is the same (taking into account the distance from the point to the line).



a) b)

a — wrong; b — correct

Figure 1.5 — Do not connect but take into account!

Experience demonstrated that all students had got this line quite well.

Exercise 1. Peter says: "Since a straight line is always drawn through two points, at least two experimental points always lie on the experimental line!". Maria objects: "As a rule, no experimental point lies on the experimental line!" Who is right?

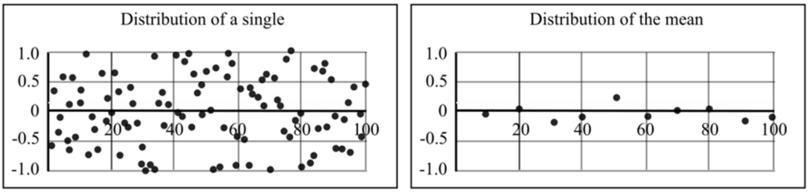
Answer: Definitely, Maria! The experimental line is not drawn through the experimental points, it should display the mean value of the experimental points. It turns out that in physics straight lines are drawn by points without connecting the points themselves.

4. *Uncertainty of a Measurement and Uncertainty of the Mean*. So, the mean value is not scary at all! Now about a really complex concept, about half-width of a confidence interval

$$\Delta x = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} \Delta x_i^2}.$$

This equation may seem complicated to us, but if we show it to mathematicians, they would say: "There's nothing complicated here! It consists of two factors: the factor $\sqrt{\sum_{i=1}^{n} \frac{\Delta x_i^2}{(n-1)}}$ and the factor $\frac{1}{\sqrt{n}}$. The first factor is denoted by δx and is called absolute uncertainty of a single measurement (standard deviation, standard)[4]. It shows how intensely the results of measurements “dance” relative to the true value. And the equation itself shows that the mean value of several results also deviates from the true value but in $\sqrt{n}$ times weaker. Just like in life: sometimes a little over, sometimes a little under, but on average, everything balances out!”

We were so impressed with these words that we used *Excel* for generating 100 points which were randomly scattered on the segment $-1 \leq y \leq 1$. Figure 1.6a illustrates the seemingly chaotic “dance” of these points. Next, we grouped these points into sets of 10, and calculated the mean value of the coordinates $\bar{y}$ for each group. As you can see (Fig. 1.6b), the “jumps” in the data are significantly smoothed out.



a) b)

a — chaotic “dance” of 100 points; b — the mean values of sets

Figure 1.6 — Illustration of the motionless of the mean

Therefore, we consider it is proven that the mean values are always closer to the true value, than individual measurements, and we adopt this idea that is expressed by the equation

$$\Delta x = \frac{\delta x}{\sqrt{n}}. \tag{1.6}$$

Here Δx is the absolute uncertainty of the mean value (half-width of the confidence interval), $\delta x = \sqrt{\frac{\sum_{i=1}^{n} \Delta x_i^2}{(n-1)}}$ is the standard deviation of a singular measurement, n is the number of measurements. It can be shown that δx does not increase and does not decrease as the number of measurements increases. Instead, it “dances” around some definite value.

In our opinion, Equation (1.6) is a cornerstone of probability theory.

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[4] Absolute uncertainty is a term of experimental physics; standard deviation (standard) is a term of probability theory.

It gives us hope that our World is cognizable: although there is always an uncertainty in each individual measurement that separates us from the true value, by increasing the number of measurements and calculating the mean, we reduce the value of the random deviation and are getting closer and closer to the true value!

Exercise 2. There is hope, but we will need to work! How many measurements need to be taken so the half-width of the confidence interval Δx is thousand times smaller than the random deviation of the one measurement δx?

Answer: a million.

The significance of the mean has long been appreciated. The ancient philosopher Aeschylus saw the love of the gods for the mean, while Confucius regarded it as the greatest perfection of the World. The concept of the mean played a significant role in in the evolution of astronomy: when Tycho Brahe applied the mean of multiple observations in his astronomical calculations, he achieved a far higher level of accuracy. The mathematical proof of the fact of the mean immobility was later provided by the mathematician Karl Gauss. *Mais, retour a` nos moutons!* (But, back to the main topic!)

5. *Graphical Definition of Absolute Uncertainties (Random Deviations)*. The mean value can be observed on a graph immediately, but we cannot directly read off the half-width of the confidence interval (the absolute uncertainty of the mean). This requires two steps.

We can find only a random deviation of a *singular measurement* δx directly from the graph (Fig. 1.7b). In order to find δx, it is necessary to place the experimental line upward, parallel to itself, until roughly 1/6 points left above it. The magnitude of the line's offset will represent the random deviation of a singular measurement.[5]

After that, dividing δx by $\sqrt{n}$, i.e. using Equation (1.6), we find a random deviation of the mean value (half-width of the confidence interval). Figure 1.7 illustrates this sequence of steps.

Note that you can move the experimental line downward, or both downward and upward, and then take the mean.

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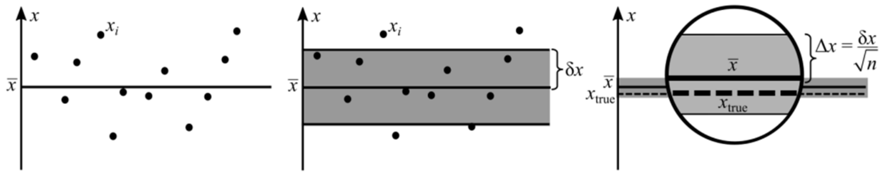
[5]*For the curious ones*. This recipe is one hundred percent valid for normal distribution. For an even distribution 1/5 of the points should be left over the line. Because there is no need for high accuracy for uncertainty, for practical use, this recipe can be considered quite acceptable.



a — single measurements; b — the random deviation; c — the absolute uncertainty

Figure 1.7 — From single measurements to the random deviation of a single measurement, and then to the absolute uncertainty of the mean

That covers the main steps to remember. Now, processing the results of the experiment, we will still use calculations. But next time, when we feel like “old birds”, we will construct a confidence interval by using just a ruler.

6. *Instrumental Uncertainty*. No measuring device is perfect, just as there are no ideal observers. Therefore, instrumental uncertainty is always present in measurements. At least because we cannot read values on a device’s scale with arbitrarily high accuracy.

Probably you know how to behave in such a situation. In school laboratory work, you were likely taught the rule: “Assume the instrumental uncertainty δ is equal to half of a scale division”. You may also recall that if there is no instrumental uncertainty then we put a simple point on the graph (Fig. 1.8a). However, if an instrumental uncertainty is present and equals δ then we represent the measurement result not only by a point but by a vertical error bar of length 2δ (Fig. 1.8b). Why is this necessary?

Remember the following rule of the geometrical method: *if an experimental line can be carried out so that it passes through all error bars, then we can assume that experimental uncertainty can be explained by presence of the instrumental uncertainty. Otherwise, it cannot!*

The main property of the instrumental uncertainty. Unlike the resulting uncertainty, Δx which is determined after the experiment (a posteriori), the instrumental uncertainty Δx_{inst} is determined before the experiment (a priori). In the simplest case, you only need to look at a scale and apply the school rule. For more complex cases, testing the measuring system becomes necessary.

Now we will measure time, and here is something to think about.

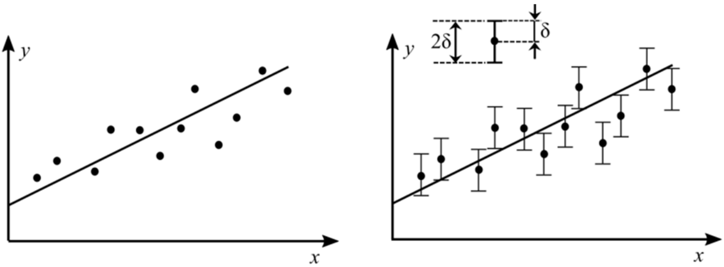
– If we use a regular clock with a second hand, the absolute instrumental uncertainty is equal…?

– Half a second!

– Correct, $\delta = 0.5$ s. And if we use a modern electronic stopwatch, which displays, for example, the value 12.04 s?

– Then the scale marking is equal to 0.01 s, and the instrumental error is $\delta = 0.005$ s.

– It is correct! And here is the key question for you: If Peter measures the time object is falling using an electronic stopwatch, will he be right if he says that the instrumental uncertainty of his measurements is equal to $\delta = 0.005$ s?



a) b)

a — if the instrumental uncertainty is small then the point marked by a pencil completely covers it; b — if the instrumental uncertainty δ is significant it is displayed using the segment of height *2*δ the random deviation

Figure 1.8 — Instrumental uncertainty

Microexperiment. Take a regular ruler and hold it vertically. Ask a volunteer to try to catch the ruler as soon as you release it. Measure the distance the ruler falls before the volunteer catches it.

Discussion of the microexperiment. If you do not warn the volunteer, the ruler usually falls down about 15 cm before being caught. However, if the ruler is released after a countdown “one, two, three”, the result improves to about 7 cm. We will use this case to estimate the reaction time of an observer during time measurements. We obtain that the reaction time of an observer is equal to $\delta = \sqrt{\frac{2h}{g}} = 0.12\ \mathrm{s} \approx 0.1\ \mathrm{s}$. This is fully consistent with the data of physiology.[6] And it is clear that the instrumental

[6]After we decide to move our fingers, the nerve impulse begins to move from the brain to muscles of our fingers. The speed of nerve impulse propagation is about 10 m/s; for distance of 1 m we obtain 0.1 s.

uncertainty of a complex measuring system “Peter with an electronic stopwatch” will be defined by this value.

Now we will work with a system that gives well-reproducible results, i.e. for which $\Delta x_{\text{ran}} = 0$. So, it is reasonable to assume that the instrumental uncertainty contributes the main part to the uncertainty. Under this assumption it is reasonable to expect that the absolute uncertainty of a singular measurement will be about $\delta t = 0.2\ \text{s}$ (taking into account that we introduce error twice: when starting and stopping the stopwatch). It will be interesting to compare this value with the measured one.

Remark. It is necessary to interpret the term “instrumental uncertainty” correctly. Consider a simple model of a measuring device, which we will refer to as “a digital device” (the exact measurement result is rounded to the nearest marking). In this context, the instrumental uncertainty is a random variable that evenly distributed across an interval 2δ wide. Therefore, 2δ is a base of evenly distributed random variable, and the value Δx_{inst}^2 that should be substituted in Equation (1.5) is a variance of the random variable, the *instrumental uncertainty*. Probability theory gives that $\Delta x_{\text{inst}}^2 = \frac{\delta^2}{3}$. The same principle applies to the rounding error of a tabular value.

2. Recommendations for the Laboratory Work

This session serves as a bridge between the calculation method, which is traditionally used for construction of a confidence interval for a physical quantity, and the geometrical method, which is traditionally used to work with linear relationships. The main methodological part of this session is to clearly distinguish between two concepts: the concept of absolute uncertainty of a singular measurement and the concept of absolute uncertainty of the mean value. Without such distinction, the geometrical method cannot be applied correctly.

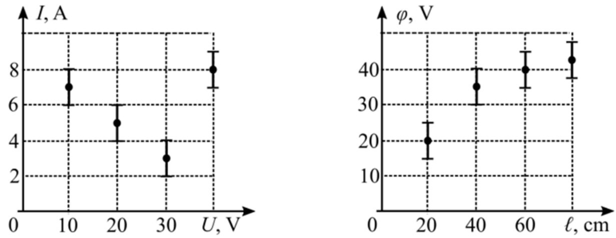
3. General Discussion “*Measurement of Time*”

Usually, we discuss here the question of why, as far as time is concerned, the number 60 appears: an hour is 60 minutes, a minute is 60 seconds? And, if time permits, we also discuss why the hands of all clocks rotate in what we call the “clockwise” direction?

4. Example of the Quiz

1. Figure 1.9a shows the results of an experimental study, including the instrumental uncertainty. *A*) What is the value of the instrumental uncertainty δ_I? *B*) What is the value of the current measured during the experiment when the voltage equals 40 V? *C*) What is the relative uncertainty of the current ε_I when the voltage equals 40 V?

[*Answer*: *a*) δ_I = 1 A; *b*) I = 8 A; *c*) ε_I = 13%.]



a) b)

a — for question 1; b — for question 2

Figure 1.9 — Results of experimental studies for the quiz

2. Figure 1.9b illustrates the results of an experimental study with the instrumental uncertainty. *A*) What is the value of the instrumental uncertainty δ_φ? *B*) What is the value of the electric potential at a distance of 60 cm from the voltage source? *C*) What is the relative uncertainty ε_φ in this measurement?

[*Answer*: *a*) δ_φ = 5 V; *b*) φ = 40 V; *c*) ε_φ = 13%.]

5. Additional Tasks

1. We used the distance the ruler fell to determine the reaction time. The opposite can also be done: using accurate time measurements to determine distances. Try using the GPS system on your mobile phone to determine … radius of planet Earth!

2°. Write a program to determine whether it is possible to draw a straight line through all error bars. Input data: table of measurements (x_i, y_i) and the value of the instrumental uncertainty δ_y. Consider the following cases:

a) the first special case of a linear relationship $y = b$;

b) the second special case of a linear relationship $y = kx$.

TOPIC 2. LINEAR RELATIONSHIP

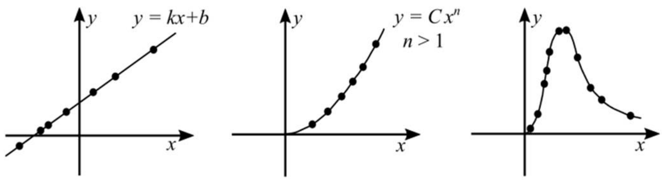
Session 3. First Acquaintance with a Linear Relationship. Plotting a Linear Relationship in Laboratory Work 3 "Measurement of the Universe Age"

In this session, students will explore the concept of a linear relationship for the first time. They will plot an experimental straight line using adapted experimental data from E. Hubble observations. The primary goal of this work is to determine the slope of the experimental line and calculate the age of the Universe. The final result will include a confidence interval for the lifetime of the Universe, derived according to Hubble's methodology. To construct this interval, students will use the given value of the relative measurement uncertainty.

1. Theoretical Background. *Linear Relationship*

In this laboratory work we shift our focus from physics objects and their properties to concepts of a completely different logical nature – linear relationship. Physics objects possess specific physics properties, and these properties are characterized through measurements. Similarly, relationships (such as physics laws) also have certain properties, which can be identified in much the same way – by analyzing measurement results. In this laboratory work, we will discuss the characteristics of linear relationships and how these characteristics (parameters) can be determined experimentally.

1. *Uniqueness of a Linear Relationship.* Having obtained experimental data, a researcher records it in a table. However, simply viewing the data in a table does not reveal any laws. To uncover the relationships, the laws, the experimental data must be plotted. A well-constructed graph allows us to interpret information far more efficiently – millions of times faster, in fact – than by studying a table of numbers.



a) b) c)

a — linear; b — power; c — more complex

Figure 2.1 — The types of graphs

Experimental graphs can be different: linear, power or more complex (Fig. 2.1). We will refer to the last ones as general relationships.

Among all possible experimental relationships, the linear relationship stands out as the most fundamental (Fig. 2.1a). It serves as the basis for understanding and analyzing other types of relationships.

Here are four unique properties that distinguish a linear relationship:

1) It is *easy to find* this relationship on the graph: A linear relationship is straightforward to recognize on a graph. You only need a ruler, which acts as a universal tool for verifying linear relationships.

2) It is *easy to* “*pack*” this relationship in a mathematical formula.

3) A linear relationship allows to make *extrapolation* – to find the value of the measured quantity in those areas where we cannot carry out direct measurements (e.g., superintense electric or magnetic fields, extremely high or low temperatures, etc.). To extrapolate, simply extend the line in required direction on the graph.

4) A linear relationship allows to perform *interpolation* – to find the value of the measured quantity between measured points.

Given these advantages, it is reasonable to plan the experiments so that their results perform a linear relationship.

2. *Definitions*.

A linear function[7] is a function in which the independent variable x is only to the first power (on the right part). For example:

$$y = kx + b,$$

where k is the slope (the angular coefficient, the gradient), and b is the y-intercept.

This function got its name “linear” because its graph is a straight line.

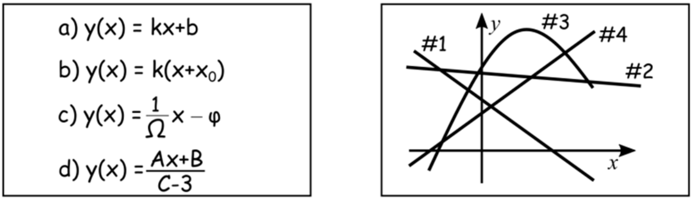
This concept is fundamental to what we will discuss next. So, let’s check your understanding of this idea.

Exercise 1. *a*) Which of the relationships in Figure 2.1a are linear? *b*) Which lines in Figure 2.2b are graphs of linear relationships?

To answer the first question, we need to focus on the variable x. It is

[7] Mathematicians prefer the word “function” to the word “relationship”.We will use both terms.

the “figure” (the main protagonist), while all the other variables serve as the “background” (decorations and costumes). What is the power of variable x on the right part of our equations? The power is 1! Hence, all these equations represent linear relationships.



a) b)

a — for question a; b — for question b

Figure 2.2 — Where is a linear relationship?

As for the second question, the answer is straightforward: all *straight lines* in Figure 2.2b represent graphs of linear relationships. However, the parabola (curve №3), is clearly not a graph of a linear relationship.

Additional Definitions. We will refer to an equation given in mathematics textbooks as $y = kx + b$ the *standard form of a linear relationship*. If an equation is written as $y = k(x + x_0)$ then we will call it the *linear relationship with an offset argument*.

3. *Definition of Parameters*. How can we derive a mathematical equation for a linear relationship based on an experimental line?

A straight line is determined by two points. This means we can choose any two points on the line and determine all parameters of the line by their coordinates (Fig. 2.3a). Corresponding formulae have the form:

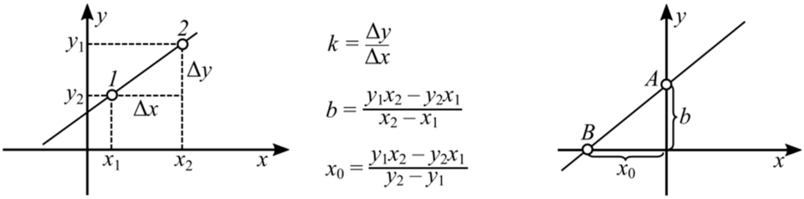
$$k = \frac{\Delta y}{\Delta x}, \qquad b = \frac{y_1x_2 - y_2x_1}{x_2 - x_1}, \qquad x_0 = \frac{y_1x_2 - y_2x_1}{y_2 - y_1} \tag{2.1}$$

where $\Delta y = y_2 - y_1$, $\Delta x = x_2 - x_1$, and (x_2, y_2), (x_1, y_1) are coordinates of the second and first points, selected on the experimental line (Fig. 2.3a).

It is essential to memorize these formulas and learn how to apply them.

Exercise 2. Derive these formulae (2.1). *Hint*: If the point lies on a straight line, its coordinates satisfy the equation of the line.

Let's clarify one important point: the equation used to calculate the slope is simple and can be easily memorized[8]. However, the formulae for *y*-intercept and the offset are quite complex. They are usually replaced by simpler rules: the parameters b and x_0 are equal to the lengths of the corresponding segments, which the experimental line cuts off on the axes (Fig. 2.3b).



a) b)

a — general rules; b — specific cases

Figure 2.3 — Definition of a straight line parameters

Exercise 3. Demonstrate that these separate rules for b and x_0 follow from the general formulae. *Hint*: for point *A* (Fig. 2.3b) we have $x_A = 0$, for point *B* $y_B = 0$.

Exercise 4. Why are general formulae (Fig. 2.3a) "stronger" than individual formulae (Fig. 2.3b)?

Answer: First of all, the general formulae contain specific ones; secondly, they remain valid even when only a small piece of a graph is available, and thirdly, they automatically provide the correct sign of the *y*-intercept and the offset.

4. Three Essential Notes for a Novice Experimenter.

1) Due to uncertainties, the experimental points are always shifted relative to their actual position. Therefore, you should not expect them to lie perfectly on a straight line.

You can approach this situation in different ways. One option is to simply connect neighboring points in series, as children often do (Fig. 2.4a). While this method might be excusable for children, who tend to

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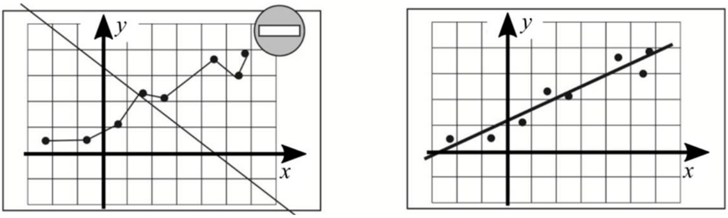
[8]It is often read as follows: the slope is equal to the tangent of the angle between an experimental straight line and the horizontal but this is not entirely correct from a physics point of view. Could you say why?

think in terms of simple pairwise relationships; for us, such an approach is a significant error!



a) b)

a — wrong; b — correct

Figure 2.4 — Two different types of thinking

As physicists, we must think in terms of multiple relations and grasp all points at once. Therefore, it is necessary to gain some courage and draw *ad libidum* one (only one!) straight line that "justly" takes into account locations of all experimental points (Fig. 2.4b). Try to make it in a way that the number of points above and below the line be approximately the same (taking into account the distance to the line). Note that it is entirely possible that none of the experimental points lies on the line.

2) Two terms should be distinguished: "experimental points" and "points on the experimental line". Let us emphasize this again: to calculate the parameters of an experimental line, it is necessary to take two points on the experimental line (instead of two experimental points)!

3) Select these two points away from each other and the reading uncertainty will be less.

5. Now, let us construct a real experimental straight line and measure its slope for the first time. The subject of our study will be a linear relationship with an a priori zero $y = kx$. We define a linear relationship as *a linear relationship with an a priori zero* as one which is known in advance, i.e. before the experiment. It is clear that an experimental line must pass through the origin. Mathematicians call this special case as *direct proportionality*. Direct proportionality $y = kx$ is a special case of a linear relationship $y = kx + b$ and has only one parameter, the slope k.

In order you will be able to get an answer to the laboratory work as it should be, in a form of a confidence interval, we provide the relative uncertainty of the final result ($\varepsilon = 3\%$).

2. Recommendations for the Laboratory Work

1. In this laboratory work the *adapted* Hubble's data of measured galaxies speeds are processed.

2. To ensure the accuracy of graphical constructions, we calculated the confidence interval parameters using statistical formulae and obtained:

for the Hubble constant

$H = (1.64 \pm 0.05) \cdot 10^{-7} \mathrm{s}^{-1}$ with the relative uncertainty $\varepsilon_H = 3\%$,

for the lifetime of the Universe

$T = (6.08 \pm 0.18) \cdot 10^{16} \mathrm{s} = (1.93 \pm 0.06)$ billions of years with the relative uncertainty $\varepsilon_T = 3\%$.

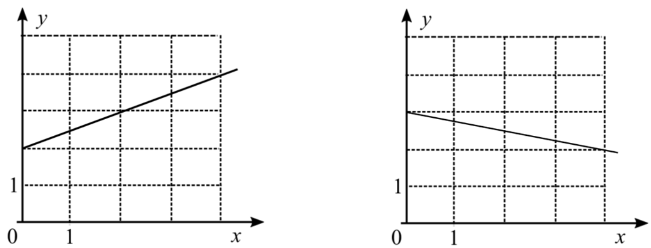
3. General Discussion "*The Center of the Universe*"

In this section, we typically discuss whether the scattering of galaxies discovered by E. Hubble proves that planet Earth, the Solar system, or our galaxy occupies the center of the Universe.

4. Example of the Quiz

1. Figure 2.5a shows a graph of the linear function with positive values of the slope and the y-intercept. Find: a) the value of the slope k; b) the value of the y-intercept b; c) write the equation of the line and calculate the value of y at $x = 8$ (i.e. perform extrapolation).

Answer: a) $k = 0.5$; b) $b = 2$; c) $y(x) = 0.5x + 2$, $y(8) = 6$.]



a) b)

a — for question 1; b — for question 2

Figure 2.5 — Graphs of the linear function for the quiz

2. Figure 2.5b shows a graph of a linear function with one negative

parameter. Find: *a*) the value of the slope k; *b*) the value of the *y*-intercept b.

[*Answer*: *a*) $k = -0.25$; *b*) $b = 3$.]

5. Additional Tasks

1. We consider this case as "a linear relationship with a priori zero". The following formulae are offered to calculate the confidence interval for the slope:

$$\bar{k} = \frac{1}{n}\sum_{i=1}^{n} k_i, \qquad \Delta k = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}\left(k_i - \bar{k}\right)^2}, \tag{2.2}$$

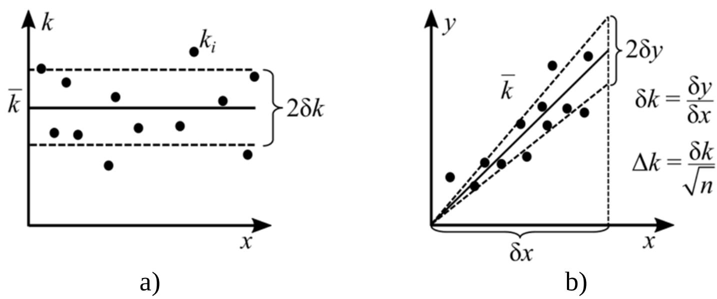
where $k_i = \frac{y_i}{x_i}$.

a) Write a program that calculates the parameters of the confidence interval of the slope in this case.

b) Use the program to calculate the mean value of the Hubble constant H and compare this result to the value obtained in the laboratory work by the graphical method.

Remark after the Lecture "Graphical Interpretation of Formulae (2.2)".

When assuming a priori zero, each measurement directly provides a value for the slope $k_i = \frac{y_i}{x_i}$. Formulae (2.2) are completely similar to the formulae we used in the previous laboratory work: the first formula determines the mean value of the slope $\bar{k}$, and the second is the absolute uncertainty (standard deviation) of the mean value Δk.



a — data on the plane (x, k); b — data on the plane (x, y)

Figure 2.6 — The slope and its absolute uncertainty on the different planes

However, the graphical method for determining these parameters requires a slight modification. On the plane (x, k), as before, the parameters $\bar{k}$ and Δk can be determined by drawing the corresponding horizontal lines according to the rules formulated in the previous laboratory work (Fig. 2.6a). In ordinary coordinates (x, y) these parallel lines correspond to a beam of lines coming from the origin[9] (Fig. 2.6b).

Therefore, the rules for the graphical determination of the slope can be reformulated as rules of "swinging" of the experimental line: *the mean value* $\bar{k}$ *is determined by drawing a line that divides the experimental points into two equal groups. To determine the standard deviation of a singular measurement you should draw lines in a such way that above/below remains 1/6 of the total number of points.*

Furthermore, it is essential to take as an inviolable rule: using a graph we can determine only the standard deviation of a singular measurement δk; the half-width of a confidence interval for the slope Δk (the standard deviation of the mean) should be calculated using the formula (1.6): $\Delta k = \frac{\delta k}{\sqrt{n}}$.

[9]In the affine transformation of the plane, lines extending from one point remain straight lines coming from one point. For parallel lines the general point is infinity, for the corresponding ones "new lines" it is the origin.

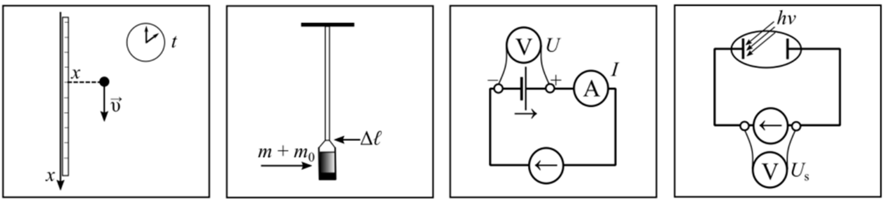
Session 4. How Not to Get Lost in Symbols. Plotting a Linear Relationship in Laboratory Work 4 "Measurement of the Liquid Viscosity by Stoke's Method"

In this session, students will build a graph of a linear relationship for uniform motion of a small ball in oil and determine its slope (velocity of the ball). The velocity will then be used to find the dynamic viscosity of the liquid. As in previous laboratory work, students will use the given relative uncertainty of measurements.

1. Theoretical Background. *"Why do we sometimes say 'speed' instead of 'slope' and talk about the x-coordinate instead of the y-coordinate?"*

Our main task during this laboratory work is to build a graph of a linear relationship for the uniform motion of a small ball in oil. However, in the theoretical part we have *pons asinorum*[10].

1. *What Physicists Study and What Mathematicians Think About It.* Physicists study various phenomena that lead to linear relationships (Fig. 2.7).



a) b) c) d)

a — free fall; b — elongation of a wire; c) electrical current; d) photoelectric effect

Figure 2.7 — Phenomena studied by physicists

They do not spare letters when denoting physical quantities. In physics, each physical quantity is assigned a unique name and a specific symbol (Fig. 2.8).

a) $x(t) = x_0 + vt$ *c*) $eU_s = h\nu - A$

b) $U(I) = \mathscr{E} - rI$ *d*) $\Delta\ell = \frac{4\ell_0 g}{\pi E d^2}(m + m_0)$

Figure 2.8 — Formulae written by physicists

[10]Pons asinorum (Latin), le pont aux ânes (French) is the "bridge of donkeys", the material is simple in itself, but causes difficulties for beginners. "Stumbling block" is the synonym.

Mathematicians:

1) Reduce all linear physics relationships to one formula $y = kx + b$.

“It is more correct”, they explain, “because a linear relationship is the only one, and physics laws are simply its various realizations.” It is difficult to disagree with this statement.

2) Teach students that one of the changing quantities should be called *independent* or *controlled*, denoted by the letter x. Its value should be plotted along the horizontal axis. The second quantity should be called a *dependent* or *measured* value, denoted by the letter y, and plotted along the vertical axis.

3) Give a universal recipe for finding the slope: $k = \frac{\Delta y}{\Delta x}$.

2. *Different Views*. The perspective of mathematicians on dependent and independent variables have become so ingrained in our thinking that we tend to consider it to be the most natural and universal way of reasoning. However, this is not entirely accurate. When studying physical phenomena, there are situations where this approach can lead to a` la confusion[11].

For example, how to understand which quantity is independent (the cause) and which one is dependent (the consequence)? What causes what: voltage the current or current the voltage? Everyone can have an opinion and we will not find arguments pro et contra. In reality, we are not limited to studying only those physical quantities that are directly related as cause and effect. For instance, we could analyze how the number of students on a beach correlates with the number of cherries in a garden. These quantities clearly do not cause one another; rather, they are both influenced by the same external factor – an increase in sunlight.

The same principle applies to the concept of controllability. It may seem that by connecting various current sources with labels “5 V” or “10 V” to an electrical circuit, we change the voltage in a controlled way. In fact, the voltage at the source terminals depends on the resistance. There are no sources of fixed voltage in Nature, just as there are no sources of fixed current, fixed force, or temperature. Therefore, you should not rely on the mathematical method of reasoning if you feel that it stops working. In this case it is better to move to the correct physics way of thinking.

3. *The Correct Way of Thinking in Physics*. In physics, we must approach problems in a particular way: *In an experiment, the relationships*

[11]La confusion (fr.) is ambiguity, confusion.

between different physical objects are studied, and the measured physical quantities do not cause one another. Instead, they simply characterize the interaction of objects from two different perspectives.

Simply put, we are free to choose which value to plot along the horizontal axis and which one to plot along the vertical axis. To emphasize this way of thinking in laboratory work, we will refer to the corresponding values as *horizontal variables* and *vertical variables*.

4. *Generalization.* Now that we understand that a variety of physical quantities can be plotted along the axes, we can generalize the standard formula for the slope $k = \frac{\Delta y}{\Delta x}$ in case of arbitrary notation. Let's do it like this:

$$k = \frac{\Delta y}{\Delta x} \qquad \Rightarrow \qquad k = \frac{\Delta(\text{vertical})}{\Delta(\text{horizontal})}$$

Now we are ready to work with any notations.

5. In the practice of processing experimental data, we can distinguish three cases where there is a well-established convention regarding which quantities should be considered as horizontal variables and as vertical ones.

1) *Tradition.* Time is always set aside as something primary[12]. Therefore, for example, in a study of uniform motion, time is usually considered as the horizontal variable, while the coordinate is treated as the vertical variable (Fig. 2.9a).

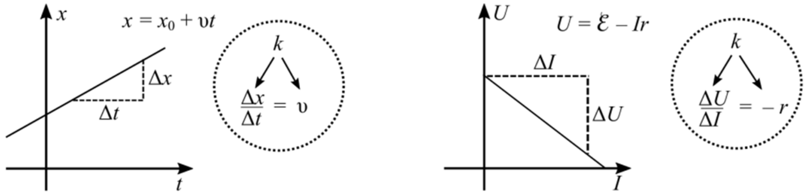
Exercise 1. *a*) How can the formula for the slope be written in this case? *b*) How does the slope relate to the speed in this case? *Note*: The standard notation for the law of uniform motion has the form $x = x_0 + \upsilon t$.
[*Answer*: *a*) $k = \frac{\Delta x}{\Delta t}$; *b*) $k = \upsilon$ (the slope is numerically equal to the speed).]

2) *Convenience.* It is convenient to choose the horizontal and vertical variables so that the slope is numerically equal to the value we want to measure. This approach allows us to read the answer and the measurement uncertainty directly from a graph. In the previous case, it turned out that the slope is equal to the measured value (speed). However, usually you have to take care in advance.

Exercise 2. The relationship $U = \mathscr{E} - rI$ is investigated experimentally. The purpose of the measurements is to determine the

[12]In Greek mythology, the god of time, Cronus, was considered the first by birth and absolutely inexorable.

internal resistance of the battery r. What variables should be plotted as horizontal and vertical ones so that the slope is numerically equal to the internal resistance?

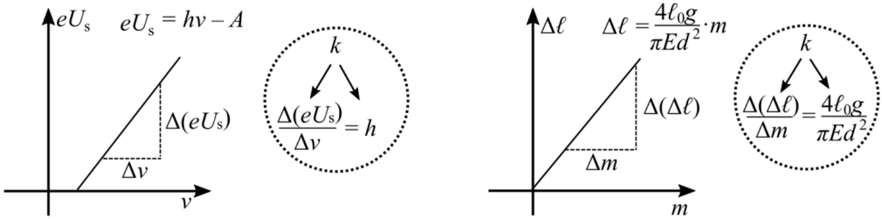


a) b)

a — for exercise 1; b — for exercise 2

Figure 2.9. The slope from the point of view of an experiment and a theory

Solution. We start with Ohm's law $U = \mathscr{E} - rI$ for two measurements, and subtract one equation from the other. Then we get $r = \left|\frac{\Delta U}{\Delta I}\right|$. For this equation to be the slope, I should be plotted along the horizontal axis and U along the vertical axis (Fig. 2.9b). With this choice of axis, the equality $|k| = r$ will hold.



a) b)

a — for exercise 3; b — for exercise 4

Figure 2.10 — Graphs plotted by physicists

3) *Control*. In some cases, we can control one of the quantities. In such situations it is natural to choose it as the horizontal variable.

For example, when studying the Einstein equation for the photoelectric effect: $eU_\text{s} = h\nu - A$ we control the frequency of the incident light. Therefore, it is natural to choose this value for the horizontal variable (Fig. 2.10a).

Exercise 3. Write down the formula of the slope if values of the frequency ν are plotted along the horizontal axis and the values of eU_s are plotted along the vertical axis. How is the slope related with the physical constants included in Einstein's equation?

Solution. The formula for the slope has the form $k = \Delta(eU_s)/\Delta\nu$. From the Einstein's law for the photoelectric effect $eU_s = h\nu - A$, it follows that: $|\Delta(eU_s\ )\ /\ \Delta\nu| = h$. By comparing both expressions, we find that for this choice, the equality $k = h$ holds.

6. And finally, one more problem. It often happens that the slope itself does not have a special name, it is simply referred to as "the slope", and that's it. However, the physical quantities, through which it is expressed, have their own names. Therefore, you need to be able to express a constant to be determined in an experiment, through the slope obtained from the experimental graph.

Exercise 4. Hooke's law $\Delta\ell = \frac{4\ell_0 g}{\pi E d^2} m$ for a wire is studied experimentally (Fig. 2.10b). The load mass *m* is chosen as the horizontal variable and the wire elongation as the vertical one. Write down the formula for the slope. How is the slope related to characteristics of the wire? How can the Young's modulus E, knowing the slope, be calculated?

Solution. The formula for the slope has the form $k = \frac{\Delta(\Delta\ell)}{\Delta m}$. From the Hooke's law follows that $\frac{\Delta(\Delta\ell)}{\Delta m} = \frac{4\ell_0 g}{\pi E d^2}$. Comparing both equations, we find $k = \frac{4\ell_0 g}{\pi E d^2}$. From this equation we obtain $E = \frac{4\ell_0 g}{\pi d^2}\frac{1}{k}$.

In conclusion, we offer the following recommendation.

When starting to plot a graph for a laboratory work, first decide how a formula of the slope will look like in chosen horizontal and vertical variables. Then identify how the physical quantity you aim to define will be expressed using the slope.

2. Recommendations for the Laboratory Work

This laboratory work implements Stoke's method for determining the coefficient of dynamic viscosity by measuring the speed of a ball falling through oil. In the standard version of the laboratory work, the experimenter records the moments of time the ball passes markings on a ruler using an electronic stopwatch. We propose recording slow-motion video of the ball's movement by a mobile phone camera and then use several shots from this video for analysis.

Based on the obtained experimental data, students plot an experimental line and determine its slope (velocity of the ball). After that they calculate the coefficient of dynamic viscosity of the oil. As in previous works, the value of the relative uncertainty of the final result is given to students. The outcome of the work is a confidence interval for the coefficient of dynamic viscosity of the oil.

For Instructors. 1) Before starting the experiment, it is necessary to find out the slowing coefficient of the video camera. To do this, record any process whose duration can be simultaneously measured with an ordinary stopwatch. By comparing indicators of the stopwatch to the time measured from the last video shot, you can determine the slowing coefficient. Usually, it is 4. It means that if the ball position corresponds to a time interval of 16 seconds, the real time for this interval is equal to 4 seconds.

2) You can verify the quality of the graphical method using analytical calculations according to the formulae provided in paragraph 5.

3. General Discussion *"In What Time Will a Ball Twice as Large Pass 20 cm?"*

At the general discussion, students are asked to determine, using their graphs, the time taken by the ball in the experiment ($d = 2$ mm) passes a distance of 20 cm, and calculate how long it will take for the ball twice as large ($d = 4$ mm). In other words, they are asked to solve a comparison problem. The answers are checked through direct measurement on an experimental setup.

4. Example of the Quiz

1. In a laboratory work, students were asked to check the validity of the relationship $U = \mathcal{E} - rI$ for a battery. To check this ratio, students connected various current sources to the battery (Fig. 2.7c) and measured the current in the circuit and the voltage across the battery in each case.

How are the parameters of the linear relationship k and b related to the battery parameters $\mathcal{E}$ and r, if: *a*) the current I is chosen as the horizontal variable, the voltage U as the vertical one; *b*) the voltage U is chosen as the horizontal variable, and the current I as the vertical one?

[*Answer*: *a*) $k = -r, b = \mathcal{E}$; *b*) $k = -1/r$, $b = \mathcal{E}/r$.]

2. In a laboratory work, the validity of Hooke's law $\Delta\ell = \frac{\ell}{E\cdot S}F$ for a wire was checked. Here $\Delta\ell$ is elongation of the wire, F is the value of the tension force, ℓ, S, E are parameters of the wire (its length, cross-sectional area, and Young's modulus of the material from which the wire is made). Suggest which values should be placed along the horizontal and vertical axes so that the slope equals to the Young's modulus E of the wire material.

Hint: In the theory of elasticity such physical quantities as mechanical stress $\sigma = \frac{F}{S}$ and relative elongation $\varepsilon = \frac{\Delta\ell}{\ell}$ are very popular, and for these quantities, Hooke's law has especially simple form $\sigma = E\varepsilon$.

[*Answer*: (ε, σ).]

5. Additional Tasks

1. Determine the viscosity coefficient of unsweetened honey.

2. We classified this case as "linear relationship with the assigned zero" (it is proposed to accept the first experimental point as zero in the laboratory work). For this case, we propose the following formulae for calculating the confidence interval for the slope:

$$\bar{k} = \frac{1}{I_c}\sum_{i=1}^{n}(x_i - x_c)^2 k_i, \tag{2.3}$$

$$\Delta k = \frac{1}{\sqrt{I_c}}\sqrt{\frac{1}{(n-2)}\sum_{i=1}^{n}(x_i - x_c)^2(k_i - k_c)^2}, \tag{2.4}$$

where $x_c = \frac{1}{n}\sum_{i=1}^{n} x_i$, $\bar{y}_c = \frac{1}{n}\sum_{i=1}^{n} y_i$ are the coordinates of the "center of mass" of the system of experimental points, $I_c = \sum_{i=1}^{n}(x_i - x_c)^2$ is the "moment of inertia" of the system of experimental points relative to its "center of mass", $k_i = \frac{(y_i - \bar{y}_c)}{(x_i - x_c)}$.

Formulae (2.3-2.4) are standard for determining the mean and the standard deviation of the slope (see, for instance, [2, 4-5]).

a) Write a program to calculate the confidence interval parameters for the slope.

b) Use the program to calculate the mean value of the velocity of the ball and compare it to the value obtained using the graphical method.

c) Calculate the relative uncertainty of the velocity in your experiment and compare it to the one that was offered to you as a known quantity.

TOPIC 3. LINEAR RELATIONSHIP AS AN OBJECT OF MEASUREMENT

Session 5. Building a Confidence Interval for Intercepts of a Linear Relationship. Application in Laboratory Work 5 “Hook’s Law. Measurement of the Mass of a ‘Black’ Load”

In this session, we have two main objectives:
1. *To compile all previously learned methods for processing experimental data into a single complex (the Small Diagram).*
2. *To generalize the method for constructing a confidence interval for intercepts in the general case.*

1. Theoretical Background. *A Small Diagram*

Today, we will carry out real research. Specifically, we will determine the mass of the “black” load. Are you familiar with the concept of a “black box”? In aviation, it refers to a special orange device that records all events occurring on a plane.

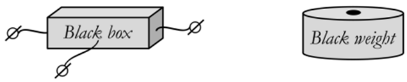


Figure 3.1 — A “black” box and a “black” load

At physics sessions a black opaque box with an electrical circuit inside (with only contacts exposed) is referred to as a “black” box (Fig. 3.1). Your task is to determine what is inside this box using only measurements. Similarly, a “black” load refers to an object of unknown mass. Today your objective is to determine the mass of the “black” load without using scales.

1. *Reflection on Previous Laboratory Work*[13]. Let us take a look at what we have already known:

– So, what is the answer to a laboratory work?

– The answer to a laboratory work is an interval (a confidence interval) $x = \bar{x} \pm \Delta x$!

---

[13]Reflection is the process of thinking about one’s own thinking. In this contest, it refers to the analysis of previously studied material.

– What value is characterized by the quality of our measurements?

– The quality of our measurements is characterized by the relative uncertainty $\varepsilon = \frac{\Delta x}{\bar{x}}$!

– So, an experimenter must be able to find three quantities using the measurements results: $\bar{x}$, the mean value (the middle of the confidence interval), Δx, the half of the width of the confidence interval (absolute measurement uncertainty), and ε, relative uncertainty.

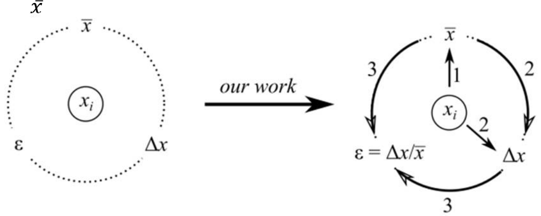
And now let's recall how we found these quantities in previous laboratory works.

We started with the case when the purpose of our measurements was to find the value of a *physics quantity* (for example, the time of load lowering in Laboratory Work 2). The *diagram of our activities* is illustrated in Figure 3.2a. We placed the results of our measurements in its center and there are three quantities which needed to be found on the outer circle. Let us draw the sequence of steps we moved during the laboratory work.

– The first step we did was determination of a confidence interval midpoint (mean value) $\bar{x}$ (Fig. 3.2b). It was equal to the arithmetic mean of all measurement results $\bar{x} = \frac{(x_1+x_2+\cdots+x_n)}{n}$.

– In the second step, we determined the absolute uncertainty Δx. To find it, we needed to use both, the measurement results themselves x_i and $\bar{x}$. Thus, to illustrate the second step, we use two arrows (Fig. 3.2b).

– And in the third step, we calculated the relative uncertainty using the formula $\varepsilon = \frac{\Delta x}{\bar{x}}$!



a) b)

a — the beginning; b — the end

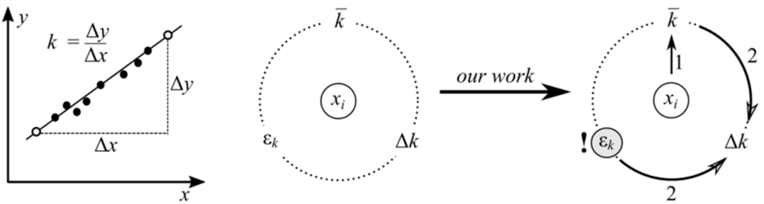
Figure 3.2 — The scheme of processing the results of physical quantity measurement is logically complete

– Excellent! Our method for processing the results of physical quantity measurements has no defects. And now let's trace the course of our actions at processing *linear relationships*.

Recall, for instance, our previous laboratory work, where we determined the viscosity of a liquid. Plotting the experimental points on the graph, we drew the experimental straight line and chose two convenient points on it (Fig. 3.3a). And how did we define the three quantities $\bar{k}$, Δk and ε_k for the slope k?

– In the first step, we found $\bar{k} = \frac{\Delta y}{\Delta x}$. This is the slope of the experimental straight line (Fig. 3.3b). But how did we find the two other quantities …?

– Let us remind you. That work was your first encounter with a linear relationship. To avoid distracting you from the main task, we directly provided the value of the relative uncertainty ε (ε = 3%). Using this value, you calculated $\Delta k = \varepsilon\bar{k}$ (Fig. 3.3b).



a) b)

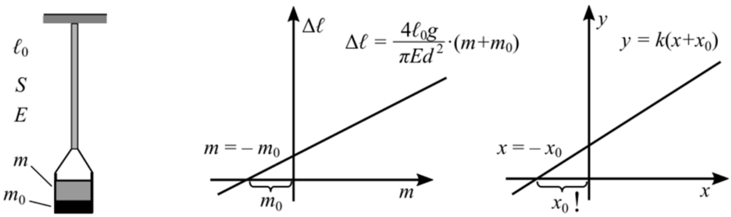
a — data on the graph; b — definition of $\bar{k}$, Δk and ε_k for the slope k?

Figure 3.3 — “Primitive” diagram of linear relationship processing. The scheme is logically open!

Obviously, no one will give us such clues in real research. It is essential to learn how to find everything we need from an experimental plot independently. *We must learn to construct confidence intervals for the parameters of a linear relationship*. This is our main task for today and the upcoming sessions. Today, we will consider how to build a confidence interval for intercepts, and in the next session, how to do it for the slope.

2. *Linear Function with an Offset Argument*. In this section, we will consider a case when a linear relationship is written in the form of a linear relationship with an offset. What do we mean? By default, in mathematics textbooks, a linear relationship is written as $y = kx + b$, so we agreed to call such a form the standard form. If you put the factor k outside the parentheses and denote $x_0 = \frac{b}{k}$, then you get a new form of a linear relationship: $y = k(x + x_0)$. In the third session, we agreed to call this form a *linear relationship with an offset argument*.

The linear relationship with an offset argument naturally arises in cases where, besides a researcher, an independent external factor also influences the system under study. Today, we will encounter this scenario. Specifically, we will investigate the process of wire elongation under loads (Fig. 3.4a). You will add the loads of known mass m to a stand attached to the wire and measure the elongation of the wire $\Delta\ell$ using a high-precision device (clock-type indicator). However, at first an instructor will fix a "black" load of unknown mass m_0 on the stand. Let us find out how wire elongation in our experiment is related to mass m.



a) b) c)

a — setup; b — physics; c — mathematics

Figure 3.4 — The process of wire elongation

Robert Hooke's law states that the elongation of the wire $\Delta\ell$ is related to force F acting on it by the law $F = k\Delta\ell$. Here $k = \frac{ES}{\ell_0}$ is the stiffness the of the wire, which depends on its length ℓ_0, cross-sectional area S (we will know the diameter of the wire d), and the Young's modulus E of the material from which the wire is made. Assumed that the tension force is equal to the weight of all loads, for the force F we can write $F = (m + m_0)g$. Using this, we derive the law of elongation of the wire. We obtained the linear relationship with an offset argument:

$$\Delta\ell = \frac{4\ell_0 g}{\pi E d^2}(m + m_0).$$

The obtained relationship looks as shown in Figure 3.4b. Will you guess where the mass of the "black" load is represented on this plot?

The mass of the "black" load corresponds to the length of the segment that is directly cut off on the axis of masses (Fig. 3.4b). Try to justify this conclusion in the general case (Fig. 3.4c). And we move on to solving our main issue.

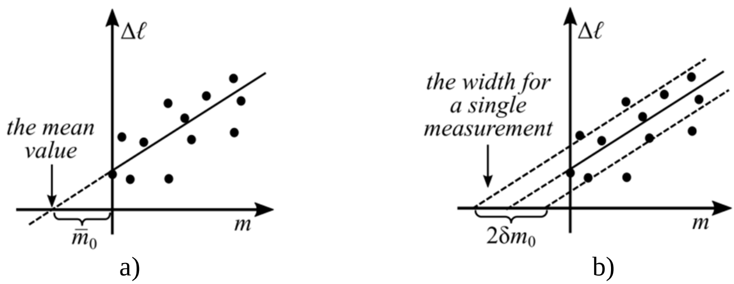
3. *Rules for Constructing a Confidence Interval for Intercepts*. We will formulate rules for a specific example (Hooke's law), and in the exercises we will ask you to write them in general notation.

1) The mean value of the confidence interval $\bar{m}_0$ can be found by extending the experimental line to the intersection with the horizontal axis.

Draw an experimental line using the experimental points (Fig. 3.5a). Remember, this should be done in such a way that "no one is offended", i.e. that the number of experimental points above and below the line should be approximately the same. Continuing it to the intersection with a horizontal axis, we obtain a segment which length is equal to m_0.

2) The absolute uncertainty of the intercept can be found by the method of parallel displacement of the experimental line.

The scattering of the experimental points gives a small backlash for the experimental line. Let us shift the experimental line parallel to itself slightly upwards, so that almost all points appear under it (or rather 5/6 of the total number), and then shift it downwards similarly. After that, these lines will cut a segment on a horizontal axis (Fig. 3.5b), the length of which is equal to the width of the confidence interval of a single measurement ($2\delta m_0$).



a — the mean; b — the width of the confidence interval of a single measurement

Figure 3.5 — Graphical definition of confidence interval parameters

To find the half-width for the *mean value of* m_0 (absolute uncertainty of a series of measurements) it is necessary to divide δm_0 by $\sqrt{n}$.

As a result, we obtain:

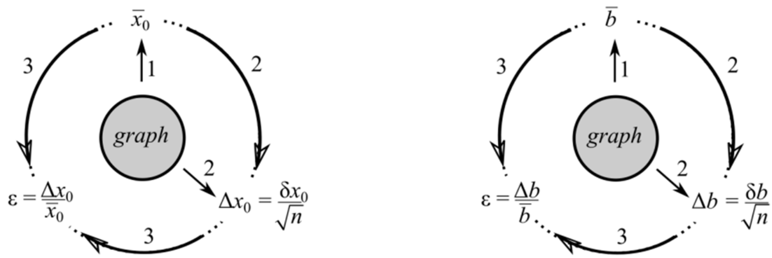
$$\Delta m_0 = \frac{\delta m_0}{\sqrt{n}}.$$

We hope that you have recognized the Confucius rule, which we formulated in the second session: the average always "dances" with the amplitude $\sqrt{n}$ less than a single!

3) The relative uncertainty of the measurement can be found by the standard formula:

$$\varepsilon_{m_0} = \frac{\Delta m_0}{\bar{m}_0}.$$

Let us summarize. In Figure 3.6a, we have compiled all the steps necessary to build the confidence interval for the parameter x_0 in a single complex called the Small Diagram.



a) b)

a — for the parameter x_0; b — for the parameter b

Figure 3.6 — The main result of this lesson is the Small Diagram

Figure 3.6b illustrates a similar set of actions for constructing a confidence interval for parameter b of the standard notation of a linear relationship.

Exercise 1. Find out which segments on the experimental plot correspond to the parameters $\bar{b}$ and δb of the confidence interval for b.

Discussion. The procedure is entirely analogous to the one described for x_0, but here the answers should be read along the vertical axis. The quantity $\bar{b}$ is equal to a segment which the experimental line cut off on vertical axis, and the segment $2\delta b$ is cut off on the vertical axis by two offset lines.

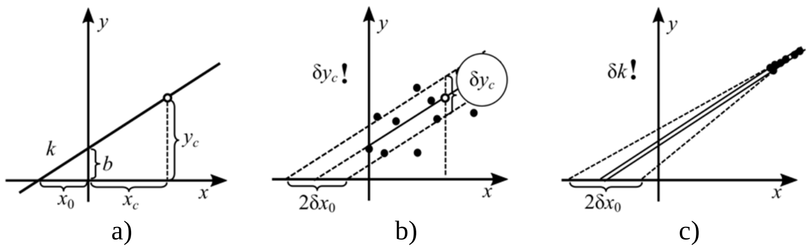
2. Recommendations for the Laboratory Work

1) At the beginning of the work, an instructor sets the "zero" point on

the indicator that measures elongation of the wire. Then, a “black” load[14] is fixed on the stand. The elongation of the wire shown by the indicator gives students the first experimental point for $m = 0$.

2) *On the content of the geometric construction in this laboratory session*. If for mathematicians the “natural” parameters of a linear relationship are the parameters k and b (or k and x_0), for the physics laboratory workshop “natural” parameters are the slope of an experimental line k and coordinates (x_c, y_c) of the center of mass of experimental points. Therefore, the rules for determining the confidence interval parameters for k and y_c will be outlined below.

Using this approach, the quantities b and x_0 should be considered as functions of the main parameters ($b = y_c - kx_c$ and $x_0 = \frac{y_c}{k} - x_c$), and the confidence interval must be constructed for them as indirectly measured quantities (Fig. 3.7a). Importantly, a contribution to the uncertainty of b and x_0 will give both, the absolute uncertainty of the slope Δk and the uncertainty of the height Δy_c.



a — ideal situation; b — extrapolation in the “close” zone;
c — extrapolation in in the “far” zone

Figure 3.7 — What does contribute to the uncertainty of b and x_0?

The geometrical method presented in this book estimates only one contribution: the uncertainty in determining the height Δy_c. From a methodological perspective, it is quite reasonable during an initial introduction of the method to limit the analysis to just one term. From a practical point of view, the uncertainty determined in this way makes a

[14]In our laboratory work, a “black” load is three ordinary loads that are join together with adhesive tape.

major contribution to the case where the extrapolation is performed in the "close" zone (Fig. 3.7b). This scenario is the focus of the current laboratory work. In a case where the extrapolation is carried out in the "far" zone, the main contribution arises from the uncertainty of the slope Δk (Fig. 3.7c). We will consider this case in the laboratory work 9.

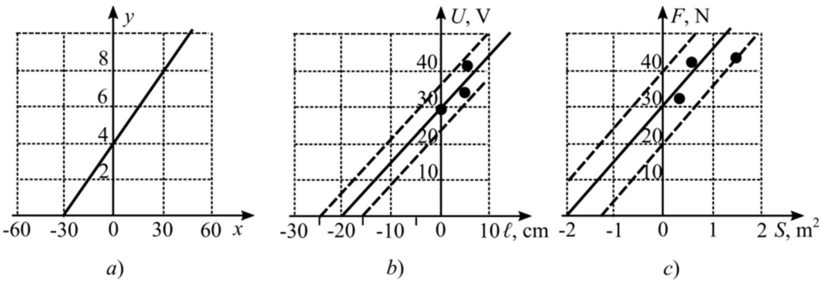
3. General Discussion *"Determining the Winner in Finding the Mass of a 'Black' Load"*

The microgroups write their results for the masses of "black" loads on the board, followed by a discussion of the results. An instructor announces which intervals "captured" the "true" value. Members of the winning teams receive additional points.

Interestingly, there is almost always a team that aims to use the field of the plot in the most efficient way (commendable!), and chooses a non-zero value to start the vertical axis. At the end of the work, they find themselves unable to read the result from the plot because they do not have the point $\Delta\ell = 0$ on the graph! What would you recommend they do in such a case? Build a new plot or …?

4. Example of the Quiz

1. Using the experimental plot (Fig. 3.8a) find: *a*) what the value of x_0 is; *b*) what the value of *b* is. [*Answers*: *a*) $x_0 = 30$; *b*) $b = 4$.]



a) b) c)

a — for question 1; b — for question 2; c — for question 3

Figure 3.8 —Experimental plots for the quiz

2. Using the experimental plot (Fig. 3.8b), construct the confidence interval for x_0. The total number of measurements is equal to $n = 25$.

[*Answer*: $\ell_0 = (20 \pm 1)$ cm]

3. Using the experimental plot (Fig. 3.8c), construct the confidence interval for b. The total number of measurements is equal to $n = 25$.

[*Answer*: $F_0 = (30 \pm 2)$ N]

4. Draw the sequence of actions on the blank of the Small Diagram *a*) for a linear relationship with an offset argument (Fig. 3.6a); b) for a linear relationship in standard form (Fig. 3.6b).

5. Additional Tasks

1. Estimate the stiffness of an eraser "to the touch".

2. We classified this case as "a general case of linear relationship". The parameters of the confidence interval are determined by the formulae:

a) for the height of the center of mass y_c:

$$\bar{y}_c = \frac{1}{n}\sum_{i=1}^{n} y_i\,, \Delta y_c = \sqrt{\frac{1}{(n-2)}\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}_c - \bar{k}(x_i - x_c)\right)^2}, \quad (3.1)$$

b) for the indirectly measured quantity $x_0 = \frac{y_c}{k} - x_c$:

$$\bar{x}_0 = \frac{1}{\bar{k}}\bar{y}_c - x_c, \qquad \Delta x_0 = \frac{1}{\bar{k}}\Delta y_c. \quad (3.2)$$

Here $x_c = \frac{1}{n}\sum_{i=1}^{n} x_i$, and $\bar{k}$ is determined by formula (2.3) from the previous session.

Attention! The formula for Δx_0 is given here for a special case of extrapolation in the "close" zone, where only the contribution due to the uncertainty of the parameter y_c is taken into account. The geometrical method used in this laboratory work corresponds exactly to this case. The formula for the general case is given in Appendix.

a) Write a program that calculates the parameters of the confidence interval for x_0.

b) Use it to calculate the confidence interval for the mass of a "black" load and compare it with the interval constructed by the geometrical method.

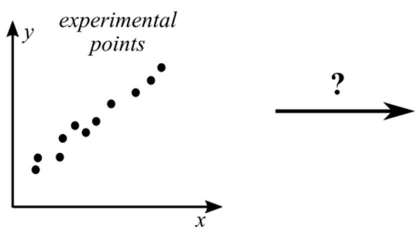
Session 6. Building a Confidence Interval for the Slope of a Linear Relationship and for an Indirectly Measured Quantity. Laboratory Work 6 "Hook's Law. Measurement of the Young's Modulus"

In this session, we will build the Great Diagram for working with indirectly measured quantities and get acquainted with the geometrical method for constructing a confidence interval for the slope.

1. Theoretical Background. *Great Diagram*

Today, we will consider the second part of our problem: we will get acquainted with the rules of a confidence interval construction for the slope of a linear relationship and for an indirectly measured quantity. Using these rules, we will define the Young's modulus of a wire and, with the help of special tables, identify the material it is made of.

1. *Directly and Indirectly Measured Quantities*. If a physical quantity is measured directly by a measuring instrument, then such a quantity is called a *directly measured* quantity. If the value of a physical quantity is found by calculation using a formula, such a physical quantity is called an *indirectly measured* quantity. For example, if we measure the sides of a rectangle with a ruler, the side lengths are directly measured values. If after that we calculate the area of the rectangle, then it is an indirectly measured value.



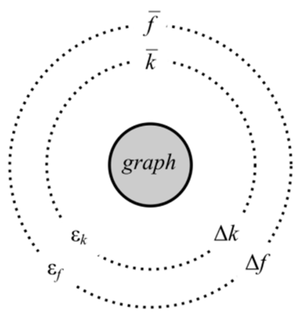


Figure 3.9 — Our task today is to learn how to fill both circles

Let us now look at linear relationships from this point of view. We will consider the slope of the line to be a directly measured quantity by agreement. This is justified because we directly determine it using a plot.

Therefore, there are always directly measured values in our laboratory works. And what about indirectly measured values?

Let us recall the laboratory works we have already done. Was the slope ever the final result for which we conducted the experiment? Or was it an intermediate result used to calculate something else?

– In Laboratory Work 3, we found the slope (Hubble constant *H*), but did not stop there. We used it to determine the lifetime of the universe using the formula $T = \frac{1}{H}$.

– In Laboratory Work 4, we determined the viscosity of the liquid using the Stokes method. We found the slope (velocity of the ball) using the graph, and then by formula $\eta = \frac{(\rho_{\text{ball}}-\rho_{\text{liq}})gd^2}{18} \cdot \frac{1}{v}$ calculated the viscosity coefficient of the liquid.

– All right! And today we will do something similar. Using a plot, we will determine the slope $k = \frac{4\ell_0 g}{\pi E d^2}$ of the linear relationship $\Delta\ell = \frac{4\ell_0 g}{\pi E d^2}(m + m_0)$, and then we calculate the Young's modulus of the wire material using the formula $E = \frac{4\ell_0 g}{\pi d^2} \cdot \frac{1}{k}$.

Did you catch the idea? As a rule, the goal of a laboratory work is not to define the slope k (directly measured quantity) itself but rather to find another quantity $f(k)$, which is calculated through it (indirectly measured quantity). Therefore, today, we will learn how to build a confidence interval for both the slope and an indirectly measured value. Naturally, the diagram of our activity will now expand (Fig. 3.9). It will include two circles: the first (internal) is for the slope k, and the second (external) is for the function $f(k)$.

The plan of our work is as follows: we learn how to fill the internal circle, "circle of the slope", and then we formulate rules for filling the external circle, "circle of an indirectly measured value".

Exercise 1. Above, we provided three equations for calculating physical characteristics using the slope. What type of functional relationship do these equations represent?

Answer: In all cases, this is a power relationship $f(k) = ax^{\alpha}, \alpha = -1$.

2. *Filling the Internal Circle*. Let us formulate the rules for finding the internal circle values (see Fig. 3.10): the mean value of the slope $\bar{k}$, the half-width of a confidence interval for the slope (absolute uncertainty of measurements) Δk, and the relative uncertainty of measurements for the slope ε_k.

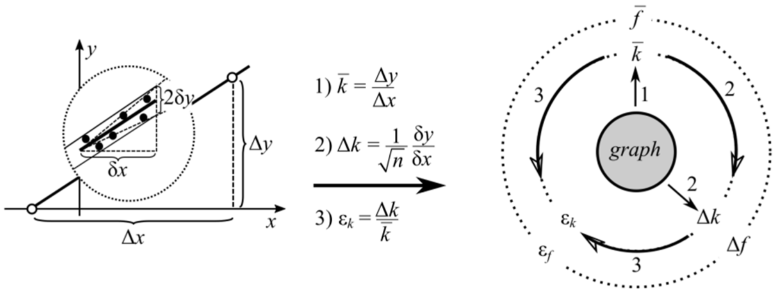


Figure 3.10 — The internal circle is filled!

1) We have already learned how to determine the mean value of the slope $\bar{k}$ by the geometrical method: using the experimental points, it is necessary to draw a straight line so that "none will be insulted", and then find its slope. *The mean value of an experimental line slope is equal to* $\frac{\Delta y}{\Delta x}$.

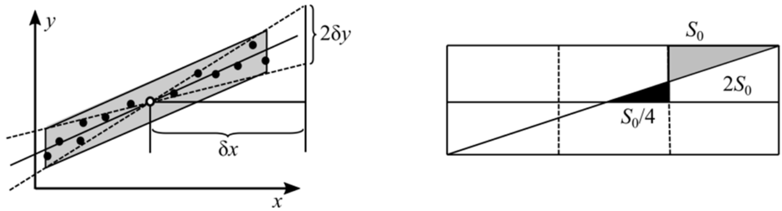
2) *The half-width of the confidence interval for the slope (absolute uncertainty) is determined by the method of swinging the line.*

Let us recall two main ideas of the geometrical method. Firstly, using a graph the *absolute uncertainty of a single measurement* δk can be determined, and the *absolute uncertainty of several measurements* Δk can be determined according to the "Confucius's rule":

$$\Delta k = \frac{\delta k}{\sqrt{n}}.$$

Secondly, the uncertainty of a single measurement is determined by a backlash of the experimental line. In Laboratory Works 2 and 5 we worked with segments and moved a straight line parallel to itself so that only 1/6 part of experimental points remained above or below it (method of parallel displacement of an experimental line). It is clear that for determining the slope uncertainty we must swing an experimental line within certain limits. Naturally, this raises two important questions: relatively to which point the line should be swung and how far? The answers to these questions are provided by the "Pencil-box Rule".

"Pencil-box Rule". Let us "pack" the experimental points into a thin, long pencil-box (Fig. 3.11a). Then the center of mass of the pencil-box (the central point) becomes the center of swings, while the sides of the pencil-box determine the amplitude of swinging $2\delta y$. The magnitude of the uncertainty of a single measurement δk is equal to $\delta k = \frac{\delta y}{\delta x}$, where δx is the "base" of swings.



a) b)

a — packing the points into a pencil-box; b — geometrical hint for the rule

Figure 3.11 — "Pencil-box Rule"

How can this rule be justified? We must proceed as we did in the second session. It is necessary to see this rule in the formula (2.4).

3) *The relative measurement uncertainty of the slope is determined by the standard way:* $\varepsilon = \frac{\Delta k}{\bar{k}}$.

Once the inner circle is filled (Fig. 3.10), we move on to filling the outer one.

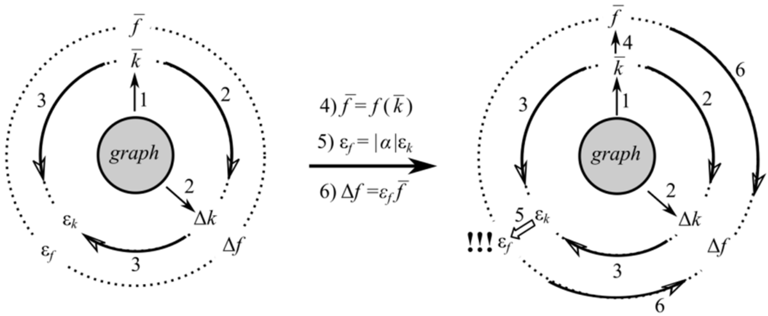


Figure 3.12 — The main result of this section: the Great Diagram

3. *Filling the Outer Circle*. Now, we will explain how the sequence quantities of the outer circle should be calculated.

4) The mean value of the measured quantity $\bar{f}$ is calculated using the known mean value of the slope $\bar{k}$: $\bar{f} = f(\bar{k})$.

Thus, the first quantity of the outer circle is determined (Fig. 3.12). Next, we need to determine once and for all which of the two remaining quantities of the outer circle, ε_f or Δf, we need to determine next. Note that once we know any two quantities of the external circle, the third one is determined automatically by the standard formula $\varepsilon_f = \frac{\Delta f}{\bar{f}}$.

So, let us remember! The fractional uncertainty ε_f is always determined by the next!

5) *The relative uncertainty of an indirectly measured quantity* ε_f *is uniquely determined by relative uncertainty of the slope* ε_k*, they are connected by formula (3.3)!*

$$\varepsilon_f = \left|\frac{d\ln f}{d\ln k}\right| \varepsilon_k \Rightarrow \varepsilon_f = |\alpha|\varepsilon_k \Rightarrow \varepsilon_f = \varepsilon_k. \qquad (3.3)$$

Some may argue that the formula on the left is terribly complicated! Maybe, but look how it simplifies (central formula) for the most common relationship: power relationship $f = ak^{\alpha}$. And if we take todays and previous laboratory works, where $|\alpha| = 1$, then we get the right formula, a really simple one!

Exercise 2. Prove formula (3.3) for the general case and obtain formulae for some simple cases from it. *Hint*: $\Delta f = \frac{df}{dk}\Delta k$.

6) *The absolute uncertainty of an indirectly measured quantity is then found by means of the standard formula* $\Delta f = \varepsilon_f \Delta k$.

The outer circle is filled! That is all!!!

2. Recommendations for the Laboratory Work

Students do not take new measurements in this session. Instead, they process the experimental data obtained in the previous session.

3. General Discussion *"How to Do it Without the Outer Circle?"*

The brilliance of the Great Diagram is impressive! Therefore, it is time to ask the question, "Is it possible to do it without the Great Diagram at all?" The answer to this question is positive, "Yes, absolutely! If you choose the 'horizontal' and 'vertical' variables appropriately, a Small Diagram will suffice!" For example, in this laboratory work, if we take the relative elongation $\varepsilon = \frac{\Delta\ell}{\ell_0}$ as the "horizontal" variable and the mechanical stress $\sigma = \frac{mg}{S}$ as the "vertical variable", the slope will be numerically equal to the Young's modulus. The second circle is not required!

4. Example of the Quiz

1. To determine the area of a square, its side was found, $a = (10 \pm 1)$ cm. Using the fact that the area of a square is calculated by the formula $S = a^2$, find all quantities of the Great Diagram in this case. Record the confidence interval for the area S.

[*Answer*: $\bar{a} = 10$ cm, $\Delta a = 1$ cm, $\varepsilon_a = 0.1$, $\bar{S} = 100$ cm^2, $\varepsilon_S = 0.2$, $\Delta S = 20$ cm^2; $S = (100 \pm 20)$cm^2.]

2. To determine the volume of a sphere, its diameter was found, $d = (10.0 \pm 0.1)$cm. Using the fact that the volume of a sphere is calculated by the formula $V = \frac{\pi d^3}{6}$, find all values of the Great Diagram in this case. Record the confidence interval for the volume V.

[*Answer:* $V = (524 \pm 16)$cm^3.]

3. Draw the sequence of steps on a blank of the Great Diagram (Fig. 3.9).

5. Additional Tasks

1. Estimate "to the touch" the Young's modulus of an eraser.

2. In this case, the confidence interval for the slope is calculated using the formulae (2.3-2.4) of session 4.

a) Write a program that calculates the parameters of a confidence interval for an indirectly measured value in the case of power dependence.

b) Use this program to calculate the confidence interval for the Young's modulus. Compare it with the interval determined using the geometrical method in this laboratory work.

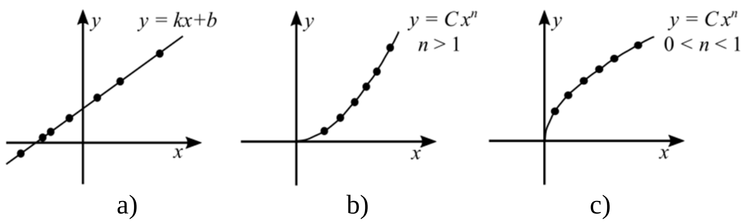
TOPIC 4. POWER RELATIONSHIP

Session 7. "Straightening" Power Relationship. Performing this Operation in Laboratory Work 7 "Stefan – Boltzmann Law"

In this session, we will learn how to transform power relationships into linear ones using logarithmic coordinates and "extract" the Stefan-Boltzmann's law of thermal radiation from the experimental data obtained by P. Dulong and A. Petit.

1. Theoretical Background. *Straightening a Power Relationship*

1. *Acquaintance with a Power Relationship.* A power relationship is a relationship of the form $y = Cx^n$. Many laws in physics take this form. Just like a linear relationship, a power relationship has two parameters: the exponent n and the constant C. Among these, the exponent is more significant: a physics law is considered to be discovered if the value of n is determined.



a — straight line; b — power function for $n > 1$; c — power function for $0 < n < 1$

Figure 4.1 — There is no ruler-like pattern for power dependences!

The power relationship can be visualized on a graph. When it appears as shown in Figure 4.1b, while for $0 < n < 1$ it looks like in Figure 4.1c. However, proving that it is exactly this relationship, and not the more complex one is not that simple. You cannot attach a ruler here. Additionally, there is no universal template for all power relationships. Moreover, you also will not read immediately from the graph the values of the parameters n and C.

To learn how to handle the power relationship, you need a new idea!

2. *A New Idea*. The power relationship should be transformed into a linear relationship!

– Is it possible?

– It is. If you can find on a calculator a function $f(x)$ which has the following property[15]:

$$f(ab) = f(a) + f(b).$$

Two exercises will help us to understand what is special in this property.

Exercise 1. Using the property $f(ab) = f(a) + f(b)$, express $f(x^2)$, $f(x^3)$, $f(x^n)$ in terms of $f(x)$.

Exercise 2. Consider what happens if we take as the argument of the function f the power function $y = Cx^n$. In other words, what is the expression $f(y) = f(Cx^n)$ equal to?

If you completed these exercises, then in the first case you got $f(x^n) = nf(x)$, and in the other

$$f(y) = f(Cx^n) = f(C) + f(x^n) = f(C) + nf(x).$$

Did you understand what happened? If *x* and *y* are connected by a power relationship, then the new values $X = f(x)$ and $Y = f(y)$ are connected by a linear relationship

$$Y = nX + f(C).$$

Therefore, if we add to two columns to the table containing our experimental data, $X_i = f(x_i)$ and $Y_i = f(y_i)$, these new experimental points will lie on a straight line. This will allow us to determine both, the exponent *n* and the coefficient *C* using a usual ruler!

3. *Search for the Magic Function*. So, what is this mysterious function? Which button on a calculator corresponds to it? Let us attempt to build a “facial composite” of our stranger and try to guess what kind of *incognito function* is. To build a “facial composite”, we will use three “points” by evaluating the function at specific arguments: unity, infinity, and zero.

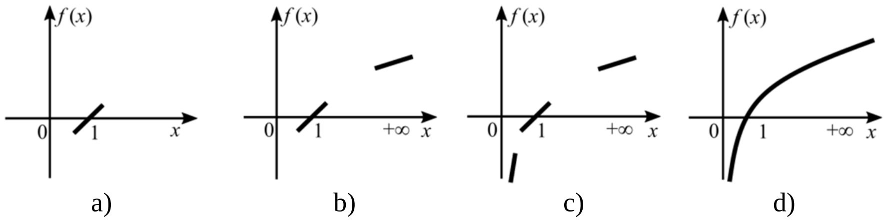
[15]Such equations are called functional equations.

What is the function $f_2(x)$ equal if $x = 1$? Let $a = 1$ in the functional equation. Then we obtain: $f_2(1 \cdot b) = f_2(1) + f_2(b)$.

From here, we get the first point of our "facial composite":

$$f_2(1) = 0!$$

Thus, our function equals zero if $x = 1$ (Fig. 4.2a)! By the way, we forgot about the main thing: How many different functions satisfy equation $f(ab) = f(a) + f(b)$? One, two, ...? Or perhaps there are an infinite number of such functions?



a — the first point; b — approaching infinity; c — the last "point"; d — the plot

Figure 4.2. Building a "facial composite" of our stranger using three "points"

You know, there is indeed an infinite number of such functions, a whole family. And they differ from one another only by a multiplier! Therefore, it's possible that there are a few "magic" buttons on the calculator!

Exercise 3. Prove that if a function $f(x)$ has the property (*A*), then the function $F(x) = \gamma f(x)$, where γ is an arbitrary factor, it also has property (*A*). *Hint*: $F(ab) = \gamma f(ab) = \ldots$

In order to distinguish a specific function from this family, mathematicians agreed to indicate the value *a* of the horizontal coordinate where the specific function equals unity. This value is written as an index in the function's notation, denoted $f_a(x)$. Thus, if you are asked what $f_{1979}(1979)$ is equal to, answer without hesitation: unity! And we get a "free" point $f_a(a) = 1$ for the graph.

Exercise 4. The appearance of indices leads to a rather interesting technique of rearranging indices and values.

Prove that:1) $f_2(10) \cdot f_{10}(2) = 1$; 2) $f_{10}(x) = \frac{1}{f_2(10)} f_2(x)$.

Now, let's see what turns out to be at infinity. Calculate the value of the function $f_a(x)$ for the series a, a^2, a^3, … We get: 1, 2, 3, … It turns out that our function at infinity approaches infinity, although very, very slowly (Fig. 4.2b). Build the last "point" in the "facial composite" on your own (Fig. 4.2c).

Exercise 5. Consider the sequence $\left(\frac{1}{a}, \frac{1}{a^2}, \frac{1}{a^3}, \dots\right)$ and make sure that at zero our function approaches negative infinity (Fig. 4.2d).

So, we think that everyone has already recognized our stranger or, better to say, a stranger. Certainly, this is a ... logarithm (see Fig.4.3)!

4. *Designations*. Let us recollect designations accepted in mathematics.

The standard notation of "logarithm to the base a of x" is $\log_a x$. However, there are two exceptions to this rule:

1) the decimal (common) logarithm does not denote $\log_{10} x$ but $\lg x$ (in the US use the notation $\log x$);

2) the natural logarithm (logarithm to the base $e = 2.718281828\ldots$) does not denote $\log_e x$ but $\ln x$.

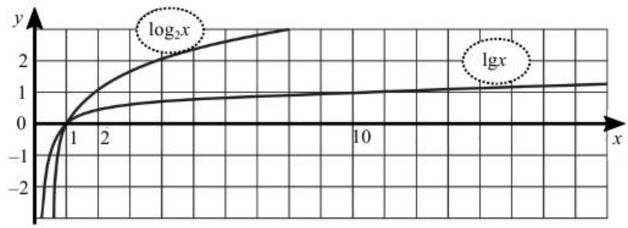


Figure 4.3 — Meet the family of logarithms, the best friends of an experimenter!

We need to decide which kind of logarithms we will use in our laboratory work. We propose using the natural logarithm. It is given to us by Nature itself. And its name comes from the fact that it appears most naturally when describing various natural phenomena.

Exercise 6. At what point is $\ln x$ equal to zero? To unity?

Know and learn to appreciate logarithms! In the pre-computer age, they increased the speed of calculations by hundreds of times. Today they will help us to straighten a power relationship. Logarithmic coordinates will always allow you to place experimental points which values differ by a hundred, a thousand, a million, or even a billion times on one plot!

5. *Algorithm for Processing Power Relationship.*

1. Complete the data table with two additional columns $X_i = \ln(x_i)$ and $Y_i = \ln(y_i)$.

2. Plot the experimental points on the plane (X, Y). If the investigated relationship is a power relationship, then the experimental points will lie on the line $Y = nX + C$.

3. The slope of this linear relationship is numerically equal to the exponent. It is determined in a standard way by measuring the slope of an experimental line:

$$n = \frac{\Delta Y}{\Delta X}.$$

4. By measuring the value of the Y-intercept b graphically we can restore the value of power relationship constant: $C = \exp(b)$.

2. Recommendations for Laboratory Work

1. Logarithmic coordinates are very capricious: the slightest inexactness in constructions leads to significant changes in the final results. Therefore, it is crucial to verify the quality of geometrical constructions using numerical calculations. We conducted such calculations by standard formulae (2.3-2.4) to calculate the parameters of the confidence interval. The results are presented in Table 4.1.

Table 4.1 — Results of the Calculation of the Confidence Interval

#	Parameter	Confidence interval	Standard deviation of the mean	Relative uncertainty
1	exponent	$n = 4.042 \pm 0.015$	$\Delta n = 0.015$	$\varepsilon_n = 0.4\%$
2	constant, 10^{-8} W/m^2·K^4	$\sigma = 4.2 \pm 0.5$	$\Delta\sigma = 0.5$	$\varepsilon_\sigma = 12\%$

2. A distinctive feature of the logarithmic coordinates method is its relatively low accuracy in determining the constant of a power relationship. In our case $\varepsilon_C = 12\%$. This is explained by the fact that the intermediate parameter

b and the final value C are related by exponential function $C = \exp(b)$. Consequently, based on the formula $\varepsilon_f = \left|\frac{d\,ln\,f}{d\,ln\,k}\right| \varepsilon_k$ we have $\varepsilon_C = \left|\bar{b}\right| \varepsilon_b$. In our case $\left|\bar{b}\right| = 17$, so there is a huge increase in relative uncertainty, from $\varepsilon_b = 0.7\%$ to $\varepsilon_C = 12\%$.

Remember! At the first stage of studying a new class of physical phenomena, the method of logarithmic coordinates has no alternatives. However, once the law is discovered (an exponent is defined), to determine the constant's value it is better to use *power straightening* coordinates. This approach will be discussed in the next session.

3. This laboratory work introduces a new logical type of experiment. It is an example of laboratory work where *simple physical hypotheses* are put forward and tested.

The purpose of previous laboratory works was to build a confidence interval for the true value of a certain continuous physical quantity y. That is why we calculated the mean value $\bar{y}$ and the half-width of the confidence interval Δy. Now the logic of working with these quantities will be different. Both quantities must be interpreted using the point paradigm: the $\bar{y}$ as an estimate of the true value, and the Δy as the standard deviation from the true value. Further considerations should be as follows.

A priori we assume that the exponents are integers (or prime fractions) in the fundamental physics laws. Then, according to the results of the measurements we can formulate three simple hypotheses[16]: the null hypothesis H_0 "the exponent is equal to four, $n_0 = 4$", the first alternative hypothesis H_1 "$n_1 = 5$" and the second alternative hypothesis H_2 "$n_2 = 3$". We use as the criterion for choosing between these hypotheses the probability that a game of chance could lead to the measurement result deviation $d_j = \left|n_j - \bar{n}\right|$ from the true value.

The deviation from the hypothetical mean is $d_0 = 2.8\Delta n$ for the null hypothesis, it is $d_1 = 64\Delta n$ for the first hypothesis, and it is $d_2 = 70\Delta n$ for the second hypothesis. Using methods from probability theory (see [4]) $P_j = \frac{2}{\sqrt{2\pi}} \int_{d_j/\Delta n}^{\infty} \exp\left(-\frac{x^2}{2}\right)$, we obtain that the probability of the

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[16]A simple hypothesis is a hypothesis about a numerical value; a term of mathematical statistics.

first deviation is $P_0 = \frac{1}{170}$, the probability of the second deviation is $P_1 = = 10^{-900}$, and the probability of the third deviation is $P_2 = 10^{-1100}$. These numbers give us every reason to assume that the null hypothesis $n = 4$ is true.

Note: The relatively low value of the probability P_0 indicates, in our opinion, the presence of a systematic error in the Dulong-Petit's experiments.

3. General Discussion *"This Awful Cosmic Cold"*

Additional points can be earned by those students who can estimate the power of heat losses to radiation using the obtained results. Usually debates smoothly flow into the discussion of the issue: Will you freeze to death in space?

4. Examples of the Quiz

1. Figures 4.4a-c show three experimental graphs plotted in logarithmic coordinates. Establish the correspondence between the graphs and the following physical laws: I) $y = C\sqrt{x}$; II) $y = Cx^2$; III) $y = Cx^3$.

[*Answer*: I) – *b*; II) – *c*; III) – *a*.]

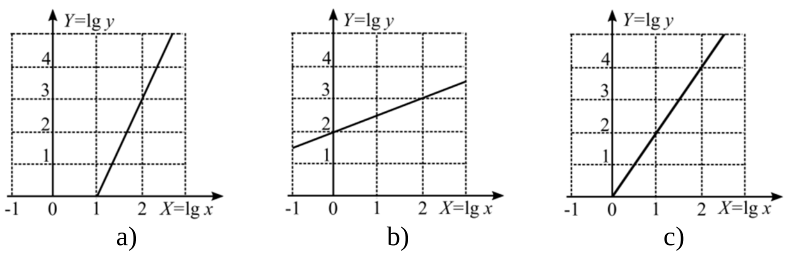


Figure 4.4 — Experimental graphs in logarithmic coordinates

2. Determine the values of the following expressions: $\log_5 5$, $\log_5 25$, $\log_{2019} 1$.

[*Answer*: 1, 2, 0.]

3. Which of the graphs presented in Figure 4.4 corresponds to the power relationship $y = Cx^n$ with the constant C equal to unity?

[*Answer*: graph (*c*).]

5. Additional Tasks

1. Estimate how long it will take for your body temperature to drop by 5°C if you leave to take a walk in open space without a protective suit.

2. We characterized this case as a general case of a linear relationship. Calculation of the confidence interval parameters for the slope (the exponent) should be carried out here according to the following formulae:

$$\bar{k} = \frac{1}{I_m}\sum_{i=1}^{N} m_i (X_i - X_m)^2 k_i, \tag{4.1}$$

$$\Delta k = \sqrt{\frac{1}{(N-2)}\frac{1}{I_m}\sum_{i=1}^{N} m_i (X_i - X_m)^2 \left(k_i - \bar{k}\right)^2}, \tag{4.2}$$

where
$m_i = y_i^2$ are the “masses” of the experimental points on the (X, Y)-plane,
$M = \sum_{i=1}^{N} m_i$ is the total “mass”,
$X_m = \frac{1}{M}\sum_{i=1}^{N} m_i X_i$ and $\bar{Y}_m = \frac{1}{M}\sum_{i=1}^{N} m_i Y_i$ are the coordinates of their “center of mass”,
$I_m = \sum_{i=1}^{N} m_i (X_i - X_m)^2$ is the “moment of inertia”,
$k_i = \frac{(Y_i - \bar{Y}_m)}{(X_i - X_m)}$.

a) Calculate, using formulae (4.1-4.2), the values of $\bar{n}$ and Δn, and determine the correct probability of the measurement result deviation from the null hypothesis.

b) Is the suspicion about the systematic error still valid?

Session 8. Dimensional Analysis. Power Straightening Coordinates. Laboratory Work 8 "Simple Pendulum"

In this session, we will learn how to straighten power relationships by using power coordinates. The dimensional analysis will help us to choose the necessary power coordinates.

1. Theoretical part. "*Dimensional Analysis*"

1. *Anticipation.* Human thought possesses a remarkable ability: anticipation. *Anticipation* is a power to predict outcomes.

When working in a garden, we quite clearly imagine the results we aim to achieve. And when we hammer a nail the result of our activity is clear to us in advance. Even as we head to a small river to fish, we know not to bring oversized fishing tackle, though we might take some extra equipment, just in case. Hope is another remarkable capacity of human cognition[17].

In everyday life, anticipation is always with us. Yet, for some reason, anticipation disappears when it comes to physics! We solve problems and perform laboratory works as if blindfolded, without even trying to imagine the outcomes. This approach is unprofessional. Experts always envision the expected results. Solving a problem then becomes a matter of clarifying some details. Don't you believe it? Then listen to a true story about a boy named Petryk.

2. *A True Story about Petryk.* At the physics exam Petryk got a task to calculate the centripetal acceleration. Unfortunately, he could not recall the formula.

"Alright", Petryk thought, "let's figure it out logically. Two letters are given in the problem, υ and R. Therefore, they must be included in the formula, otherwise, why would they be given? Adding them doesn't seem right, but I can try multiplying and dividing. I will write the formula in following way

$$a = \upsilon^{\alpha} R^{\beta}.$$

Now, the challenge is determining the exponents α and β for velocity and radius. Well, the dimensions of the left and right parts must be

---

[17]But computers have neither anticipation nor hope!

the same. So, I'll write the dimensions of the left and right parts:

$$\frac{m}{s^2} = \frac{m^\alpha \cdot m^\beta}{s^\alpha}.$$

Here I focus on the letter "s". On the left it is in the denominator and has the power "2", and on the right "s" is also in the denominator but has the power "α". In the correct formula the dimension of the left and right parts must coincide, so $\alpha = 2$. Now let's look at the letter "m". On the left it has the power "1", and on the right it has the power "$(2 + \beta)$". So, $\beta = -1$, and I restored the formula for the centripetal acceleration,

$$a = v^2 R^{-1} = \frac{v^2}{R}.\text{"}$$

Petryk got an excellent grade. And we have a topic to discuss: how should we evaluate Petryk's actions?

You can hear a variety of opinions on this matter.

– Wow! You can learn nothing!

– No, it's essential to study physics and learn about dimensions.

– It is somehow dishonest to get answers from the air, or more precisely, from the analysis of dimensions. Problems should be solved honestly!

Let's summarize. Our perspective on this topic is as follows: dimensional analysis is a powerful tool for obtaining answers, and we definitely take it up!

Any physical research should begin by considering what answers are possible. The primary tool here is dimensional analysis.

3. *Dimensional Method*. First, it is crucial to understand the logic of the dimensional method. Let's work through an example.

Exercise 1. In class, students solved a problem about a body falling from a height h, and they all got different answers. Nicholas got $t = \frac{h}{g}$, Alexander got $t = \sqrt{2hg}$, and Mary got $t = \sqrt{\frac{2h}{g}}$. Using the reasoning with dimensions, determine which students were wrong.

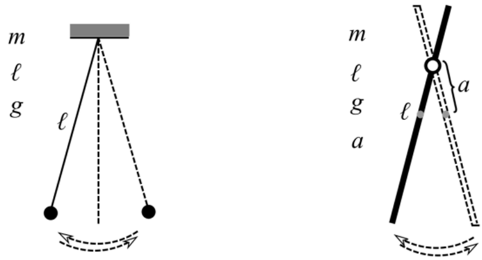
Discussion. It's clear that Nicholas and Alexander provided incorrect answers. For example, Nicholas's result instead of seconds gives a second squared, $[t] = \left[\frac{h}{g}\right] = 1\,\frac{\mathrm{m}\cdot\mathrm{s}^2}{\mathrm{m}} = 1\,\mathrm{s}^2$.

However, everything is fine with the dimension in Mary's answer, $[t] = \left[\sqrt{\frac{2h}{g}}\right] = 1\sqrt{\frac{\mathrm{m}\cdot\mathrm{s}^2}{\mathrm{m}}} = 1\sqrt{\mathrm{s}^2} = 1\,\mathrm{s}$.

And now, the main question. Did we prove that the boys' answers were wrong? Yes, we did!

Did we prove that Mary's answer was correct? No, we did not! Why? Imagine that, instead of 2 in the numerator Mary had 5. Would the answer still pass the dimension test? It would! But it would still be wrong!

Do you see the logic of the dimension method? It helps to restore only the main, dimensional part of an answer. However, to find the coefficient you need to solve the problem "honestly", by performing mathematical calculations and physical measurements!



a) b)

a — a simple pendulum; b — a physical pendulum

Figure 4.5 — Parameters which could define phenomena

If you understand this, let's learn how to find the main part of an answer by using dimensional analysis. To find the main part of an answer by dimensional analysis, it is necessary to write down all quantities the answer could depend on. Then, compose them in such a way that the dimension of the obtained combination is equal to the dimension of the desired quantity.

Exercise 2. Obtain the formula for the period of a simple pendulum oscillations by dimensional analysis. In order to get the perfect answer, we will provide the value of the coefficient, which gives the “honest” mathematical solution: $k = 2\pi$.

Discussion. What quantities can the period of a simple pendulum’s oscillations depend on (Fig. 4.5a)? Everyone agrees: its mass m, length ℓ, and on the free fall acceleration g! After the list is made, we do the same as Petryk did and write:

$$T = k \cdot m^{\alpha} \cdot \ell^{\beta} \cdot g^{\gamma}.$$

Maybe, what Petryk did is not very simple? Maybe it’s right that he got an excellent grade? Try to repeat his actions and get the answer yourself!

If you did everything right, you get: $\alpha = 0$ (the period does not depend on the mass!), $\beta = \frac{1}{2}$ and $\gamma = -\frac{1}{2}$. Here we received the answer without solving the problem:

$$T = 2\pi\sqrt{\frac{\ell}{g}}.$$

3. *A Priori and a Posteriori Laboratory Works*. There are two different, from the logic point of view, types of laboratory works. We will refer to the works of the first type as *a priori* works (a priori (Latin) means “before an experiment”), and works of the second type as *a posteriori* works (a posteriori (Latin) means “after an experiment”).

When G. Galileo, R. Hook, and J. Stefan conducted their pioneering researches, there were no theories of the phenomena under study. There was neither acceleration of free fall nor Young’s modulus, nor Planck’s constant. And, of course, they could not write the answer using dimensions. They had to derive a physical law from their measurements “honestly”. We classify such researches *a posteriori*, emphasizing that their logical meaning is to find a law that was not known in advance (before an experiment). We have already noted in the previous work that, in such cases, the universal tool of a researcher is the use of logarithmic straightening coordinates.

We will call a laboratory work *a priori* work if the law under the study is known in advance. For example, if it can be restored (up to a coefficient) using dimensional analysis. It is clear that we can do this only when the theory of the phenomenon is fully developed, and we have all necessary constants.

Exercise 3. Get the Stefan-Boltzmann law of thermal radiation using dimensional analysis $\left(k = \frac{\pi^2}{60}\right)$. Note that this task involves concepts from molecular physics (Boltzmann's constant, k_B), radiation (speed of light, c), and quantum mechanics (Planck's constant, $\hbar$). [*Answer*: $R = \frac{\pi^2}{60}\frac{k_B^4}{\hbar^3 c^2}T^4$.]

A natural question arises: why carry out laboratory work, if everything is already known? Why rediscover established laws?

The answer is as follows: the purpose of such research is to clarify the values of fundamental constants or find the values of coefficients in new conditions. The law itself becomes a tool in this case.

For example, today we will use formula $T = 2\pi\sqrt{\frac{\ell}{g}}$ to determine the free fall acceleration in our area. Someone might object, saying, "It has been known for us already, it is written in all textbooks that $g_{\text{theor}} = 9.81$ m/s^2!" Not quite.

Free fall acceleration is not a fundamental constant. Its value depends on general characteristics of a planet, geographical coordinates, altitude, and even on mineral deposits in the area. So, it can be very useful to compile an atlas of values of the free fall acceleration in your property estate. Maybe it is a gold deposit ($g > g_{\text{theor}}$) under us, or a huge underground sea of the purest artesian water ($g < g_{\text{theor}}$).

5. *Power Straightening Coordinates.* In a priori laboratory work you must select power coordinates as straightening coordinates.

Usually, they could be selected in several ways. For example, the power relationship $T = 2\pi\sqrt{\frac{\ell}{g}}$ we will work today with, turns into a linear (directly proportional) relationship $Y = kX$ for each set of coordinates: $\left(X_1 = \sqrt{\ell}, Y_1 = T\right)$, $(X_2 = \ell, Y_2 = T^2)$, $\left(X = \frac{T^2}{4\pi^2}, Y = \ell\right)$.

In our laboratory work we will use the last set of power coordinates. Why? Try to guess!

Exercise 4. For each set of power straightening coordinates, find the formula that connects the measured value of g with the slope k of the experimental line.

[*Answer*: 1) $g = \frac{4\pi^2}{k_1^2}$; 2) $g = \frac{4\pi^2}{k_2}$; 3) $g = k$.]

2. Recommendations for the Laboratory Work

When using dimensional analysis, keep in mind that some cases can be more complex, when "an undefined part" is not a *number* but a *function* of a dimensionless parameter.

For instance, consider a physical pendulum (Fig. 4.5b), which involves two quantities with a length dimension: the length of the rod ℓ and the distance from the pivot to the center of mass a. Here, for dimensional reasons, we can write only $T = 2\pi\sqrt{\frac{\ell}{g}} \cdot f\left(\frac{a}{\ell}\right)$ and the aim of our study is to restore the dimensionless function of the parameter $\frac{a}{\ell}$.

Remember, that in general, the purpose of a posteriori works is to restore dimensionless functions of dimensionless parameters.

3. General Discussion "*Weighing the Moon*"

In this section, we usually discuss whether it is possible to detect the gravitational influence of the Moon on objects around us using the pendulum.

4. Example of the Quiz

1. Use dimensional analysis to derive the formula for the speed of sound c_{sound} in solids ($k = 1$). The quantities it can depend on are the density ρ and the Young's modulus of elasticity E.

[*Answer*: $c_{\text{sound}} = \sqrt{\frac{E}{\rho}}$.]

2. Astronomers study the dependence of the period T of satellites rotation around a star on radii R of their orbits: $T = 2\pi\sqrt{\frac{R^3}{GM}}$. Here, G is the gravitational constant, M is the mass of the star.

a) Suggest power straightening coordinates for this dependence;

b) Determine how is the mass of the planet related to the line slope in your choice?

[*Answer*: *a*) for example, $(X = R^3, Y = T^2)$; *b*) $M = \frac{4\pi^2}{G}\frac{1}{k}$.]

5. Additional Tasks

1. Obtain the formula for the velocity of gravitational waves on a water surface and estimate the corresponding coefficient by observing the waves traveling on the surface of rivers or seas.

2. We characterize this case as a linear relationship with a priori zero. The relevant formulae for calculating the parameters of a confidence interval of the slope are:

$$\bar{k} = \frac{1}{I_0}\sum_{i=1}^{n} x_i^2 k_i, \quad \Delta k = \frac{1}{\sqrt{I_0}}\sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n} x_i^2 \left(k_i - \bar{k}\right)^2} \qquad (4.3)$$

where $k_i = \frac{y_i}{x_i}$, $I_0 = \sum_{i=1}^{n} x_i^2$ is the “moment of inertia” of the experimental points relative to zero.

Write a program that calculates the parameters of the confidence interval for the slope in the case of a linear relationship with an a priori zero and apply it to our laboratory work.

TOPIC 5. STRAIGHTENING COORDINATES

Session 9. Building Straightening Coordinates in a General Case. Laboratory Work 9 "Measurement of the Lifetime of an Incandescent Lamp"

In this session, one of the main ideas of an experimental research is presented: the key to find straightening coordinates is the mathematical model of a phenomenon under study.

1. Theoretical Background. *General Case*

We already know how to handle linear relationships and how to convert power relationships into linear ones. Today, we will explore how to transform experimental relationships of the general kind into linear ones.

For a physical situation, we take a technical problem of determining the lifetime of an incandescent lamp. This problem is not easy: some light bulbs burn out almost instantly, while others last for decades. For instance, in the USA, there is an incandescent lamp in a mine that has been burning for over a century! It has even become a famous tourist attraction. However, our focus lies elsewhere. The main thing for us is the following: when some value can change thousands or even millions of times, it is certainly neither linear nor a simple power relationship. This is what we need: an example of a general case! We begin its consideration and immediately inform the main principle for handling such cases: *to find the straightening coordinates, we must use theoretical reasoning to hypothesize the type of relationship involved.*

1. *Mathematical Question.* Can any mathematical relationship be straightened? For example, is it possible to straighten the dependence

$$y = A\exp\left(\frac{\beta}{x}\right)?$$

You can straighten any monotonous relationship! For the given example it can be done as follows: take the logarithm of both sides of this equation and obtain $\ln y = \ln A + \frac{\beta}{x}$. This shows that for variables $Y = \ln y$ and $X = \frac{1}{x}$ the relationship is converted into a linear relationship

$$Y = kX + b,$$

where $k = \beta$, $\beta = \ln A$.

2. *Accelerated Reliability Test.* Imagine you are a director of a company that produces a very important product. The product is of high quality, exceptionally durable, capable of functioning without failure for a year, or even ten years. A customer is interested in purchasing your product but insists on an official statement of its *lifetime*.

Do you understand what the problem is? Determining the lifetime by simply waiting for the product to fail is impractical. Testing products for several years would be inefficient and costly. Not the best option!

This is where accelerated reliability testing comes in. In an *accelerated reliability test*, several sets of the product are subjected to stress levels far beyond the expected operating conditions (e.g., high voltage, high speed, or strong vibration, etc.). This forces the product to fail much faster. And we only need to find such straightening coordinates that the experimental points lie on a straight line. Then it will be enough to take a ruler, draw a line and read the result in the area where we did not measure, at regular voltage values. Extrapolation!

Today, we will apply this principle to our poor incandescent lamps. We will test them at voltages much higher than regular 3.5 V. Then they will burn out fast, and this will allow us to collect data on their lifetimes under stress conditions. We will make a table for their lifetime at high voltage. All that remains is to find the straightening coordinates and, after extrapolating, read the lifetime of the lamp at regular voltage.

3. *Developing a Theoretical Model for the Process of Lamp Burning out*. Why do electrical appliances burn out? The failure occurs primarily due to overheating, caused by the electric current passing through them. Conductors made of common metals (copper, nichrome etc.) simply melt when overheating. However, the tungsten coil of an incandescent lamp "burns out" differently. Tungsten is a very refractory metal. Its melting point is equal to $T_{\mathrm{melt}} = 3695$ K.

So, at operating temperatures of the lamp ($T_{\mathrm{work}} = 2000 - 2400$ K) it does not melt, it evaporates (sublimates).

According to the molecular kinetic theory (MKT), evaporation always occurs because the chaotic thermal motion can give some atoms of a body surface enough energy to break free from the body. This allows us to derive the first formula for the lifetime τ of an incandescent lamp. Let the total number of atoms, a spiral consists of, be equal to N_{total}, and the number of atoms that fly out of it surface in a second is equal to N_1, then the lifetime of the lamp is equal to:

$$\tau = \frac{N_{\text{total}}}{N_1}. \tag{5.1}$$

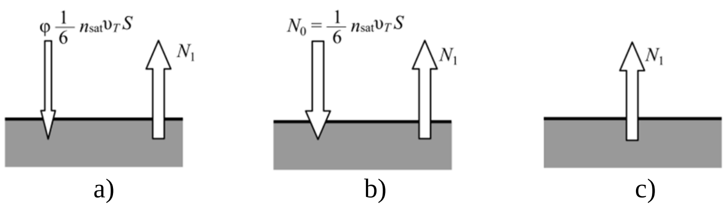
The first step was taken: we managed to reduce the question about the lifetime of the lamp to a well-known phenomenon – evaporation. Each of us has observed more than once during a lecture how water evaporates after a blackboard is wiped with a wet cloth. And how many molecules fly out of the surface per unit of time?

– ...?

– It seems that we should revise what MKT, a science that studies the motion of atoms, tells us. First, one of the primary results is the formula for the root mean square velocity of thermal motion of atoms $\upsilon_T = \sqrt{\frac{3RT}{\mu}}$. Second, MKT provides a formula for calculating the number of atoms of an ideal gas that fall on the surface of the area S every second: $N_0 = \frac{n\upsilon_T S}{6}$.

However, MKT does not provide a formula for the number of atoms that escape (evaporate) from a body into a gas. The reason is that MKT only applies to the simplest system of atoms – an ideal gas. The motion of atoms in a solid and in a liquid does not yet lend itself to theory study. But we have a way out! Let's think: does the wiped blackboard always become dry?

– No, not always! It will not dry out if the water vapor in the classroom is saturated!



a – relative humidity $\varphi < 1$; b — relative humidity $\varphi = 1$; c — only evaporation stream

Figure 5.1 — Evaporation and condensation

Let's explain what was said (Fig. 5.1). Besides the flow of atoms leaving the body into the gas (evaporation), there is also a reverse flow of atoms coming from the gas back to the body (condensation). If the air is sufficiently dry (relative humidity $\varphi < 1$, $n = \varphi \cdot n_{\text{sat}}$), then the flow-evaporation prevails (Fig. 5.1a), and the blackboard dries. If the water vapor is saturated ($\varphi = 1$, $n = n_{\text{sat}}$), then both flows are equal (according to the definition), and the board remains wet (Fig. 5.1b). And if there is no vapor at all, as it should be in the bulb of an incandescent lamp, then there is only one stream, evaporation stream (Fig. 5.1c).

Take a close look at the pictures! Have you noticed the most important thing? Although the evaporation stream of atoms from the body is unknown to us, we do know that it is exactly the same as the flow of condensation in case of saturated vapor, $N_1 = N_0$. And for this situation MKT gives us the formula $N_0 = \frac{n_{\text{sat}} v_T S}{6}$. Thus,

$$N_1 = \frac{n_{\text{sat}} v_T S}{6}. \tag{5.2}$$

We are halfway there. Let's pause briefly to deepen our understanding of evaporation processes and at the same time explore why engineers prefer to fill incandescent bulbs with inert gases and halogens.

Exercise 1. Using formulae (5.1-5.2), calculate how long it takes for water to evaporate from a saucer. Assume the height of the water layer is $h =$ $= 1$ cm, the relative humidity in the room is equal to $\varphi = 0.6$, the temperature is 300 K, the density of saturated water vapor at this temperature is equal to $\rho_{\text{sat}} = 26$ g/m^3, the density of water is $\rho = 1000$ g/m^3.

[*Answer*: $\tau = \frac{1}{1-\varphi} \frac{\rho}{\rho_{\text{sat}}} \frac{h}{v_T} = 1.5$ s, think about this answer!]

And now, the most important thing! Which of the values in formulae (5.1-5.2) is the key parameter that determines the lamp lifetime? You will never guess! This is the density of saturated tungsten vapor.

The density of saturated vapor of a substance is not constant; it changes with temperature! And it changes catastrophically!

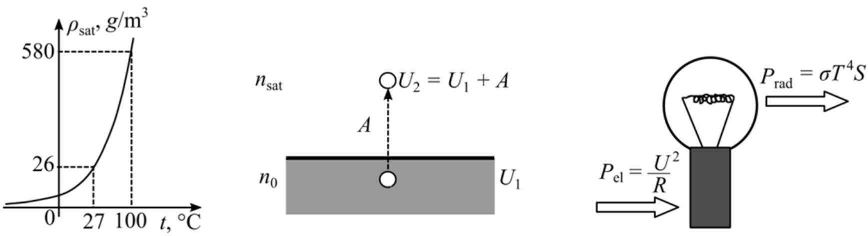
While we could not find a table specifically showing the dependence of saturated tungsten vapor density on temperature, this is not an issue. You can consider any graph of this type, for example, the dependence of the density of saturated water vapor on the given temperature which is commonly found in each textbook (Fig. 5.2a). See, as the temperature increases from 0°C to 100°C (absolute temperature increased by only 37%), the density of saturated vapor increases more than 120 times!

An expert might say: “This is not a linear relationship, and not even a power relationship. This is an exponential relationship!”

And the expert would be correct. It is an exponent due to the Boltzmann distribution $n = n_0 \exp\left(-\frac{W}{kT}\right)$ which describes the distribution of molecules in a potential field. Here W is the potential energy of the atom (molecule), k is the Boltzmann constant.

Look, to remove the atom from the condensed body and convert it to the free state, it is necessary to perform some positive work A (Fig. 5.2b). Therefore, the potential energies of the atom in condensed (W_1) and free (W_2) states will be linked by the formula $W_2 = W_1 + A$. According to the Boltzmann distribution at thermal equilibrium (i.e. in the case when the vapor is saturated) the concentration of atoms in the free state n_{sat} (saturated vapor concentration) will be related with the concentration of atoms in the condensed state n_0 by the ratio:

$$n_{sat} = n_0 \exp\left(-\frac{A}{kT}\right). \qquad (5.3)$$



a) b) c)

a — the dependence of saturated water vapor density on temperature;
b — where the potential energy of atoms is greater, where their concentration is lower; c — the balance of the two powers only evaporation stream

Figure 5.2 — Developing a theoretical model

Combining formulae (5.1-5.3), we get the dependence of the lamp’s lifetime on its operating temperature:

$$\tau = \frac{6N_{total}}{n_{sat} v_T S} = \frac{6N_{total}}{n_0 v_T S} \exp\left(\frac{A}{kT}\right).$$

Note that the numerical value of the work A is very close to the value of the latent heat of vaporization of a substance per atom.

And one last point. In this laboratory work, we will measure not the temperature of the filament T, but the voltage U applied to the lamp. The Stefan – Boltzmann Law allows us to find a connection between these quantities.

The heat the filament emitted in the form of light energy: $\frac{U^2}{R} = \sigma T^4 S$ (Fig. 5.2c). Hence, we obtain $T = (\sigma RS)^{-\frac{1}{4}}\sqrt{U}$, and the final formula for the lifetime of the lamp is:

$$\tau = \frac{6N_{\text{total}}}{n_0 v_T S}\exp\left(\frac{A(\sigma RS)^{\frac{1}{4}}}{k}\frac{1}{\sqrt{U}}\right).$$

4. *Straightening Coordinates*. The final formula we obtained looks very scary. Let’s simplify it. Do you understand that if a constant is multiplied by another constant, the result is still a constant? With that in mind, there are only two constants in our formula: one is in front of the exponent function and the other is in the exponent itself. Therefore, we finally obtain the formula that very clearly demonstrates the structure of the dependence of the lamp lifetime on applied voltage:

$$\tau = A\exp\left(\frac{\beta}{\sqrt{U}}\right).$$

Will you be able to construct straightening coordinates for this relationship? Remind that if we take the logarithm of both sides of this equation, we obtain $\ln\tau = \ln A + \beta\frac{1}{\sqrt{U}}$. For variables $X = \frac{1}{\sqrt{U}}$ and $Y = \ln\tau$ the proposed relationship is transformed into a linear relationship $Y = kX + b$, where $k = \beta$, $b = \ln A$.

2. Recommendations for the Laboratory Work

We typically perform measurements for a pocket incandescent lamp with the following characteristics. Technical characteristics of this bulb: operating voltage, current and resistance are 3.5 V, 0.26 A, and 13.46 Ω respectively. We perform accelerated reliability tests at voltages from 5.2 V to 7.6 V, while the lifetime of the lamp varies from 780 s to 6 s.

Since the laboratory work is associated with certain expenses (lamps burn out for real), we generally use only one set of equipment. Naturally, we give the opportunity to burn out at least one lamp for each student group.

We have not previously discussed the process of constructing a confidence interval for a quantity determined by extrapolation. Therefore, students need to use already known general concepts to estimate the absolute uncertainty of the measurement.

Due to the fact that logarithmic coordinates are used for straightening, the accuracy of the result is usually low. However, the outcome typically coincides well with the data given in technical reference books (nominal lifetime for the lamp we use is equal to 20 hours, for a 100-watt incandescent lamp it is 1000 hours).

If necessary, you can check the accuracy of the geometrical method result using analytical formulae given below.

3. General Discussion *"Centennial Light Bulb"*

This section typically involves a discussion about the question about a more appropriate method for operating lamps: should we use "underburning" or "overburning"? In the first case, we increase the lifetime of the lamp, but reduce its light radiation. In the second case the opposite is true. Usually debates flow smoothly into the discussion about so called Centennial Light.

4. Example of the Quiz

1. The process of free falling is investigated. The purpose of the study is to find the law connecting the distance traveled h and the travel time t. Tasks: *a*) offer a mathematical model of the phenomenon under the study; *b*) suggest straightening coordinates; *c*) how is the free fall acceleration related to the slope of the line in the coordinates you suggested?

[*Answer*: *a*) $h = \frac{gt^2}{2}$; *b*) $X = \frac{t^2}{2}$, $Y = h$; *c*) $g = k$.]

2. The process of discharging a charged capacitor through a resistor is investigated. The purpose of the study is to derive the law that links the discharge current I to the discharge time t. Known parameter is the resistance R, unknown parameter is the capacitance C. Tasks: *a*) propose a mathematical model of the phenomenon under the study; *b*) suggest

straightening coordinates; *c*) explain how the unknown parameter (capacitor capacity) is related to the slope of the line in the coordinates.

[*Answer*: *a*) $I = I_0 \exp\left(-\frac{t}{\tau}\right)$, where $\tau = RC$;
b) $X = t$, $Y = \ln I$; *c*) $C = -\frac{1}{kR}$.]

5. Additional Tasks

1. A strong dependence on temperature reveals itself in many chemical reactions. For example, in reactions which make borscht sour. Find out how the "lifetime" of borscht depends on the storage temperature.

2. We characterize this case as a general case of linear relationship. Required parameters of confidence intervals of directly measured quantities are calculated by the formulae (4.1-4.2) of Session 7, $\bar{Y}_m = \frac{1}{M}\sum_{i=1}^{n} m_i Y_i$.

To construct a confidence interval for a lamp lifetime at operating voltage, it is necessary to use the rules for the indirectly measured quantity twice (Session 6).

Firstly, for the relationship $Y_{\text{oper}} = Y_m + k(X_{\text{oper}} - X_m)$, and secondly, for the relationship $\tau_{\text{oper}} = \exp(Y_{\text{oper}})$.

The parameters of the confidence intervals are given by the formulae:

a) for the auxiliary value Y_{oper}:

$$\bar{Y}_{\text{oper}} = \bar{Y}_m + k(X_{\text{oper}} - X_m),\ \Delta Y_{\text{oper}} = (X_{\text{oper}} - X_m)\Delta k. \qquad (15)$$

b) for the lifetime at operating voltage τ_{oper}:

$$\bar{\tau}_{\text{oper}} = \exp(\bar{Y}_{\text{oper}}), \quad \Delta\tau_{\text{oper}} = \exp(\bar{Y}_{\text{oper}})\,\Delta Y_{\text{oper}}.$$

Warning! The formula for ΔY_{oper} is written for a special case of extrapolation in the far zone, when only the contribution due to the uncertainty of the parameter k is taken into account. The geometrical method used in this laboratory work corresponds exactly to this case. The formula for the general case is given in Appendix A.

Use these formulae to calculate the confidence interval for the lamp lifetime and compare it with the interval constructed by the geometrical method.

Session 10. Experimental Proof of Hypotheses. Laboratory Work 10 "Temperature Dependence of Semiconductor Resistivity"

In this session, students will perform two new operations: put forward a hypothesis and test it experimentally.

1. Theoretical Background. *"Formulation and Testing of Hypotheses"*

In this session you should: 1) offer a formula for the dependence of semiconductors resistance on temperature (i.e., offer a theoretical model of the studied phenomenon); 2) obtain experimental data, that is, to measure the resistance of a semiconductor at various temperatures; 3) use the experimental data show that the experiment confirms your hypothesis.

1. *How Can Hypothesis Be Proven?* The experimental verification of hypotheses involves the following steps: you should offer the straightening coordinates for your proposed theoretical formula $R = R(T)$ and plot your experimental data using these coordinates. The hypothesis is considered experimentally proven if the experimental points lie *well* on the straight line.

2. *How are Hypotheses Formulated?* Hypotheses appear as a result of such a mental action as *induction*. This means you consider a few facts that you found interesting, identify a pattern or trend, and ... *guess* what conclusion they lead to. The proof of your guess is an experiment.

Today you will perform this mental operation yourself. To help you, we will first introduce two intriguing facts.

3. *The First Interesting Fact*. There have been three revolutions in the science of the direct current. The first one occurred in 1827, when Georg Ohm put into a single formula all the variety of phenomena associated with the electric current. He wrote this formula as follows: $I = \frac{U}{R}$ This equation, universally known as Ohm's law. In addition, Ohm discovered the dependence of the resistance on the geometric dimensions of the conductor: $R = \rho\frac{\ell}{S}$, here ℓ is the length of the conductor, S is the cross-sectional area, ρ is the specific resistance of the substance the conductor is made from.

The second revolution took place in 1900, three years after the discovery of the electron. Paul Drude created the electronic theory of electrical conductivity. He applied classical mechanics to the motion of electrons in a conductor and obtained an expression for the current I that flows through to a conductor at the applied voltage U:

$$I = \frac{e^2 n_{\mathrm{free}} \tau}{2m} \frac{S}{\ell} U.$$

Here e is the electron charge, m is the electron mass, τ is the free travel time (relaxation time), average time between electron impacts on crystal lattice ions, n_{free} is the concentration of *free* electrons.

Exercise 1. What laws does the formula obtained by Drude contain?

Solution. First, it is Ohm's law, second, it is the dependence of the resistance on the geometric dimensions, and third, it is the dependence of the resistivity on physical characteristics of a substance. As homework, justify these points with mathematical and conceptual reasoning. Now, we pass to discussion of the last Drude's result.

Thus, according to Drude, the specific resistance of a substance depends on its physical characteristics as follows:

$$\rho = \frac{2m}{e^2 n_{\mathrm{free}} \tau}.$$

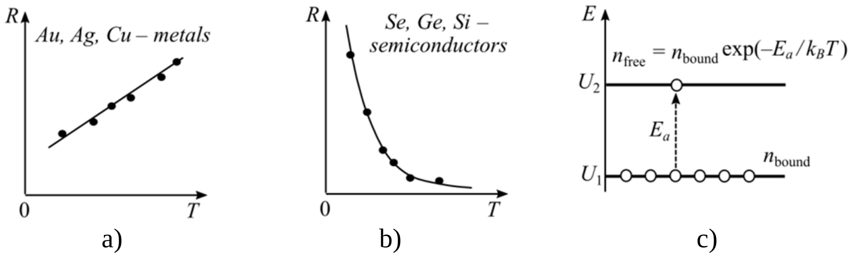
If you ask an educated person which of the quantities included in this formula will change when a substance is heated, the person will undoubtedly answer: "The value that will vary is the free run time τ. As the temperature increases, the ions of the crystal lattice begin to oscillate more and more intensely. This increases the disorder in the crystal, and as a result, collisions between electrons and ions occur more and more frequently. Therefore, as the temperature rises, the free-run time decreases, and the resistivity of the substance increases".

This hypothesis – that the resistivity of a substance increases with temperature – was tested experimentally on materials like gold, silver, copper, iron, etc (Fig. 5.3a). The results confirmed this behavior ...

The third, very subtle revolution in physics took place during the hot summer of 1873, when the telegraph connection failed in one of the counties of England.

This event was puzzling because a new telegraph line, equipped with modern insulators made of high-quality selenium material, had just been installed. In a laboratory Selenium proved to be an excellent insulator.

The investigation revealed that selenium is not an excellent insulator but an excellent deceiver! It is a good insulator at low temperatures; however, at high temperatures it becomes ... a good conductor (Fig. 5.3b). Later, germanium, silicon, and their numerous compounds were added to selenium.



a — it has always been this way;
b — but for some reason, it sometimes turns out that way; c — alternative state

Figure 5.3 — $R = R(T)$ and energy diagram

In order to distinguish between these two cases, ordinary and unusual, for substances whose resistivity increases when heated the name *metals* was left, while materials like selenium, which can switch between “insulators-conductors” were called *semiconductors*.

4. *The Second Interesting Fact.* All substances contain electrons, but not every substance conducts current. For conductivity, the presence of *free* electrons is essential.

Electrons are part of the atom and are held within it by Coulomb force from the positively charged nucleus. These electrons are naturally called bound electrons. Atoms can combine to form a solid. If electrons remain bound to its atoms, the substance will not conduct electricity. Scientists will call such substances an insulator.

However, as every child knows, things can be broken. You can also have an electron torn off from its atom. Then it will become free and will be able to move in the crystal, that is, to conduct electricity.

Electrons can be torn off individually. For example, an electron can gain enough energy from an external electric field that hitting an oxygen atom will break it into parts. The ejected electrons themselves accelerate, breaking up new atoms and … This avalanche-like transformation of an insulator into a conductor is what we call a lightning.

The second way of electrons separation from their atoms is the separation due to thermal motion. There is always a probability that due to random chaotic motion an electron will gain additional energy (Fig. 5.3c) and escape its atom.

So, how many electrons are detached from their atoms in the state of thermal equilibrium? The Boltzmann distribution provides the answer to this question

$$n_{\text{free}} = n_{\text{bound}} \exp\left(-\frac{E_a}{k_B T}\right).$$

Now you know everything and here is an exercise for you.

Exercise 2. Propose a formula for the dependence of the resistance of a semiconductor on temperature, and identify the corresponding straightening coordinates.

5. *Remarks for Instructors*. The energy ΔE required to move a particle into a free state is called differently in various fields of physics. In atomic physics, it is known as the ionization energy; in semiconductor physics, it is the width of the forbidden zone; in the physics of chemical processes and in nuclear physics, it is referred to as the binding energy.

One of the most intriguing scientific mysteries is connected with this quantity. The values of ΔE obtained in experiments on the conductivity measurement were always twice less than the values obtained in direct optical measurements of light absorption.

The mystery was resolved by M. Saha, who demonstrated that in the process of gas ionization where the transition occurs to a state with a continuous spectrum, the correct application Boltzmann distribution leads to the answer in which the exponent is the energy ionization divided by two:

$$n_{\text{free}} = n_{\text{bound}} \exp\left(-\frac{E_i}{2k_B T}\right).$$

Similar considerations for a semiconductor give

$$n_{\text{free}} = n_{\text{bound}} \exp\left(-\frac{\Delta E}{2k_B T}\right),$$

where ΔE is the band gap.

In this course, we chose not to delve into these details and instead used the concept of activation energy E_a.

2. Recommendations for Laboratory Work

This session is the final one in a series devoted to exploring the laws of physics. Summing up, we provide students with a table of straightening coordinates (see Table 5.1).

Table 5.1 — Standard Straightening Coordinates

1	Linear relationship $y = kx + b$				
1a	1st special case $k = 0$	$y = b$	X arbitrarily $Y = y$	$Y = b$	
1b	2nd special case $b = 0$	$y = kx$	$X = x$ $Y = y$	$Y = kx$	
1c	general case	$y = kx + b$	$X = x$ $Y = y$	$Y = kX + b$	
2	Power relationship $y = Cx^n$				
2a	logarithmic coordinates	$y = Cx^n$	$X = ln\,x$ $Y = ln\,y$	$Y = kX + b$	$k = n$ $b = \ln C$
2b	power coordinates	$y = Cx^n$	$X = x^n$ $Y = y$	$Y = kX$	$k = C$ $b = 0$
2c	complex power coordinates	$y = Cx^n$	$X = x^{n\cdot m}$ $Y = y^m$	$Y = kX$	$k = C^m$ $b = 0$
3	Exponential relationship				
3a	exponential damping	$y =$ $= y_0 \exp(-\alpha t)$	$X = t$ $Y = \ln y$	$Y = kX + b$	$k = -\alpha$ $b = \ln y_0$
3b	thermodynamic dependence	$y = A \exp\left(\frac{\beta}{T}\right)$	$X = \frac{1}{T}$ $Y = \ln y$	$Y = kX$	$k = \beta$ $b = \ln A$

3. General Discussion *"Expected Semiconductor Resistance at 0°C"*

Usually, a competition is held here. Students try to predict, using the best-fit experimental line, the value of the semiconductor resistance at 0°C. The winner is determined by direct measurement: the semiconductor is cooled in a refrigerator freezer and its resistance is measured.

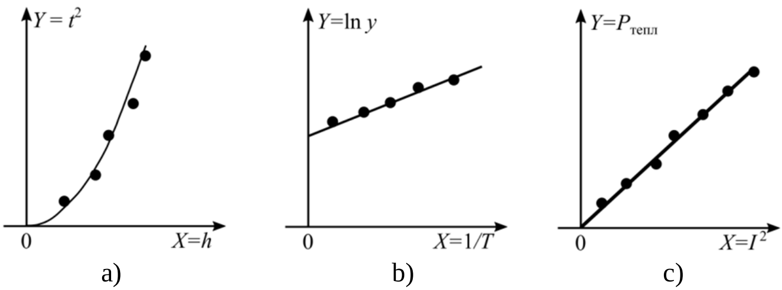
4. Examples of the Quiz

1. Figure 5.4a displays the results of an experimental study in coordinates $Y = t^2$ and $X = h$. Do these results prove the hypothesis that the square of the fall time is directly proportional to the height traveled?

[*Answer*: no.]

2. Figure 5.4b presents the results of an experimental study in coordinates $Y = \ln y$ and $X = \frac{1}{T}$. Which of the two hypotheses is proven by these experimental data: *a*) $y = A\exp\left(\frac{\beta^2}{T}\right)$; *b*) $y = A\exp\left(\frac{-\beta^2}{T}\right)$?

[*Answer*: Hypothesis "*a*".]



a — for question 1; b — for question 2; c — for question 3

Figure 5.4 — Results of experimental studies in different coordinates

3. Figure 5.4c shows the results of measuring the heat generated by a resistor when a current flows through it. $Y = P_{\text{heat}}$ and $X = I^2$ are chosen for the straightening coordinates, where P_{heat} is the heat power, I is the current. Formulate a hypothesis which is proven by these experimental data.

[*Answer*: "Heat power is directly proportional to the square of the current".]

5. Additional Tasks

1. In our laboratory work, the values of semiconductor resistance were read from the digital ohmmeter at random moments of time, more precisely, at those moments when the digital thermometer showed a new temperature value. Suggest a method to increase the accuracy of resistance measurement by video recording.

2. In session 2, we used a numerical method to determine whether a line passes through all experimental segments or not for two special cases of a linear relationship. For the general case of a linear relationship, the whole thing is no longer so simple. Still, suggest an algorithm that could be used to answer this question in the general case.

a) Write a program that implements your algorithm.

b) Investigate the data of this laboratory work using your program.

TOPIC 6. THE BASIS OF DATA MINING

Session 11. Basic Concepts of Intelligent Search in Databases: Trends and Physical Laws. Laboratory Work 11 “Dulong-Petit Law”

In this session, students get acquainted with the basic concepts of intellectual knowledge extraction from databases. In the practical part, they are offered to find the Dulong-Petit law for the molar heat capacity of substances by analyzing a database. The search strategy involves identifying a promising trend and selecting the appropriate dependency within it.

1. Theoretical part. *Intellectual Search*

To say that Nature deliberately hides its laws from us would probably be anthropomorphism[18]. Yet, it often seems that this is the case. The data we obtain from experiments appear to be intentionally encrypted. If that were true, it would be useful for us to get acquainted with the basics of encryption.

1. *Symmetric and Asymmetric Coding, Hashing, and Mining*. How old is cryptography? Probably, as old as writing itself. Encrypting a message can be as simple as rearranging letters or replacing them with numbers. Figure 6.1a shows what we obtain when each letter in a message is replaced with the next letter in alphabetical order.

Sasha loves Marichka $\Longrightarrow$ *tbtib mpwft mbsjdilb*

? $\Longleftarrow$ *mbsjdilb mpwft tbtib*

a)

$$\begin{bmatrix}1&2&3\\4&5&6\\7&8&9\end{bmatrix}\cdot\begin{bmatrix}7&8&9\\4&5&6\\1&2&3\end{bmatrix}=\begin{bmatrix}18&24&30\\54&66&84\\90&144&138\end{bmatrix}$$

$$? \Longleftarrow \begin{bmatrix}72&56&40\\69&56&43\\111&95&79\end{bmatrix}$$

b)

a — symmetric coding; b — asymmetric coding

Figure 6.1 — Examples of coding

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[18] Anthropomorphism is the interpretation of nonhuman things or events in terms of human characteristics (sky was frown, a crocodile felt offended, etc.). Anthropomorphism is a characteristic of primitive and early childhood thinking.

Can these records be decrypted? As long as we are dealing with such a primitive cipher, a well-known principle holds: "What one person can encrypt, another can always decrypt". For details, refer to Edgar Allan Poe's story "The Gold Bug". Naturally, decrypting becomes easy if you succeed in learning the encryption rule.

Exercise 1. Decipher the second message in Figure 6.1a, assuming the encryption rule is the same as for the first message.

[*Answer*: "Marichka loves Sasha".]

If encryption and decryption follow the same rule, the encryption is called *symmetric encryption*. A revolution in cryptography, and in modern life, occurred in 1976 when Whitfield Diffie and Martin Hellman invented *asymmetric encryption*. This mathematical breakthrough enabled the development of secure payment systems and credit cards. Asymmetric encryption has a key characteristic: knowing how to encrypt a message does not allow you to guess how to decrypt it (*asymmetric encryption of the first type*). Or, conversely, knowing how to decrypt a message does not reveal how to encrypt it (*asymmetric encryption of the second type*).

Exercise 2. You have invited your business partners to share their intentions for a new project. So, they have to be able to encrypt their messages and only you should be able to read their responses. For this you need to publish the rule of asymmetric encryption of … type.

[*Answer*: first.]

Exercise 3. To ensure that your business partners can verify that a message truly came from you, you need to publicly disclose the asymmetric encryption rule of ... type.

[*Answer*: second, such encryption is called an electronic signature.]

For people who are familiar with mathematics, the following example might help in understanding asymmetric encryption (Fig. 6.1b). It is easy to multiply two matrices of a special kind. However, performing the inverse transition (decomposing the matrix into the product of two matrices, each containing all numbers from 1 to 9) is in 10^{11} times more complicated!

Moreover, it is possible to encode in a way that decoding will be not in a million but in 10^{80} times more difficult. And here the naive rule of decoding turns into a much more pessimistic one: "There exist encryption methods for which decryption becomes a computationally impossible task!"

An example of such complex coding is hashing.

Hashing is a mathematical operation that converts an input text into a number of a given length.

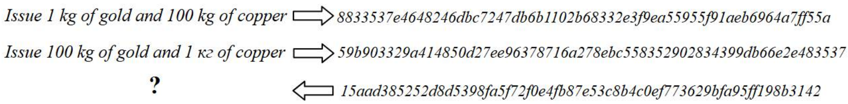


Figure 6.2 — The number of characters in real and fake messages is the same, but their hashes are completely different

Hashing was invented to protect documents from unauthorized modifications. Of course, you can count the number of characters of each type in the message, but this does not always help against dishonest people. Hashing, however, always helps!

To see hashing in action, open the calculator at https://www.infobyip.com/verifyemailaccount.php and select "Hash calculator". Get hash coding for the two phrases in the left side of Figure 6.2. The result we got is shown in the right side of Figure 6.2.

Exercise 4. How many different numbers can this hash function generate? Note that *a*, *b*, *c*, *d*, *e*, *f* are not letters but numbers 10, 11, 12, 13, 14, 15 in the hexadecimal number system.

[*Answer*: $2^{256} \approx 10^{77}$.]

Now, is it clear how difficult the task of decryption can be? To determine the original text corresponding to the last hash in Figure 6.2 ("*Hello World!*"), it is necessary to generate and hash 10^{77} texts.

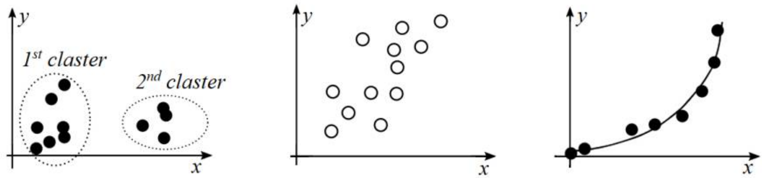
Such calculations are very similar to a mine work that shovels thousands and thousands of tons of empty rock in order to find a small diamond crystal. That is why large-scale calculations, where countless variants are "stupidly" sorted out, are called *mining*. Mining is used in both scientific and financial calculations.

In science, mining is used in bioinformatics to restore the human genome. Experimenters can cut DNA by chemical substances into separate parts. The task of computer scientists is to find the correct combination of countless pieces into a single integrity.

In financial calculations, mining is used to extract bitcoins. A bitcoin is a beautiful hash that looks special, such as four zeros at the end. In order to find such a hash and get a reward, you need an enormous amount of computation. Since a single computer is insufficient for this task, miners combine their computers into large "farms". This is a serious industry.

Today, the amount of electricity consumed by bitcoin mining is already comparable to the energy consumption of an entire country, such as Cyprus. However, a small part of bitcoins, one satosi[19] (1 satosi = 10^{-8} bitcoins), one can get it relatively easily. Usually, it is enough to complete a simple task on the Internet.

The main conclusion for us is as follows: searching for information in a database of experimental research may require a large number of calculations.



a) b) c)

a — clusters; b — trends; c — a dependence

Figure 6.3 — Good luck if you can find clusters, trends or dependencies in a database

2. *Intelligent Search in Databases*. Mining can be made more effective by incorporating intelligence. The intelligent search for knowledge in a database is called *Knowledge Discovery in Databases*. The most interesting thing is that an intelligent search is often carried out even when it is unclear whether the data contain any useful information or if they are merely an "empty breed". It's hard to look for something when you don't even know what you are looking for. Therefore, "miners" often look for patterns.

A *pattern* is something that can be extracted from a database and presented in a form that is understandable to a person. Patterns are usually presented as drawings of a certain type. The most important patterns are *clusters, trends, dependencies*.

A cluster is a breakdown of database elements into groups where a

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[19] Satoshi Nakamoto is the inventor of Bitcoin. However, no one knows who this person is. A lot of people have claimed to be Satoshi, but none have been able to prove it. Others have been suspected of being Satoshi, yet all such suspicions have turned out to be unfounded. The mystery remains …

common dependency is likely to be found. For example, if we take a table of the resistivity of different substances and represent each element as a point on a graph, we will observe two large groups (two clusters): conductors and insulators (Fig. 6.3a). It is clear that each cluster will likely follow its own law, so it is useful to consider them separately.

A trend is a general tendency showing, for example, that an increase in one quantity x leads to an increase in another quantity y. On a plot a trend appears as a "cloud" of experimental points (Fig. 6.3b). For example, this is how the experimental data illustrating the dependence of a store's profit on its retail area might appear. Finding a trend always gives researchers the opportunity to discover a new law. To do so, they need to find a "hidden" parameter z, which transforms the trend into a physics relationship $y = f(x, z)$ (Fig. 6.4).

Dependencies are narrow trends where we can recognize a physics relationship "blurred" due to measurement uncertainties (Fig. 3*c*). You already have a full arsenal of tricks to discover such patterns and turn them into physical laws.

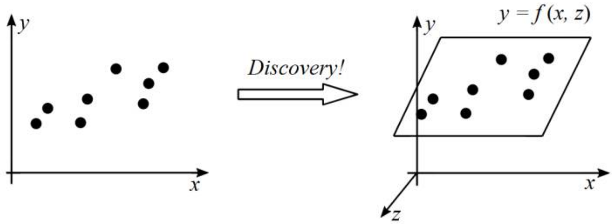


Figure 6.4 — Special luck if you can find the hidden parameter and turn the trend into the physical law

Let's formulate a rule that allows us to identify cases with hidden parameters.

If uncertainties of a physical quantity measurement cannot be explained by the instrumental uncertainty, the set of experimental data is a trend, and it makes sense to find a hidden parameter.

You will need to apply this rule completing your homework.

2. Recommendations for Laboratory Work

Students use Excel in this lab, build five plots and carry out the necessary calculations. The work consists of three stages. At the stage of preliminary analysis, students choose one of two trends for further research. In the second stage (the main study), they carry out a complete processing of the selected dependence and formulate the Dulong – Petit law. At the final stage, they receive a task to find out whether lithium and diamond obey the found law.

For Instructors. Lithium provides strong confirmation of the found law. It lies on the experimental line perfectly and at a very distant point. However, diamond completely refutes the Dulong-Petit law.

3. General Discussion "*The Problem of Diamond*"

Diamond stubbornly refuses to lie on the received experimental straight line. What should we do with this fact? Should we force it to "squeeze" into the Dulong-Petit law, or ...? This is the beginning of a story "Hidden parameters in physics and Debye theory of specific heat capacity of solids".

4. Examples of the Quiz

1. The resistivity ρ and the molar mass μ of twenty different substances were measured in the laboratory. The measurement results are plotted on the graph in coordinates $X = \lg \rho$ and $Y = \lg \mu$ (Fig. 6.5a), the molar mass was measured in g/mol.

a) What objects do make sense to consider together when searching for a physical law?

b) In what range do the values of the substance resistivity classified as "poor conductors" lie?

c) In what range do the values of substances molar mass classified as "good conductors" lie?

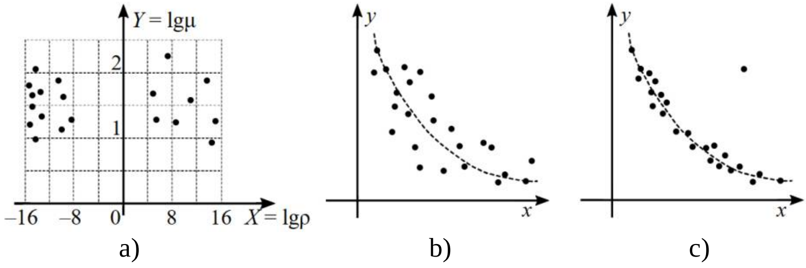
[*Answer*: *a*) a group of substances that conduct electricity well (12 samples) and a group of substances that conduct electricity poorly (8 samples); *b*) $(10^4 - 10^{16})$ Ohm·m; *c*) $(10 - 100)\cdot 10^{-3}$ kg/mol.]

2. Which of the trends shown in Figures 6.5b-c is "promising"?

[*Answer*: trend *c*.]

3. Experimenters study the dependence $y = 0.2x + 0.5z$. However, they are unaware of the dependence of the value *y* on the parameter *z* and plot the results of measurements on (x, y)-plane. What width of the "cloud" will be obtained if the measurements were made for the following values the controlled variable $x = 0, 1, \ldots 20$, and among the studied objects there are three types of objects for which the "hidden" parameter *z* has the values

$z_0 = 0$, $z_1 = 1$ and $z_2 = 2$? [*Answer*: $\delta y = 1$.]



a — for question 1; b — for question 2; c — for question 2

Figure 6.5 — Measurement results for the quiz

5. Additional Tasks

1. Investigate how the success of your classmates depends on their height. What did you get: a trend or a dependence?

2°. Diamonds do not appear in physics laboratories for no particular reason. A diamond as a counter-argument appeared at the final stage of the discussion. Firstly, the conclusion that a hypothesis "*All* substances subject to the Dulong-Petit law" should be rejected based on statistical analysis of uncertainties. We offer you to conduct such an analysis by numerical methods.

Note: The data table for specific heat is a "digital device" with a half-base equal to half of the smallest increment $\delta_c = 0.5$ J/(kg·K).

a) Check if, for all experimental points, the random deviation from the experimental line $\Delta Y_i = Y_i - \bar{k}X_i$ is greater than "instrumental uncertainty" δ_c, i.e. it is impossible to explain the deviation due to the presence of instrumental uncertainty.

b) Calculate the length of the realizations vector squared $\ell^2_{\text{real}} = \sum_{i=1}^{n} \delta Y_i^2$, which characterizes the scatter of the data in general, and compare it with the mean of this quantity, which would be observed in the presence of only the instrumental uncertainty $\langle \ell^2_{\text{ins}} \rangle = \frac{\delta_c^2}{3}(n-2)$.

c) What part of the total uncertainty is the instrumental uncertainty, and what part is random uncertainty coming from the system under the study?

Session 12. Interpretation of Experimental Patterns. Laboratory Work 12 "Electrostatic Field Simulation by Using a Conductive Paper"

This session focuses on modeling in two various aspects. First, we introduce the method of mathematical modeling of physical processes. Second, we consider a physical situation that is a model itself. Deviations occur in the case are due to the presence of hidden parameters. We begin the session by discussing the concept of experimental data interpretation.

1. Theoretical Part. "*Mathematical Modeling*"

1. *Interpretation is a treatment of facts observed in the experiment (hermeneutics).* The same set of facts can be interpreted in different ways.

For example, observing lightning, we may describe it as an electric discharge in the atmosphere, or, as the ancient Greeks believed, as magic spears thrown by angry Zeus. Similarly, we can describe planetary motion using the Ptolemaic system, where the Sun and the planets revolve around the Earth, or the Copernican system, where all planets revolve around the Sun. Both interpretations explain visible movement of planets and the Sun.

Which of the interpretations wins? Usually, the more natural does. However, it is important to understand that an interpretation is not an axiom, but rather a presumption[20]. It is impossible to formally prove that one interpretation is absolutely correct over another, and stubborn ones may remain in their opinion for as long as they like. New interpretations win not because opponents finally understand their meaning, but simply because of the natural change of generations.

In this laboratory work, you will need to interpret a very strange fact that we will find in the process of measurements. While formulating various explanations, remember the key methodological principle of scientific research, known as Occam's razor[21]: "Entities should not be multiplied beyond necessity". In simpler terms, try to find the most natural explanation.

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[20] Axiom is a statement that is considered true unconditionally; a presumption is a statement that is considered true until its falsity is unequivocally proven.

[21] William of Ockham (1285-1349) was an English philosopher and logician.

2. *Modeling is a process of physical research where an object is judged by measurements made of another object*. We distinguish between two types of modeling: physical modeling and mathematical modeling.

Physical modeling (discussed in Session 8) is a simulation that is carried out with an object that is similar to the original one, but may differ in size, be placed in a different medium, or move at a different speed. One of the earliest examples of such modeling is associated with the name of Gustave Eiffel. To determine the wind strength which will act on his tower, he did not place the actual tower in a wind tunnel but its model. In such experiments, in order to recalculate the results obtained for the model to the results for the prototype, you should use the theory of dimensions (session 8), or, in more complex cases, its generalization, the similarity theory.

Exercise 1. On the model of a car which is made in scale 1:10 and placed in wind tunnel where the speed of air is 10 m/s, acts the resistance force equal to 3 N. What will be the resistance force acting on a real car moving at the speed of 72 km/h? *Hint*: according to the dimensional analysis, the equation for the resistance force is $F_{\mathrm{res}} = C\rho \upsilon^2 d^2$, here ρ is the density of the medium, υ is the flow velocity, d is a characteristic size of the object, C is a dimensionless coefficient.

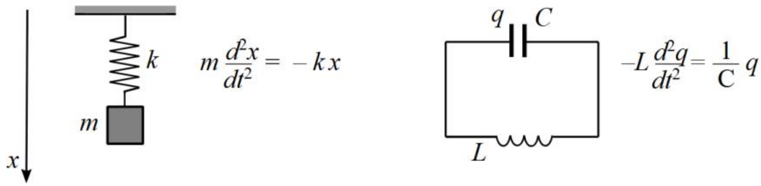
[*Answer*: 1200 N.]

3. *Mathematical modeling* is an absolutely unbelievable action for a beginner. It turns out that the way neutrons are distributed in a nuclear reactor or the way temperature is distributed in the thickness of the material cut by the laser can be found by a simple experiment. To do this, it is necessary to measure the voltage distribution in the coal (conductive) paper with current flows!

Mathematical modeling is a measurement on an object that has a completely different physical nature than the original, but described by the same mathematical equations. Note that a similar approach in theoretical problems-solving is known as the method of solving problems by analogy.

The idea of the analogy method and mathematical modeling is as follows: if two phenomena are described by the same mathematical equations, the evolution and distribution of the corresponding quantities are described by the same formulae (up to the notations substitution).

There is no need to explain anything here. We believe that this concept is *logically* clear to everyone familiar with algebra and knows that the mathematical content does not depend on the choice of a letter to denote. However, there might be a psychological barrier: it may seem unbelievable that a huge number of results in physics can be obtained "for free". Check yourself.



a) b)

a — a load on the spring; b — oscillatory circuit

Figure 6.6 — An example of mathematical modeling

Exercise 2. If a load (Fig. 6.6a) is shifted down from the equilibrium position by a distance A, and then released, its motion is described by the equation: $x(t) = Acos(\omega t)$, where $\omega = \sqrt{k/m}$. Write down the equation that describes the change in the charge of a capacitor (Fig. 6.6b), if initially the charge of the capacitor was at its maximum and was equal to q_0. The parameters of the systems, as well as the differential equations that control their evolution, are shown in Figure 6.6.

[*Answer*: $q(t) = q_0 cos(\omega t)$, where $\omega = \sqrt{1/CL}$.]

4. Application of mathematical modeling in the case of ordinary differential equations is interesting, but not revolutionary. Today, many programs (for example, MATLAB) can easily calculate changes in one or more variables over time.

A more interesting case is the use of mathematical modeling for solving equations in partial derivatives. These equations are much more complicated. They describe something that evolves at every point in space. And there are infinite numbers of such points. Therefore, the differential equations in partial derivatives are significant!

The most common in physics are the equations in partial derivatives containing *laplacian*. Laplacian is denoted by sign Δ and stands for:

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}.$$

Do not confuse the laplacian operator (Δ) with the gradient operator

(∇). When applied to a function $f(x, y)$ the gradient gives a vector $\nabla f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j}$, while the laplacian gives a scalar.

Exercise 3. Which of the functions:

a) $f = \alpha x + \beta y$; b) $f = x^2 - y^2$; c) $f = x^2 + y^2$,

satisfy the Laplace's equation $\Delta f = 0$?

[*Answer*: the first and second.]

In mathematics, the solution of equations in partial derivatives, for example, the Laplace's equation $\Delta f = 0$, is a much more complex procedure than the solution of ordinary differentials equations. However, solutions of this equation have a number of properties that make it easier to work with. If the functions $f(x, y)$ and $g(x, y)$ are solutions of the Laplace's equation, then the following functions are also solutions:

1) $f_1 = f(x, y) + const$; 2) $f_2 = \lambda f(x, y)$;

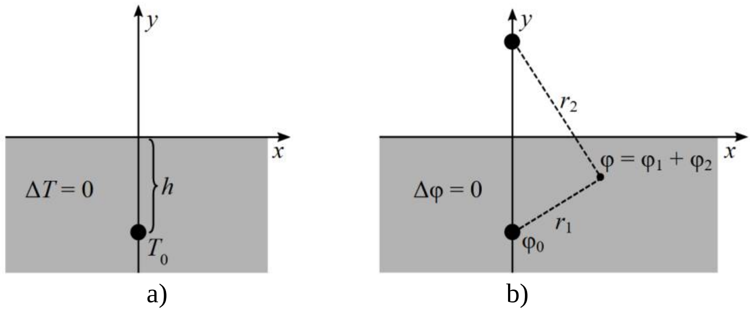
3) $f_3 = f(\lambda x, \lambda y)$; 4) $f_4 = f(x, y) + g(x, y)$.

These properties allow constructing many new solutions from already found solutions of Laplace's equations. That is why experts say that the main challenge in Laplace's equation is not to find a solution, but to find the solution that *satisfies the specified boundary conditions*.

Exercise 4. Prove the above properties of Laplace's equation solutions.

5. *In physics*, Laplace's equation describes the distribution of heat, the diffusion of matter, the distribution of electric field between charged conductors, the passage of electric current through a conductive environment, and much more. For those familiar with math, the reason for such universality is clear: flows are always proportional to gradients $\vec{q}_f = -\alpha \nabla f$, and in the stationary case, when nothing accumulates anywhere, the condition $\operatorname{div} \vec{q}_f = 0$ is satisfied. Laplace's equation $\Delta f = 0$ follows from these two conditions. It turns out that all of the above phenomena are described by the same mathematical equation and, consequently, have the same distribution in space. This is what physicists use. Let's explore how physicists solve problems involving Laplace's equation.

At the beginning of nuclear energy development, physicists had to solve the following problem. Under the ground, at a depth h is storage of radioactive materials, represented as a relatively small spherical region of radius a (Fig. 6.7a). Due to nuclear reactions, the temperature in the storage exceeds the ambient temperature by T_0. What is the temperature distribution on the surface of the earth? It is clear that observations of the temperature on the earth's surface can provide valuable information about the intensity of processes going on in the storage.



a — temperature distribution; b — potential distribution

Figure 6.7 — The problem of distributions

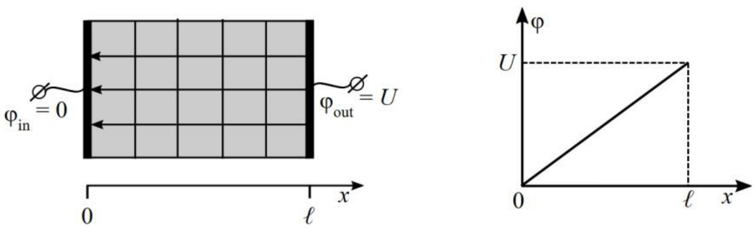
If this problem were to be solved by theoretical physicists, they would most likely think as follows: "For simplicity, let's consider the storage as a point mass (very small sphere of radius a). We need to find a solution of the Laplace's equation $\Delta T = 0$ in the lower half-space that satisfies the corresponding boundary conditions.

A similar problem has been solved in electrostatics by the method of mirror images (Fig. 6.7b). The solution to the electrostatic problem is the sum of two terms: $\varphi = \varphi_1 + +\varphi_2 = \varphi_0 \left(\frac{a}{r_1} + \frac{a}{r_2}\right)$, where φ_0 is the potential of the sphere, and r_1 and r_2 are the distances from the sphere and its image to the observation point. Then everything is obvious …"

Exercise 5. Complete the theorists' reasoning. Write expressions for temperature distribution: *a*) inside the earth; *b*) on the surface of the earth.

[*Answer*: *a*) $T = T_0 \left(\frac{a}{r_1} + \frac{a}{r_2}\right)$; *b*) $T = 2T_0 \frac{a}{\sqrt{h^2+x^2}}$.]

Experimental physicists would most likely think differently: "Before investigating a three-dimensional case, it is reasonable to watch what happens in a two-dimensional case, because it's very simple! For the two-dimensional case, we can determine the temperature distribution inside the earth by measuring the distribution of electric potential when current flows through conductive paper. Why is this possible? Because the distribution of both, potential and temperature, are described by Laplace's equation. We just have to think about the form of the electrodes, because the boundary conditions for the electric potential should coincide with boundary conditions for the temperature."



a) b)

a — equipotential surfaces and current lines for a rectangular plate temperature distribution; b — distribution of potential along the length of the plate potential distribution

Figure 6.8 — The electric current flow through a rectangular plate

Today, we will introduce this method of mathematical modeling to you. We will work with the simplest geometry: studying the electric current flow through a rectangular plate (Fig. 6.8). In this case, the distribution of the potential along the length of the plate is given by a simple formula $\varphi(x) = U\frac{x}{\ell}$, where U is the applied voltage, and ℓ is the plate length.

The aim of our study is not merely to confirm this simple distribution, but rather to explore something more profound. In this laboratory work, we will observe how hidden parameters (unknown to us characteristics of the measured system) lead to trends appearance.

2. Recommendations for Laboratory Work

The main part of the laboratory setup is a plate with three layers. The top and bottom layers are opaque covers, and a rectangular conductive carbon paper is between them. This conductive paper is turned relative to the covers by the angle $\alpha = \arctan(1/4) = 14°$. This is the hidden parameter that distorts the expected potential distribution.

The main intrigue of this laboratory work is the question of whether students will be able to understand that they are dealing with a trend, not a dependency, and guess the cause of "data blur". Instructors can assist by providing hints, for example, asking students to construct real equipotential surfaces according to measurements or just say that the paper has been.

And one more remark. When measuring, several points are in the region with no conductive paper (the value of the potential equals to zero). These points form a cluster that must be excluded from consideration.

3. General Discussion "The Anti-Trend Coordinate"

Once the hidden parameter is found, everyone discusses with great interest how, using well-known mathematics formulae of rotational coordinate transformation, find an expression for the correct anti-trend coordinate and turn the trend into a dependency.

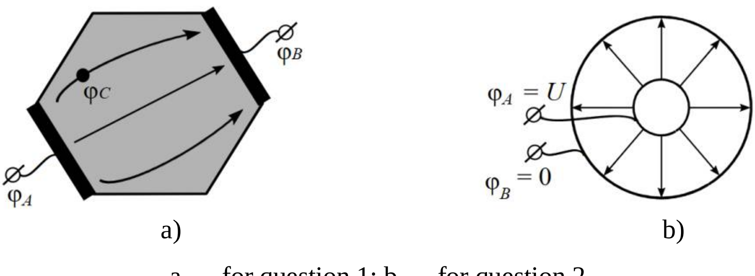
4. Examples of the Quiz

1. Figure 6.9a shows the setup used to determine the voltage distribution in a conductive paper shaped like a hexagon. Measurements showed that at electric potentials on the electrodes equal to $\varphi_A = 100$ V and $\varphi_B = 0$ V potential at the point C is equal to $\varphi_C = 78$ V. Using the properties of Laplace's equation solutions, answer the following questions:

a) what would the temperature be at point C, if the experiment were conducted to study temperature distribution, where $t_A = 100$°C and $t_B = 0$°C ?

b) what would the temperature be at point C, if the temperature of the boundaries were equal $t_A = 150$°C and $t_B = 50$°C ?

[*Answers*: *a*) $t_C = 78$°C; *b*) $t_B = 138$°C.]



a) b)

a — for question 1; b — for question 2

Figure 6.9 — Experimental setups for the quiz

2. The distribution of the potential between two cylindrical conductors (see Fig. 6.9b) is given by the formula $\varphi = U\frac{\ln(b/\rho)}{\ln(b/a)}$, where a and b are the radii of the conductors, and U is the potential difference between the conductors. Wright down the formula for temperature distribution in the space between heated cylinders of radii a and b, if the temperature of the inner cylinder is equal to T_A, and the temperature of the outer is T_B.

[*Answer*: $T(\rho) = (T_A - T_B)\frac{\ln(b/\rho)}{\ln(b/a)} + T_B$.]

5. Additional Tasks

1. Suggest an algorithm that would allow you to find hidden parameters in the trend.

2. This case is characterized as a general case of linear dependence.

a) Using formulae (2.3-2.4) and the ideas of the previous session, calculate the standard deviation of a single measurement of potential φ by the formula $\delta\varphi = \sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}\Delta\varphi_i^2}$ for usual and anti-trend coordinates, and compare it to the instrumental uncertainty of a digital voltmeter δ_φ = 0.2 V. Draw conclusions.

b) Write a program for calculating the parameters of a confidence interval for the value $y(x)$, which is determined by linear interpolation based on measured points $y_i(x_i)$.

c) Use your program to calculate a confidence interval for the potential of the “black” point and compare it with the interval constructed by the geometrical method in the laboratory work. Does the geometrical method work well in this case?

TOPIC 7. ASSESSMENT SESSIONS

Session 13. Theoretical Minimum

In the first assessment session we check students' understanding of core concepts and their proficiency in essential laboratory skills. The session is offered in the form of a written test. There are 4 possible answers to each of 25 questions. Each question has only one correct answer. A sample of the test is given below.

1. A student obtained an interval for coordinates of an object at one instant: from 53.1 cm to 53.5 cm. Express this as $\bar{x} \pm \Delta x$.

A	B	C	D
(53.1 ± 0.1) см	(53.3 ± 0.2) см	(53.1 ± 0.4) см	(53.3 ± 0.4) см

2. Find the fractional uncertainty in the following experimental result: (80 ± 4) µC.

A	B	C	D
5%	0.5%	50%	0.2%

3. Calculate the absolute uncertainty in the following experimental result: 24 ± 25%.

A	B	C	D
(24.00 ± 0.25) cm	(24.0 ± 2.5) cm	(24 ± 3) cm	(24 ± 6) cm

4. Round the following result: h = (5.032 ± 0.04329) m

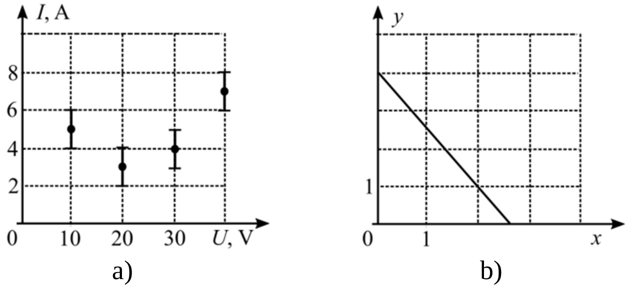
A	B	C	D
(5.032 ± 0.043) m	(5 ± 0.043) m	(5.03 ± 0.0433) m	(5.03 ± 0.04) m

5. A student measured some quantity three times, and obtained the following values: $x_1 = 13$, $x_2 = 13$, $x_3 = 10$. Present the experimental result as $\bar{x} \pm \Delta x$. *Hint:* $\Delta x = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n(n-1)}}$.

A	B	C	D
12 ± 1	12.0 ± 0.4	12.0 ± 0.8	11.5 ± 1.5

6. Figure 7.1a shows the results of an experimental study, including instrumental uncertainty. What is the value of the instrumental uncertainty δ_I?

A	B	C	D
2 A	1 A	0.5 A	0.25 A



a — for questions 6 -7; b — for question 8

Figure 7.1 — Plots for the questions 6 - 8

7. What is the fractional uncertainty of the current when a voltage equals 30 V (Fig. 7.1a)?

A	B	C	D
10%	5%	50%	25%

8. Figure 7.1b shows a graph of the linear dependence. Find the equation of the line.

A	B	C	D
$y = -2x + 4$	$y = -\frac{2}{3}x + 4$	$y = -1.5x + 4$	$y = -1.5x + 2$

9. It is known that the relationship between the resistance of a conductor and its temperature is described by a formula $R(t) = R_0 + R_0\alpha t$, where α is the temperature coefficient of resistance. How are variables of the linear relationship X, Y, and a slope k related to the variables and parameters of the given relationship?

A	B	C	D
$X = R$ $Y = t$ $k = \alpha$	$X = R$ $Y = t$ $k = R_0$	$X = t$ $Y = R$ $k = R_0$	$X = t$ $Y = R$ $k = \alpha R_0$

10. In a laboratory work, the validity of Hooke's law $\Delta\ell = \frac{\ell}{E \cdot S} F$ for a wire was checked. Here $\Delta\ell$ is elongation of the wire, F is the value of the tension force, ℓ, S, E are parameters of the wire (its length, cross-sectional area, and Young's modulus of the material from which the wire is made). The force F is chosen as the horizontal variable, the elongation $\Delta\ell$ as the vertical one. How is the slope k of the obtained linear relationship related to the parameters of the wire ℓ, S, and E?

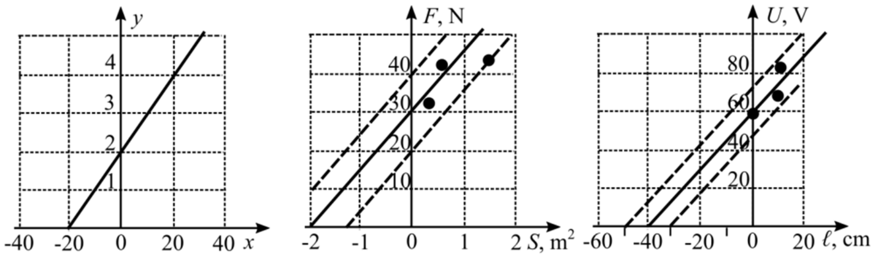
A	B	C	D
$k = \frac{ES}{\ell}$	$k = \frac{\ell}{ES}$	$k = \frac{1}{ES}$	$k = \ell$

11. Using the experimental plot (Fig. 7.2a), find the mean value of x_0 and b. *Hint*: $y = kx + b = k(x + x_0)$.

A	B	C	D
$b = 20$ $x_0 = 2$	$b = 2$ $x_0 = 20$	$b = -20$ $x_0 = 2$	$b = 2$ $x_0 = -20$

12. Using the experimental plot (Fig. 7.2b), construct the confidence interval for b. The total number of measurements is equal to 25.

A	B	C	D
$b = (30 \pm 5)$ N	$b = (30 \pm 10)$N	$b = (30 \pm 2)$ N	$b = (30 \pm 1)$ N



a) *b*) c)

a — for question 11; b — for question 12; c — for question 13

Figure 7.2 — Plots for the questions 11 - 13

13. Using the experimental plot (Fig. 7.2c), construct the confidence interval for x_0. The total number of measurements is equal to 25.

A	B	C	D
x_0 = (40 ± 5)cm	x_0 = (40 ± 2)cm	x_0 = (40 ± 10)cm	x_0 = (40 ± 1)cm

14. To determine the area of a square, its side was found:

$$a = (10.0 \pm 0.4)\ \text{cm}.$$

Using the fact that the area of a square is calculated by the formula $S = a^2$, find the fractional uncertainty ε_S of the measured area.

A	B	C	D
ε_S = 2%	ε_S = 8%	ε_S = 4%	ε_S = 16%

15. For the situation from the previous problem, construct the confidence interval for the area S.

A	B	C	D
(100 ± 16) cm^2	(100.0 ± 0.4) cm^2	(100 ± 4)cm^2	(100 ± 8)cm^2

16. Figures 7.3a-c show three experimental graphs plotted in logarithmic coordinates. Establish a correspondence between graphs and following physical laws: I) $y = Cx^2$; II) $y = Cx^6$; III) $y = C\sqrt{x}$.

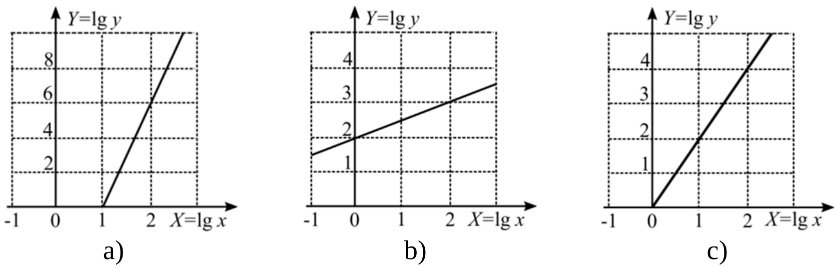


Figure 7.3 — Plots for the questions 16 - 17

A	B	C	D
I — *a*) II — *b*) III —*c*)	I — *c*) II —*b*) III —*a*)	I — *c*) II — *a*) III —*b*)	I — *b*) II — *a*) III —*c*)

17. Which of the graphs presented in Figure 7.3 corresponds to the power relationship $y = Cx^n$ with a coefficient C equal to unity?

A	**B**	**C**	**D**
a)	*b*)	*c*)	none

18. Use dimensional analysis to derive the formula for the ideal gas concentration n ($k = 1$). The quantities it can depend on are the gas density ρ and the mass of gas molecule m_0.

A	**B**	**C**	**D**
$n = m_0\rho$	$n = m_0/\rho$	$n = m_0\rho^2$	$n = \rho/m_0$

19. The dependence of the oscillation frequency f of the spring pendulum on the mass of the weight m is: $f = \frac{1}{2\pi}\sqrt{\frac{k_{spr}}{m}}$, where k_{spr} is the spring stiffness. The following straitening coordinates are suggested: $X = \frac{1}{m}$ and $Y = 4\pi^2 f^2$. How is the slope k of the linear relationship related to the spring stiffness k_{spr}?

A	**B**	**C**	**D**
$k = k_{spr}/m$	$k = k_{spr}$	$k = \sqrt{k_{spr}/m}$	$k = 1/k_{spr}$

20. The dependence of the charge q of the capacitor on time t during discharging via the resistor is $q = q_0 e^{-\frac{t}{RC}}$, where q_0 is the initial charge of the capacitor, C is its capacity, R is unknown resistance of the resistor. The following straitening coordinates are suggested: $X = t$ and $Y = \ln q$. How is the absolute value of the slope $|k|$ of the linear relationship related to the resistance R?

A	**B**	**C**	**D**
$\lvert k\rvert = 1/RC$	$\lvert k\rvert = R/C$	$\lvert k\rvert = RC$	$\lvert k\rvert = C/R$

21. The final velocity of the body υ and the height of the fall H were measured in the experiment. The measurement results are plotted on the graph in coordinates $Y = \upsilon^2$ and $X = H$. (see Fig. 7.4a). Which hypothesis do these results confirm?

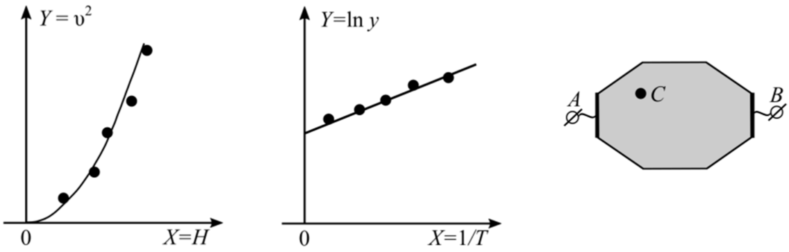
A	"The final velocity is inversely proportional to the height of the fall"
B	"The square of the final velocity is directly proportional to the height of the fall"
C	"The final velocity is directly proportional to the square of the height of the fall"
D	None of the above

22. The concentration (number density) n of molecules depends on height h above sea level and the temperature T according to $n = n_0 e^{-\frac{m_0 gh}{k_B T}}$, where n_0 is the number density at sea level, m_0 is the mass of a gas molecule. In a study, the dependence of the concentration n on the temperature T is checked (hight h is fixed). Suggest straitening coordinates X and Y for this study, and find the slope k and y-interception b of the straitened dependence.

A	**B**	**C**	**D**
$X = T$ $Y = \ln n$ $\lvert k\rvert = \frac{m_0 gh}{k_B}$ $b = \ln n_0$	$X = 1/T$ $Y = \ln n$ $\lvert k\rvert = \frac{m_0 gh}{k_B}$ $b = \ln n_0$	$X = 1/T$ $Y = \ln n$ $\lvert k\rvert = \frac{1}{k_B}$ $b = \ln n_0$	$X = 1/T$ $Y = n$ $\lvert k\rvert = \frac{m_0 gh}{k_B}$ $b = n_0$

23. Figure 7.4b presents the results of an experimental study in coordinates $Y = \ln y$ and $X = \frac{1}{T}$. Which of the hypotheses is proven by these experimental data?

A	**B**	**C**	**D**
$y = A\exp\left(\frac{T}{\beta^2}\right)$	$y = A\exp\left(\frac{\beta^2}{T}\right)$	$y = A\exp\left(\frac{-T}{\beta^2}\right)$	$y = A\exp\left(\frac{-\beta^2}{T}\right)$



a) b) c)

a — for question 21; b — for question 23; c — for question 25

Figure 7.4 — Plots for the questions 21, 23, 25

24. Experimenters study the dependence $y = 0.3x + 0.5z$. However, they unaware of the dependence of the value y on the parameter z and plot the results of measurements on (x, y)-plane. What width of the "cloud" will be obtained if the measurements were made for the following values the controlled variable $x = 0, 1, \ldots, 20$, and among the studied objects there are three types of objects for which the "hidden" parameter z has the values $z_0 = 0$, $z_1 = 1$ and $z_2 = 2$?

A	B	C	D
0.5	2	1	0.3

25. Figure 7.4c shows the setup used to determine the voltage distribution in a conductive paper shaped like a hexagon. Measurements showed that at electric potentials on the electrodes equal to $\varphi_A = 70$ V and $\varphi_B = 0$ V potential at the point C is equal to $\varphi_C = 33$ V. What would the temperature be at point C, if the experiment were conducted to study temperature distribution, where $t_A = 210°C$ and $t_B = 0°C$? Consider the setup insulated.

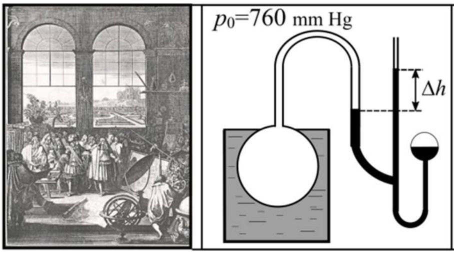
A	B	C	D
99°C	33°C	177°C	103°C

Session 14. Search for Physics Laws

At the second assessment session, the student's ability to obtain a physical law from experimental data is evaluated. Samples of examination questions and answer requirements are given below.

1. Linear Relationship

Version 1. Absolute Zero Temperature According to Amonton. In 1702, an academician of the Paris Academy of Sciences, Guillaume Amonton studied how the pressure of air changes when heating, if its volume is kept constant (Fig. 7.5).



	t,°C	Δp, mm Hg
1	0	0
2	20	58
3	40	130
4	60	183
5	80	263
6	100	313

Figure 7.5 — King Louis XIV of France visits G. Amonton's laboratory

According to the results of his measurements, Amonton found the temperature at which the air pressure would drop to zero. He called it absolute zero temperature – the lowest temperature possible in nature.

Task 1. Plot a graph showing the dependence of the air pressure in the flask on temperature and test the hypothesis: "in an isochoric process, gas pressure depends linearly on temperature."

Hint: Take into account that the value of atmospheric pressure at the moment is indicated on the experiment diagram measurements – 760 mm Hg.

Task 2. Measure the parameters of the obtained linear dependence.

Task 3. Write down Amonton's law in the form of a formula that relates pressure to temperature in degrees Celsius.

Task 4. Find the value of "absolute zero temperature according to Amonton" and compare it with the modern value (–273°C).

Version 2. Specific Resistance. The dependence of resistivity on temperature for two copper alloys (*A* and *B*) was studied in the laboratory.

A scattered scientist signed only the first two measurements in the journal of measurements (Fig. 7.6). Your task is to get the laws out of this data, despite the misunderstanding. Both alloys are metals.

#	$t, ^\circ C$	ρ, $10^{-8}\Omega\cdot m$	alloy	#	$t, ^\circ C$	ρ, $10^{-8}\Omega\cdot m$	alloy
1	20	2.80	**A**	7	250	7.23	?
2	50	4.97	**B**	8	260	6.18	?
3	80	3.94	?	9	300	8.23	?
4	100	5,26	?	10	320	6.53	?
5	120	4.81	?	11	380	9.85	?
6	230	5.85	?	12	400	7.06	?

Figure 7.6 — The journal of a scattered scientist

Task 1. Divide the data into two clusters.

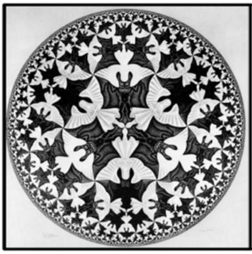
Task 2. Measure the parameters of the selected dependencies.

Task 3. Write for each alloy the "law of resistivity" in the standard form.

Task 4. Find the value of the resistivity of each alloy at 1000° C.

2. Logarithmic Coordinates

***Version 3*. *Fractals*.** The geometrical characteristics of ordinary bodies are connected by power dependences, where the powers (exponents) are integers or prime fractions. For example, the area of a circle is related to its radius by the equation $S = \pi r^2$. However, some objects exhibit far more complex structures than a circle or a cube. These are fractals (Fig. 7.7). A natural example of a fractal is … a common river system.



#	river	ℓ, km	S, km^2
1	Guyandotte	145.1	2088
2	Little Coal	90.5	984
3	Tug Duy Fork	63.7	586
4	Big Coal	56.2	449
5	Racoon	53.6	446
6	Pingeon	49.1	405

Figure 7.7 — Fractals in art and nature

The table shows data on several rivers in the USA, including their length ℓ and the areas S of their basins. Your task is to obtain the law from these data.

Task 1. Test the hypothesis: "The area of the river basin is proportional to the square of its length." Note that in this case, there is an a priori zero $S(0) = 0$: no river – no basin.

Task 2. Assuming a power dependence is valid, suggest straitening coordinates for the dependence $S(\ell)$ and find the parameters of this straitened dependence.

Task 3. Obtain from the straitened dependence Hack's law – the equation that connects the area of the river basin and its length.

Task 4. Using the obtained law, estimate the area of the Amazon River basin. The length of the Amazon River is 6400 km.

Version 4. ***Earthquakes***. Not only ordinary objects follow laws, but even large-scale natural phenomena such as earthquakes exhibit predictable patterns. An earthquake is a manifestation of hidden movements of tectonic plates occurring deep within the Earth's crust.

#	type	*M*	*N*, 1/year
1	disastrous	8	1
2	devastating	7	10
3	destructive	6	100
4	very powerful	5	1000
5	powerful	4	10 000
6	moderate	3	100 000

Figure 7.8 — Seismograph of the Chinese emperor

The table (Fig. 7.8) provides data on the number of earthquakes N occurring per year worldwide, depending on their intensity, which is characterized by magnitude M. Your task is to extract the law from these data.

Task 1. Test the hypothesis: "The number of earthquakes is inversely proportional to their magnitude."

Task 2. Suggest the straitening coordinates and determine parameters of the straitened dependence. Note that you must be able to place experimental points, which differ by five orders of magnitude, on the graph.

Task 3. Obtain from the above data the "earthquake frequency law" – the formula that relates the number of earthquakes to their intensity (magnitude).

Task 4. Figure 7.8 shows a seismograph of the Chinese emperor, invented by Chinese sage Zhang Heng in the 2nd century AD. This device was capable of detecting even weak earthquakes occurring up to 500 km away from the imperial palace. How many times did this device register an earthquake per day? A "weak earthquake" is defined as an earthquake with a magnitude of 2. The Earth's radius is 6400 km.

3. Dimensional Analysis

Version 5. Determination of the Mass of an Apatosaurus. Biologists claim that the mass *m* of a mammal is related in some way to the diameter *d* of its tibia. As evidence, they cite the table shown in Figure 7.9.

#	animal	d, cm	m, kg
1	mouse	0.05	0.02
2	cat	0.45	2.0
3	fox	1.1	12
4	wolf	2.3	50
5	buffalo	8.0	600
6	elephant	22.0	4500

Figure 7.9 —The weight of an animal and the diameter of the tibia

Task 1. Test the hypothesis that "the weight of an animal linearly depends on the diameter of the tibia." Note that in this case, there is an a priori zero $m(0) = 0$: an extremely small tibia can only support an extremely small animal.

Task 2. Obtain by dimensional analysis the main part of the dependence of an animal mass and the diameter of its tibia. Suggest the straightening coordinates. Identify parameters of the straightened dependence.

Hint: the tibia diameter may depend on the mass $[m]$ = 1 kg, the acceleration of free fall on the planet $[g]$ = 1 m/s^2, and the strength limits of bone tissue $[\sigma]$ = 1 N/m^2.

Task 3. Obtain from the above data the "law of a tibia" – the formula that connects the mass of the animal to the diameter $m(d)$.

Task 4. Why did Apatosaurs become extinct? Maybe they just didn't have enough space in Noah's Ark? Using the obtained law, determine the mass of an Apatosaurus (large herbivorous dinosaur). The diameter of

Apatosaurus's tibia is 60 cm.

***Version 6*. *Freezing of a Pond*.** "December was warm. Then, in January the frost hit. For a week the thermometer persistently showed minus twenty degrees. Every day, the ice on the river grew thicker …"

#	t, day	h, cm
1	0	0
2	1	16
3	2	22
4	3	28
5	4	34
6	7	42

Figure 7.10 — Winter activities in the Netherlands

The table (Fig. 7.10) provides data on how the thickness of ice increases over time when the air temperature remains constant at –20°C.

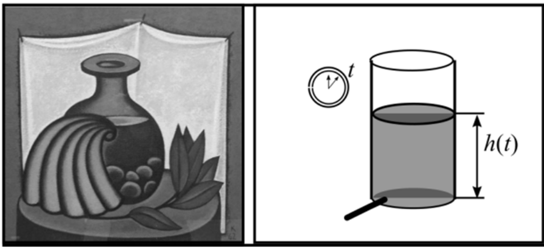
Task 1. Test the hypothesis that "the thickness of ice linearly depends on time." Note that in this case, there is an a priori zero $h(0) = 0$: water does not freeze instantly.

Task 2. Using dimensional analysis, obtain the main part of the ice thickness dependence on time and suggest straightening coordinates. Determine the parameters of the straightened dependence. Note, the thickness of ice may depend on time $[t]$ = 1 s, temperature difference $[T]$ = 1 K, and such characteristics of ice as density $[\rho]$ = 1 kg/m^3, thermal conductivity $[\chi]$ = 1 W/(m·K), and specific heat of fusion $[\lambda]$ = 1 J/kg.

Task 3. Using the data provided above, derive the "law of freezing ponds" – a formula that connects the thickness of ice with time.

Task 4. Using the received law, define how long it will take for a pond to freeze completely to the bottom if its depth is 2 meters, starting from the onset of frost?

***Version 7*. *Water Leakage*.** An installation, which is a cylindrical vessel with a cross-sectional area of 84.5 cm^2, was made in a laboratory. The vessel has a small hole with an area of 7.1 mm^2 on its side wall (Fig. 7.11). In the experiment, it was measured how the flowing liquid level h (measured from the hole level) depends on the time t. The measurement results are given in a table. Your task is to extract a law from these data.



#	t, s	h, cm
1	0	15
2	36	9
3	102	4
4	130	2
5	153	1
6	204	0

Figure 7.11 — The experiment with a liquid

Task 1. Using the given table, make a "correct" table that relates the outflow time t_f to the initial liquid level height h_0. Test the hypothesis that "the outflow time is directly proportional to the initial level height."

Task 2. Obtain by dimensional analysis the main part of the dependence of the outflow time on the initial level height. Propose the straightening coordinates for the dependence $t_f(h_0)$. Define the parameters of the straightened dependence. Note that, in this case, there is an a priori zero $t_f(0) = 0$: non-existent water will flow out instantly.

Hint: the time of liquid leakage may depend on the density of the liquid $[\rho]$ = 1 kg/m^3, free fall acceleration $[g]$ = 1 m/s^2, and the initial height of the level $[h_0]$ = 1 m.

Task 3. Obtain from the above data the "law of water leakage" – a formula that connects the time of water leakage from the laboratory vessel with the initial height of the liquid level.

Task 4. A company specializing in the storage of liquids asked the physics laboratory to find out how long it will take for all the liquid to leak out of a cylindrical tank with a height of 10 meters and a cross-sectional area of 5 m^2 if a crack with an area of 2 cm^2 is formed at the bottom of the tank. Provide an answer to this inquiry.

Hint: to answer this question, use the fact that the coefficient of the law you received contains the ratio S/s of the cross-sectional areas of the vessel S and the hole s in the first power (justify!).

4. Mathematical Models

Version 8. Problem of Radioactive Waste Storage. It is proposed to use concrete spherical containers with an inner cavity to store spent radioactive fuel. The table (Fig. 7.12) shows the values of the attenuation coefficient of radiation intensity $K(\ell) = I_0/I(\ell)$ depending on the thickness of the concrete protection ℓ (in the table the technical term *multiplicity of reduction of radiation intensity* is used). Your task is to extract the law from this data.

Task 1. Construct the "correct" table (ℓ, i), where $i = I/I_0$ is the relative intensity, and test the hypothesis that "the radiation intensity decreases linearly with increasing thickness of protection." Note that in this case, there is an a priori point $i(0) = 1$: no protection – no reduction of radiation.

#	ℓ, cm	multiplicity
1	1.9	1
2	5.2	10
3	9.9	100
4	11.7	2 000
5	18.5	10 000
6	21.7	50 000

Fig. 7.12 — Creative method of radioactive waste storage

Task 2. Suggest the straightening coordinates for the dependence $i(\ell)$ and find the parameters of the r straightened dependence.

Hint: In this case, dimensional analysis will not work. Try to build a mathematical model of the phenomenon based on the differential equation $dI = -\gamma I d\ell$. This equation fixes the following simple fact: the number of gamma quanta absorbed by an infinitely thin layer is directly proportional to both the intensity of the flow I and the thickness of the layer $d\ell$.

Task 3. Obtain from the above data the "law of protection against radioactivity" – a formula that relates the intensity of radiation to the thickness of the concrete protection.

Task 4. Using the obtained law, find the wall thickness that guarantees a reduction of intensity in a billion times.

Version 9*. *Protons' Free Path. Why do charged particles slow down as they pass through a substance? Someone can hurry with the answer: "Because they collide with atoms of matter." This is a wrong answer, as the dense part of atoms – the nucleus – is so small that direct collisions between particles and nuclei occur very rarely. Instead, a charged particle in matter slows down for another reason: as it passes near an atom, it "shakes" its electrons, and consumes energy. The table (Fig. 7.13) shows the path lengths of protons in air, depending on their initial kinetic energy. Your task is to extract the law from this data.

#	E, MeV	ℓ, m
1	25	4.9
2	50	20.0
3	75	41.3
4	100	69.1
5	125	108.6
6	150	140.9

Figure 7.13 — Everything that moves is slowed down!

Task 1. Test the hypothesis that "the path length is directly proportional to the initial kinetic energy of the particle." This hypothesis is valid if the friction force is constant.

Task 2. Suggest the straightening coordinates for the dependence $\ell(E)$ and find the parameters of the straightened dependence. Note that in this case there is an a priori zero $\ell(0) = 0$: a particle that does not move does not pass the path.

Hint: In this case, dimensional analysis will not work. Instead, try constructing a mathematical model of the phenomenon based on the differential equation: $dE = -\gamma\, d\ell/E$. This equation fixes the following fact: the energy lost by a particle when an atom is shaken is inversely proportional to the square of its speed.

Task 3. Obtain from the given data the "law of protons' free path" – a formula that relates the path length to the particle energy.

Task 4. Using the obtained law, find the energy at which protons from the Sun can pass through Earth's atmosphere. Consider the effective atmospheric thickness is 8 km.

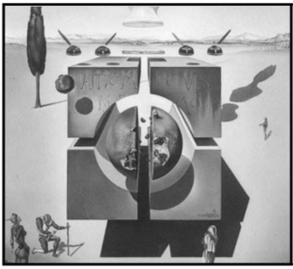
5. General Case of Straightening Coordinates

Version 10*. *Hydrogen Atom. Until 1885, physicists measured all the characteristics radiation lines of the hydrogen atom (Fig. 7.14), but they did not see a law in the data. It was Johann Ballmer, a school mathematics teacher, who first recognized a law in the accumulated data:

$$f = \frac{E_i}{h}\left(\frac{1}{n^2} - \frac{1}{m^2}\right),$$

where n is an integer, and $m = n + 1$; $n + 2$; …, E_i is the ionization energy of the hydrogen atom, $h = 6.626 \cdot 10^{-34}$ J·s is Planck's constant. The series of lines shown in the table is now called the Ballmer series.

Using data from the table, prove that they fit this law and "extract" the ionization energy from the data E_i of the hydrogen atom. The challenge lies in the fact that the hydrogen atom "did not tell us" what is the value of n in this case, i.e. which energy level these transitions correspond to.



#	line	f, 10^{14} s^{-1}
1	α-*line*	4.568
2	β-*line*	6.167
3	γ-*line*	6.906
4	δ-*line*	7.308
5	ε-*line*	7.551
6	ζ-*line*	7.709

Figure 7.14 — J. Balmer's idea is the beginning of splitting an atom

Task 1. Propose the straightening coordinates for the dependence $f(m)$ and test the hypothesis that "the Balmer series corresponds to the transitions of the electron to the ground level $n = 1$." Note that for the dependence $f(m)$ there is an a priori point $f(n) = 0$: no transition – no radiation.

Hint: To facilitate analysis, compile a table in which each line name corresponds to a specific value of m.

Task 2. Test the validity of the hypothesis that "the Balmer series corresponds to the electron transitions to the first excited level $n = 2$." Determine the parameters of the straightened dependence.

Task 3. Obtain from your data the values of the ionization energy E_i of the hydrogen atom.

Task 4. Provide an expert opinion for experimenters preparing equipment to study the Lyman series: specify the limits of the frequency range for this series. Lyman series is a series of lines corresponding to the transitions of the electron to the ground level $n = 1$.

Version 11*. *Viscosity of Glass. To create glass masterpieces, glassblowers must have a deep understanding of how molten glass behaves. And the behavior of molten glass is determined by its viscosity.

#	T, K	η, 10^6 Pa·s
1	500	1 120 000
2	600	1 250
3	700	3.90
4	800	0.160
5	1000	0.00034
6	1300	0.000021

Figure 7.15 — Molten glass behavior

The table (Fig. 7.15) provides experimental data showing how the viscosity of glass used to make TV screens depends on absolute temperature. Your task is to extract a law from this data.

Task 1. Test the hypothesis that "the viscosity of glass decreases linearly with increasing temperature."

Task 2. Suggest the straightening coordinates for the dependence $\eta(T)$ and determine the parameters of the straightened dependence.

Hint: The structure of glass is so complex that neither the theory of dimensions nor differential equations will help. There is only one reason left: given that this case is about a thermodynamic phenomenon, it would be reasonable to try a thermodynamic exponent.

Task 3. Obtain from the above data the "law of viscosity of glass": the formula that connects viscosity of glass with absolute temperature.

Task 4. Calculate the viscosity of glass at temperature T = 2000 K?

Requirements for an Answer

1) In the restoration of power dependencies, the main principle is "powers in physics dependences are integers or prime fractions", therefore in these cases the answer should be a number, not an interval (except in Version 3).

2) In all other cases, answers should be given in the form of a standard confidence interval ($t_n^\alpha = 1$).

3) To construct confidence intervals for directly measured quantities (parameters of linear dependences), it is given a priori that the fractional uncertainty of these measurement quantities is equal to 1%.

4) Confidence intervals for indirectly measured values should be constructed in the standard way.

LABORATORY MANUAL

This manual provides students with guidelines to help them perform laboratory works. We have provided here only the most essential elements: descriptions of educational objectives, sequences of main actions, and worksheets. In our opinion, these elements most adequately reflect the structure of experimental research activities. The following Table LM.1 gives a general overview of the proposed laboratory works. Here we used two new logical bases for classifying laboratory works: *the research purpose of the laboratory work* and *the role of the physical law in it*.

Table LM.1 — Classification of Laboratory Works

	Type of a Laboratory Work	**A**	**B**	**C**
1	a posteriori	3, 9, 12	7, 11	10
2	a priori	1, 2, 4, 5, 6, 8	—	—

Based on research purposes, we categorize laboratory works into three classes. Class **A** consists of laboratory works aimed at determining the value of a physical characteristic. The answer in these cases is given as a confidence interval. Class **B** consists of laboratory works aimed at determining the exponent of power dependence (formulating and confirming a hypothesis of the first kind). The answer is a number. Class **C** consists of laboratory works aimed at confirming or refuting a hypothesis of the second kind. The answer is a statement: either “hypothesis is accepted” or “hypothesis is rejected”.

Based on the role of the physical law, we classify laboratory works into two classes. *A posteriori laboratory works* involve cases where the type of physical law is unknown before conducting measurements. It is determined only after processing the measurement results. Thus, the purpose of a posteriori laboratory work is to obtain a physical law. *A priori laboratory works* involve cases where the physical law is already known before conducting measurements. Here, the law serves as a tool for gaining other knowledge, such as determining the values of physical quantities.

Remark. During the course creation, our laboratory works happened to be divided into even- and odd-numbered ones. In the odd-numbered laboratory works (except the first), the material for processing is adapted data from key physical experiments. In contrast, in the even-numbered works, we use the data from our own measurements.

Laboratory Work 1 “Buffon – de Morgan Experiment”

Purpose: To get acquainted with the procedure of constructing a confidence interval.

Equipment: 21 coins, plastic cup.

Brief Theoretical Background

It is well known that the probability of obtaining “heads” when tossing a coin is equal to 0.5. In this laboratory work, you will experimentally verify this fact.

Description of the Experimental Setup

You will use 21 coins and a plastic cup to mix them before each trail.

Measurements and Calculations

Put the coins in the plastic cup, shake them well, and pour them onto a table. Count the number of coins that land on “heads” and record this number in Table LM.2. Calculate the probability of getting “heads” using the formula: $x = \frac{N_{\text{head}}}{N}$, where N is the total number of coins. Repeat this experiment two more times.

Table LM.2 — Data and Data Processing

Test number, i	Total number of coins, N	Number of “heads”, N_{head}	Probability of getting “heads” in the i-th experiment, x_i	Deviation of x_i from the mean, $\Delta x_i = x_i - \bar{x}$	Auxiliary column, Δx_i^2
i = 1					
i = 2					
i = 3					
The mean $\bar{x} = \frac{1}{n} \cdot \sum_{i=1}^{n} x_i$				$\bar{x} =$ ____________	
Half-width of the confidence interval (the absolute uncertainty) $\Delta x = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2}$					$\Delta x =$ _________
“Raw” result of the laboratory work					_____ ± _____
The final result of the laboratory work					_____ ± _____
Relative uncertainty of the measurement $\varepsilon = \frac{\Delta x}{\bar{x}}$					$\varepsilon =$ _____ or _____%

2. Fill in Table LM.2 in the following order:

2.1. Calculate the mean value of the confidence interval $\bar{x}$ according to the formula given in the table.

2.2. Determine the random deviation of the result for the *i*-th value using the formula $\Delta x_i = x_i - \bar{x}$ and enter this number in the table.

2.3. Fill in the last (auxiliary) column.

2.4. Calculate the value of the absolute measurement uncertainty Δx.

2.5. Record the "raw" result of the laboratory work.

2.6. Round the result.

2.7. Calculate the relative measurement uncertainty ε.

3. Write your conclusion for the laboratory work in the following form:

In this laboratory work, we measured _________. It was found that the probability of getting "heads" is equal to ______±______. The relative measurement uncertainty is equal to _____%.

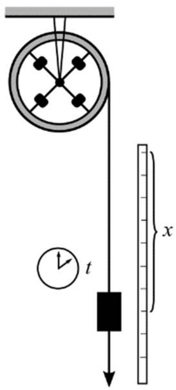
Laboratory Work 2 “Measurement of Time”

Purpose: 1) to build a confidence interval; 2) to get acquainted with the graphical method of experimental data processing; 3) to verify the consistency between geometric and analytical methods.

Equipment: Oberbeck pendulum, electronic stopwatch.

Brief Theoretical Information

In this laboratory work, the main source of uncertainties is the observer-stopwatch measuring system. In the theoretical part of the session, we assume a priori that the absolute uncertainty of a single measurement with this system is equal to $\delta_{\text{theor}} = 0.1$ s. Your task is to obtain the value of this quantity from the experimental data using both analytical and geometrical methods and draw a conclusion about the validity of our theoretical assumption. In addition, you will complete several related tasks.



LM.1 — Oberbeck pendulum

Description of the Experimental Setup

We will measure the time during which a load of the Oberbeck pendulum goes down. The Oberbeck pendulum consists of a hoop fixed on the horizontal axis with a spool of thread rigidly attached to it. A load is tied to the thread (Fig. LM.1). The system is held at rest by a mechanical brake. When released, the load descends, causing the hoop to rotate. The load is uniformly accelerated in this experiment.

Measurements and Calculations

1. Obtain the specified load height from your instructor for your measurements.

2. Carry out six measurements of the time it takes an object to fall from the given height. Record the results in Table LM.3.

3. Complete the table and obtain the confidence interval for the time.

4. Geometrical Method.

4.1. Draw horizontal and vertical axes on graph paper. You should plot the experiment number i along the horizontal axis andthe time t along the vertical axis. Plot the experimental points as vertical segments. Adjust the vertical axis scale and set the origin so that a segment representing the total instrumental uncertaintyspans two large ticks.

4.2. Draw the best-fit line. Does it pass through all the experimental segments? What does it mean?

Table LM.3 — Data and Data Processing

<table>
<tr><td colspan="2">Setup number _____</td><td colspan="2">Load height h =____ m</td></tr>
<tr><td>Test number, i</td><td>Time it takes to fall over the given height in the i-th experiment, t_i, s</td><td>Deviation from the mean, $\Delta t_i = t_i - \bar{t}$, s</td><td>Auxiliary column, Δt_i^2, s^2</td></tr>
<tr><td>$i = 1$</td><td></td><td></td><td></td></tr>
<tr><td>…</td><td></td><td></td><td></td></tr>
<tr><td>$i = 6$</td><td></td><td></td><td></td></tr>
<tr><td colspan="3">The mean $\bar{t} = \frac{1}{n} \cdot \sum_{i=1}^{n} t_i$</td><td>$\bar{t}$ =________</td></tr>
<tr><td colspan="3">Absolute uncertainty of a singular measurement $$\delta t = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} \Delta t_i^{\,2}}$$</td><td>δt =_______</td></tr>
<tr><td colspan="3">Absolute uncertainty of the mean $\Delta t = \frac{\delta t}{\sqrt{n}}$</td><td>$\Delta t$ =_______</td></tr>
<tr><td colspan="3">“Raw” result of the laboratory work</td><td>_________ ± ________</td></tr>
<tr><td colspan="3">The final result of the laboratory work</td><td>_________ ± ________</td></tr>
<tr><td colspan="3">Relative uncertainty of the measurement $\varepsilon = \frac{\Delta t}{\bar{t}}$</td><td>$\varepsilon$ =_____ or ___%</td></tr>
</table>

4.3. Determine the mean time $\bar{t}$ graphically. Enter in Table LM.4 the values of $\bar{t}$ obtained by both the analytical and geometrical methods.

Table LM.4 — Comparison of Analytical and Geometrical Methods

<table>
<tr><td></td><td>Measured quantity</td><td>Analytical method</td><td>Geometrical method</td><td>A priori value</td><td>Consistency</td></tr>
<tr><td>1</td><td>The mean $\bar{t}$</td><td></td><td></td><td>—</td><td rowspan="2">Yes/No</td></tr>
<tr><td>2</td><td>Absolute uncertainty of a singular measurement δt</td><td></td><td></td><td></td></tr>
</table>

4.4. Determine the absolute uncertainty of a singular measurement δt using the geometrical method. Enter the values of the absolute uncertainty of a singular measurement obtained by both the analytical and geometrical methods in Table LM.4. Also, enter the a priori value of δ_{theor}.

4.5. Can we conclude that the results are consistent?

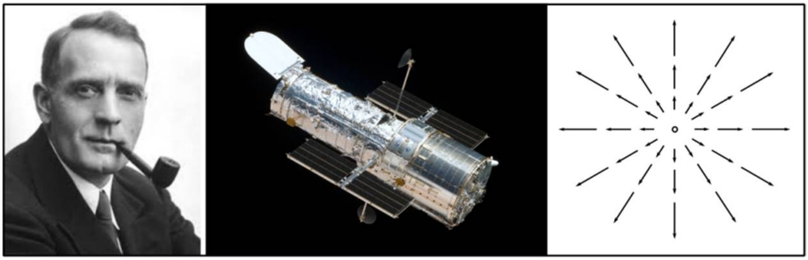
5. Formulate a conclusion for the laboratory work.

Laboratory Work 3 "Measurement of the Age of the Universe"

Purpose: 1) learn how to build an experimental line and determine its slope; 2) calculate a confidence interval for the lifetime of the Universe.

Equipment: E. Hubble's data table, graph paper.

Brief Theoretical Information



a) b) c)

a — E. Hubble, the person; b — the Space Telescope; c — scattering of galaxies

Figure LM.2 — Hubble, the person, and the Space Telescope, and the idea of scattering of galaxies

In 1929, the entire scientific world was shocked by the message of an American astronomer Edwin Hubble (Fig. LM.2): Our universe is not stationary, it is expanding! Galaxies are scattering from us in all directions, and the farther away the galaxy is, the greater is the speed of scattering. E. Hubble offered a law that describes the phenomenon he discovered: $\upsilon = H \cdot R$. Why is this lawso important? Knowing this law, you can find the lifetime of the Universe: $T = R/\upsilon = 1/H$.

In this laboratory work, you will process E. Hubble's data to determine the lifetime of the Universe according to Hubble.

Description of the Experimental Setup

The two leftmost columns in Table LM.5 contain the data obtained by E. Hubble. The first column represents the distances R from the Earth to galaxies (in parsecs), the second column lists velocities of their scattering υ (in km/s).

Measurements and Calculations

1. Convert E. Hubble's astronomical data into metric: express the distance to galaxies in meters (1 parsec = $3.1 \cdot 10^{16}$ m) and velocities of scattering in m/s (fill in the two rightmost columns in Table LM.5).

2. Draw axes (R, υ) on a sheet of graph paper and choose the scale for each axis. The axis should, firstly, be comfortable to work with, and secondly, ensure that all experimental points fit within the graph's range. Label and number both axes clearly.

Table LM.5 — Hubble's Data

i	E. Hubble's data		E. Hubble's data (in SI)	
	R, 10^6 pc	v, 100 km/s	R, 10^{23}m	v, 10^5 m/s
1	0.5	2.5		
2	5.0	29.0		
3	6.5	31.0		
4	9.1	51.0		
5	11.0	56.5		
6	14.2	75.0		
7	17.3	81.5		
8	20.0	89.5		

3. Plot the experimental points on the graph. You do not need to label their coordinates!

4. Set the a priori, the most important point $v(0) = 0$.

5. Draw the best-fit line.

6. Choose two points on the experimental line and record their coordinates in Table LM.6.

Table LM.6 — Data Processing

Points	R_i, 10^{23}m	v_i, 10^5 m/s	ΔR, 10^{23}m	Δv, 10^5 m/s	$\bar{H} = \frac{\Delta v}{\Delta R}$, 1/s
1					
2					
Age of the Universe $\bar{T} = \frac{1}{\bar{H}}$					$\bar{T} =$ ____ $\cdot 10^{[\dots]}$ s
Relative uncertainty of the measurement $\varepsilon_T = \frac{\Delta T}{\bar{T}}$					$\varepsilon_T = 3\%$
Half-width of the confidence interval $\Delta T = \varepsilon_T \cdot \bar{T}$					$\Delta T =$ ____ $\cdot 10^{[\dots]}$ s
"Raw" result of the laboratory work					$T = (\_ \pm \_) \cdot 10^{[\dots]}$ s
The final result of the laboratory work					$T = (\_ \pm \_) \cdot 10^{[\dots]}$ s

7. Complete Table LM.6. Since you are working with the linear relationship for the first time, we provide the value of relative uncertainty for the age of the Universe: $\varepsilon_T = 3\%$.

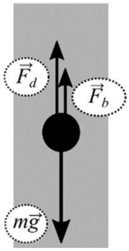
8. Formulate a conclusion for the laboratory work.

Laboratory Work 4 "Measurement of the Liquid Viscosity by Stokes Method"

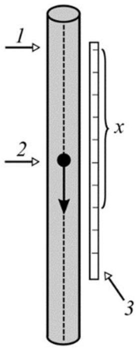
Purpose:1) draw a best-fit line and measure its slope; 2) construct a confidence interval for the coefficient of dynamic viscosity.

Equipment: Stokes device, small balls, micrometer, ruler, video camera.

Brief Theoretical Information



LM.3 — The forces



LM.4 — The setup

The determination of a liquid's dynamic viscosity coefficient using Stokes' method is based on the study of the free fall of a ball in the investigated liquid. The ball falling in the liquid gains such speed that the viscous force (drag force) $F_d = 3\pi\eta dv$ (Stokes' law) together with the buoyancy (Archimedes' law) $F_b = \rho_{\text{liq}} g V_{\text{ball}}$ begins to balance the gravity force. After that, the motion of a ball became uniform (Fig. LM.3). The equilibrium condition $mg = F_d + F_b$ allows to determine the coefficient η of dynamic viscosity of the fluid by measuring the velocity v of uniform motion of the ball. If we take into account that $m = \rho_{\text{ball}} V_{\text{ball}}$ and $V_{\text{ball}} = \pi d^3/6$ then for the coefficient of dynamic viscosity we obtain

$$\eta = \frac{(\rho_{\text{ball}} - \rho_{\text{liq}}) g d^2}{v}. \qquad \text{(LM.1)}$$

Here, d is the diameter of the ball, v is the uniform motion velocity, g is the free fall acceleration, ρ_{ball} is the density of the ball's material, ρ_{liq} is the density of the liquid.

2) The law of uniform motion has the form $x = v \cdot t$. Therefore, by measuring the slope of the experimental line, you can calculate the coefficient of dynamic viscosity of the liquid using formula (LM.1).

Description of the Experimental Setup

The experimental setup consists of a tall cylindrical vessel filled with liquid (1) (Fig. LM.4). The experimenter places a ball (2) in the upper part of the vessel and observes its fall. To determine the coordinate of the ball, a ruler (3) is attached to the cylinder. A smartphone video camera is used to record the ball motion. A slow-motion recording mode is recommended.

Measurements and Calculations

1. Determine, together with your instructor, the slow-motion factor of your video camera. Write down the value in the rightmost column of Table LM.7.

2. Measure the diameter of the ball and enter its value in the table.

3. Record a video of the ball falling in the liquid.

4. Select several time instances from the video and determine the coordinates of the ball at these moments. Fill in the first two columns of Table LM.7. These are your "raw" experimental data.

5. Data analysis will be more convenient if we redefine them a bit. Choose for the zero event the first experimental point. Fill the 3rd and 4th columns of Table LM.7. When recalculating, take into account the slowing down factor of the camera.

Table LM.7 — Data

i	t_{exp}, s	x_{exp}, s	t, s	x, 10^{-2} m	*Important data*
1			0.00	0.0	slowing down factor ___
2					diameter of the ball ______ m
…					density of the ball material 11400 kg/m^3
7					density of the liquid 960 kg/m^3

6. Plot the relationship $x(t)$ on graph paper. When drawing the best-fit line, keep in mind that you have an assigned zero point $(0{,}0)$. Did you get the proportional relationship between t and x?

7. Choose two convenient points on the experimental line and record their coordinates in Table LM.8.

Table LM.8 — Data Processing

<table>
<tr><td>Points</td><td>t_i, s</td><td>x_i, m</td><td>Δt, s</td><td>Δx, m</td><td>$k \equiv \bar{v} = \Delta x / \Delta t$, m/s</td></tr>
<tr><td>1</td><td></td><td></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>2</td><td></td><td></td></tr>
<tr><td colspan="5">The mean (see Equation (LM.1))</td><td>$\bar{\eta}$ =________ Pa·s</td></tr>
<tr><td colspan="5">Relative uncertainty $\varepsilon_\eta = \frac{\Delta\eta}{\bar{\eta}}$</td><td>$\varepsilon_\eta = 5\%$</td></tr>
<tr><td colspan="5">Half-width of the confidence interval $\Delta\eta = \varepsilon_\eta \bar{\eta}$</td><td>$\Delta\eta$ =________ Pa·s</td></tr>
<tr><td colspan="5">"Raw" result of the laboratory work</td><td>_____±_____</td></tr>
<tr><td colspan="5">The final result of the laboratory work</td><td>_____±_____</td></tr>
</table>

8. Fill in Table LM.8. Note that since this is your first experience working with a linear relationship without instructor guidance, a value of the relative uncertainty for ε_η is provided.

9. Formulate a conclusion for the laboratory work.

Laboratory Work 5 "Hook's Law. Measurement of the Mass of a 'Black' Load"

Purpose: 1) to master the first part of experimental data graphical processing; 2) to determine the mass of a "black" load.

Equipment: metal wire in a casing, indicator, platform, set of loads.

Brief Theoretical Information

Bodies elongation as a result of applied force action follows the law discovered by Robert Hooke: $F = k_s\Delta\ell$. Here $k_s = ES/\ell_0$ is the wire stiffness, E is the Young's modulus of the wire material, ℓ_0 is the initial length of the wire, $S = \pi d^2/4$ is the wire cross-sectional area, d is the wire diameter. Since we are going to change the tension of the wire (in our case $F = m_{\text{total}}g$) and measure the elongation of the wire, it will be convenient to present Hooke's law in the form he wrote it once:

$$\Delta\ell = \frac{4g\ell_0}{\pi d^2 E}(m + m_0),$$

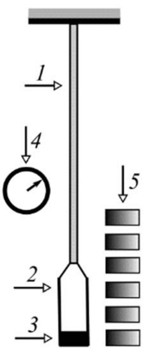
where m is the mass of the loads you put, m_0 is the mass of the unknown "black" load.

2) The experimental relationship we expect is a linear relationship with an offset argument $y = k(x + x_0)$. Having built the best-fit line, we will be able to determine the mass of the unknown load by measuring the horizontal shift and the Young's modulus of the wire material by measuring the slope of the line.

Description of the Experimental Setup

The experimental setup is schematically shown in Fig. LM.5. It consists of: 1) a metal wire (in a case); 2) a platform for placing the loads; 3) a load of unknown mass; 4) an indicator that measures the elongation of the wire (at initial position it shows the elongation of the wire under the "black" load action); 5) a set of six loads of known mass.

During the work, you will consistently add loads on the platform, increasing the total mass that stretches the wire, and measure the elongation of the wire.



LM.5 — The setup

Measurements and Calculations

1. Fill in the table with the initial elongation of the wire (elongation of the wire under the "black" load action).

2. Place the first load on the platform. Record its mass and the wire elongation in Table LM.9.

3. Carry out several measurements. Continue adding loads systematically and record all data in the second column of Table LM.9.

Table LM.9 — Data

i	Mass of a load, Δm, kg	Mass of all loads, m, kg	Wire elongation, $\Delta\ell$, 10^{-5}m	*Important data*
0	0	0		$g = 9.81$ m/s^2
1				$\ell_0 = 1100$ mm
…				$d = 0.6$ mm
6				$\pi = 3.14$

4. Plot a graph of the relationship $\Delta\ell(m)$ on a graded paper based on *seven* points. Did you get a linear relationship?

5. Extent the experimental line to the intersection with the horizontal axis. Measure the mean mass of the unknown load from the graph. Enter its value in Table LM.10.

Table LM.10 — Data Processing

$\bar{m}_0$, kg	$2\delta m_0$, kg	δm_0, kg	$\Delta m_0 = \frac{\delta m_0}{\sqrt{n}}$, kg	$\varepsilon_{m_0} = \frac{\Delta m_0}{\bar{m}_0}$
“Raw” result of the laboratory work $\bar{m}_0 \pm \Delta\bar{m}_0$				____±___
The final result of the laboratory work				____±___
Relative uncertainty of the method				____%

6. By performing a parallel displacement of the experimental line, find the width of the segment cut by the line on the horizontal axis. Enter its value in the table.

7. Calculate the following: the standard deviation of a single mass measurement, the half-width of the confidence interval for the mass (absolute uncertainty of the mean = absolute uncertainty of the measurement series), and the relative uncertainty of the method.

8. Formulate a conclusion for the laboratory work.

Laboratory Work 6 “Hook’s Law. Measurement of the Young’s Modulus”

Purpose: 1) to master the complete procedure for processing experimental data; 2) to learn how to determine the parameters of a confidence interval for the slope using a graphical method; 3) to construct a confidence interval for the Young’s modulus of the wire material.

Equipment: experimental data obtained in the previous laboratory work.

Brief Theoretical Information

In this laboratory work, we get the value of the Young’s modulus of the wire material from the experimental data obtained in the last session. We have investigated the dependence of wire elongation $\Delta\ell$ on the mass of loads m described by law $\Delta\ell = \frac{4g\ell_0}{\pi d^2 E}(m + m_0)$. The Young’s modulus E is related to the slope k of the corresponding linear relationship by the equation $E = \frac{4g\ell_0}{\pi d^2}\frac{1}{k}$. Therefore, by constructing a confidence interval for the slope$k$, we will be able to construct a confidence interval for the Young’s modulus using the rules for constructing a confidence interval for indirectly measured quantity (*Great Diagram*).

Description of the Data

The experimental plot obtained in the previous session is used.

Measurements and Calculations

Choose two convenient points on the previously constructed experimental line and record their coordinates in Table LM.11. Calculate the mean value of the slope.

Table LM.11 — The Mean of the Slope

Points	m_i, kg	$\Delta\ell_i$, m	Δm, kg	$\Delta(\Delta\ell)$, 10^{-5}m	$\bar{k} = \frac{\Delta(\Delta\ell)}{\Delta m}$, 10^{-5}m/kg
1					
2					

2. Swing the experimental line and enter in Table LM.12 a value of the “base” δm and a value of the vertical displacement $2\delta(\Delta\ell)$.

3. Find the absolute uncertainty of a single measurement $\delta k = \delta(\Delta\ell)/\delta m$ and the absolute uncertainty of the whole series of measurements $\Delta k = \delta k/\sqrt{n}$. Calculate the relative measurement uncertainty of the slope $\varepsilon_k = \Delta k/\bar{k}$.

Table LM.12 — Absolute Uncertainty of the Slope

δm, kg	$2\delta(\Delta\ell)$, 10^{-5} m	$\delta(\Delta\ell)$, 10^{-5} m	δk, 10^{-5} m/kg	Δk, 10^{-5} m/kg	$\varepsilon_k = \frac{\Delta k}{\bar{k}}$

4. Let's move on to filling the outer circle. Table 3 will help you organize your actions.

Table LM.13 — Filling the Outer Circle

E, 10^{11} Pa	ε_E	ΔE, 10^{11} Pa
"Raw" result of the laboratory work		$E =$_____$\pm$_____$\cdot 10^{[\dots]}$Pa
Final result of the laboratory work		$E =$_____$\pm$_____$\cdot 10^{[\dots]}$Pa

5. Formulate a conclusion for the laboratory work.

Laboratory Work 7 "Stefan – Boltzmann Law"

Purpose: 1) to get acquainted with logarithmic coordinates; 2) to formulate a simple physical hypothesis and perform its probabilistic estimation.

Equipment: a table with data obtained by P. Dulong and A. Petit, J. Stefan worked with.

Brief Theoretical Information

Is it easy to emit electromagnetic waves? Absolutely! There is nothing easier! Try swimming in a pond without causing waves – it is impossible. Waves will spread outwards in all directions. Similarly, try walking quietly in the woods, and it won't work either, something will definitely creak (producing a sound wave). In the same way, any motion of electric charges generates electromagnetic waves in space. Naturally, the more intense the charges motion is, the higher the temperature, the more intense the radiation they emit. Many scientists have tried to discover the law of thermal radiation. Sir Isaac Newton was only able to establish that the dependence was nonlinear. French scientists Pierre Dulong and Alexis Petit, as well as British scientist John Tyndall, obtained reliable experimental data, but were unable to turn them into a physical law. Josef Stefan, a professor at the University of Vienna, derived the empirical law of thermal radiation in 1879. In 1884, his student Ludwig Boltzmann confirmed the result of his teacher, publishing a theoretical derivation of the same law. This is why the law of thermal radiation is called the Stefan-Boltzmann law.

In this laboratory work, you will retrace J. Stefan's approach and restore the law of thermal radiation, determining how the radiating capacity of a black body depends on absolute temperature.

Definition: the radiating capacity R is the amount of energy emitted per unit time per unit surface area.

2) Laboratory work aimed at restoring exponents in physical laws (not the values of constants!) are special laboratory works. The fact is that we know in advance that the exponents in physical laws n are small integers or prime fractions. Therefore, in such works the confidence interval has only auxiliary value, and the primary focus is on its components: *the mean* $\bar{n}$ and *the standard deviation of the mean* Δn.

The measured mean $\bar{n}$ allows us to put forward several hypotheses such as "the exponent equals three" or "the exponent equals four."

The standard deviation Δn allows us to determine the probability of each of the hypotheses validity. In other words, the probability that corresponding random deviation from the true value has occurred in these series of measurements. As a rule, the probabilities corresponding to different hypotheses are different from each other in such a way that there is no doubt in the choice of the final answer.

Description of Data

You are provided with experimental data obtained by P. Dulong and A. Petit (left part of Table LM.14).

Measurements and Calculations

1. Plot the first graph in coordinates (T, R). Make sure that the experimental points *do not lie* on the straight line. This means that the relationship is nonlinear.

2. Assume the working assumption that the required relationship is a power relationship $R = \sigma T^n$. Then in logarithmic coordinates $X = \ln T$ and $Y = \ln R$ it becomes a linear relationship $Y = kX + b$, where $k = n$ and $b = \ln\sigma$. To draw the corresponding graph, calculate the values of logarithmic coordinates for the experimental points and fill in the third and fourth columns of the provided table.

Table LM.14 — Data

i	T, K	R, 10^4 W/m^2	$X = \ln T$	$Y = \ln R$
1	813	2.40		
2	927	4.19		
3	1055	7.02		
4	1183	11.10		
5	1311	16.75		
6	1439	24.31		
7	1502	28.86		
8	1566	34.67		

3. Draw the best-fit line on the $(X = \ln T, Y = \ln R)$-plane. Determine the mean value of the slope $\overline{k}$ using the geometrical method.

4. Using the line-swinging method, determine the standard deviation of the mean Δk. To do this, enter in Table LM.15: the “base” of swings δX and the value of the vertical displacement $2\delta Y$. Calculate the standard deviation of the slope mean using these values (Table LM.15).

Table LM.15 — Data Processing

<table>
<tr><td>Points</td><td>X_i</td><td>Y_i</td><td>ΔX</td><td>ΔY</td><td>$\bar{n} = \bar{k} = \Delta Y / \Delta X$</td></tr>
<tr><td>1</td><td></td><td></td><td rowspan="2"></td><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td>2</td><td></td><td></td></tr>
<tr><td>δX</td><td colspan="2">$2\delta Y$</td><td>δY</td><td>$\delta k = \delta Y / \delta X$</td><td>$\Delta n = \Delta k = \delta k / \sqrt{N}$</td></tr>
<tr><td></td><td colspan="2"></td><td></td><td></td><td></td></tr>
<tr><td colspan="5">“Raw” confidence interval</td><td>$n =$ ___ ± ___</td></tr>
<tr><td colspan="5">The final confidence interval</td><td>$n =$ ___ ± ___</td></tr>
</table>

5. Record the confidence interval for the exponent n.

6. Formulation of simple physical hypotheses and their probabilistic estimation.

6.1. Based on the obtained value of $\bar{n}$, formulate three simple hypotheses (for example, $n = 3, \ldots$) and enter them in Table LM.16.

Table LM.16 — Statistical Testing of Hypothesis

<table>
<tr><td colspan="2">Hypothesis</td><td>$n_1 = 3$</td><td>$n_2 = 4$</td><td>$n_3 = 5$</td></tr>
<tr><td rowspan="2">Deviation of the mean from hypothetical value $|\bar{n} - n_i|$</td><td>in absolute value</td><td></td><td></td><td></td></tr>
<tr><td>in Δn units</td><td></td><td></td><td></td></tr>
<tr><td colspan="2">Probability of deviation</td><td></td><td></td><td></td></tr>
<tr><td colspan="2">The final answer</td><td colspan="3">Exponent is equal to ____</td></tr>
</table>

6.2. Calculate the deviation of $\bar{n}$ from the corresponding hypothetical value: a) in absolute value; b) in Δn units.

6.3. Assuming that the measured value $\bar{n}$ is a realization of a random variable distributed according to the normal law, calculate the probability of the corresponding fluctuation (use tables or mathematical software).

6.4. Choose the most likely hypothesis and record the final answer in Table LM.16.

7. Formulate a conclusion for the laboratory work. In conclusion, formulate the Stefan – Boltzmann law of thermal radiation.

Laboratory Work 8 “Simple Pendulum”

Purpose: 1) to get acquainted with the use of power rectifying coordinates; 2) to construct a confidence interval for free fall acceleration.

Equipment: simple pendulum, stopwatch, tape-measure.

Brief Theoretical Information

The period of oscillations of a simple pendulum T is related to the length of the thread ℓ and the free fall acceleration g by the formula: $T = 2\pi\sqrt{\ell/g}$. If we choose the power straightening coordinates $X = T^2/4\pi^2$ and $Y = \ell$ for processing the measurement results, then the corresponding relationship will have the form $Y = gX$. It means that for such coordinates the equality $g = k$ holds. Therefore, if we construct a confidence interval for the slope k, it will be the confidence interval for the free fall acceleration $g$as well.

Description of the Experimental Setup

In this laboratory work you are offered to measure the oscillation period of a small ball suspended on a thread for different thread lengths. In order to reduce the measurement uncertainty of the period, you are asked to measure the time of 10 oscillations.

Measurements and Calculations

1. Select an initial value of length thread. Deflect the pendulum at a small angle and measure the time of 10 oscillations. Record the results in Table LM.17.

2. Repeat the measurement for all other thread lengths.

3. Fill in the fourth column of Table LM.17. Using these data, draw an experimental graph in coordinates ($x = \ell$, $y = T$). Did you get a linear relationship?

4. Fill in the last two columns of Table LM.18 (calculate the values in power coordinates) and draw an experimental graph in coordinates (X, Y). Note that in this case we have an a priori zero, the experimental line has to pass through the origin. Did you get a linear relationship?

Table LM.17 — Data

i	Length, ℓ, m	Time of 10 oscillations, t, s	Period, T, s	$X = \dfrac{T^2}{4\pi^2}$	$Y = \ell$
0	0.00	0.00	0.00	0.00	0.00
1	0.20				
…	…				
12	1.50				

5. Construct a confidence interval for the free fall accelerations.

Table LM.18 — Data Processing

Points	X_i	Y_i	ΔX	ΔY	$\bar{g} = \bar{k} = \Delta Y / \Delta X$
1					
2	0	0			
δX	$2\delta Y$		δY	$\delta k = \delta Y / \delta X$	$\Delta n = \Delta k = \delta k / \sqrt{N}$
"Raw" result of the laboratory work					$g =$ ___ ± ___
The final result of the laboratory work					$g =$ ___ ± ___
Relative uncertainty $\varepsilon_g = \Delta g / \bar{g}$					$\varepsilon_g =$ ____%

6. Formulate a conclusion for the laboratory work, compare the obtained result with the generally accepted value $g_{\text{accep}} = 9.81$ m/s^2.

Laboratory Work 9 "Measurement of the Lifetime of an Incandescent Lamp"

Purpose: 1) to master the procedure of linear relationship extrapolation; 2) to get acquainted with an accelerated reliability test; 3) to construct a confidence interval for the lifetime of an incandescent lamp at operating voltage.

Equipment: ten miniature incandescent lamps, source of stable controlled voltage, switch, wires, stopwatch.

Brief Theoretical Information

The theory proposes the following formula for the dependence of an incandescent lamp's lifetime on the applied voltage ($\beta > 0$):

$$\tau = A\exp(\frac{\beta}{\sqrt{U}}).$$

The straightening coordinates for this dependence will be $X = 1/\sqrt{U}$ and $Y = \ln\tau$. In these coordinates, suchrelationship has the form of increasing linear relationship $Y = \beta X + \ln A$.

Description of the Experimental Setup

The scheme of the experimental setup is shown in Fig. LM.6. The light bulb (2) is screwed into the holder (1). After closing the switch (3), the light comes on. Using the stopwatch (4), the time until the lamp burns out is measured. Measurements are performed sequentially for ten light bulbs at increasing voltage values (typically ranging from $1.5U_{\text{oper}}$ to $2.2U_{\text{oper}}$).

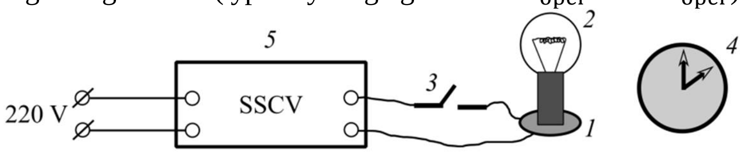


Figure LM.6 — The experimental setup

The key component of the setup is the source of stable controlled voltage (5). Standard power sources are not suitable for this job. They do not allow conducting an experiment at a given, strictly fixed value of the voltage. When a load (the lamp) is connected, the previously set voltage decreases sharply. In addition, during operation, the lamp's resistance changes, which also leads to "floating" working voltage. In SSCV used in this work, voltage stability is achieved using a special electronic circuit.

Measurements and Calculations

Read the values of operating voltage and current on the lamp base. Calculate the lamp's operating resistance ($R = U/I$) and its operating power ($P = UI$). Enter all values in Table LM.19.

Table LM.19 — Technical Characteristics of an Incandescent Lamp

Marking	U_{oper}, V	I_{oper}, A	R_{oper}, Ω	P_{oper}, W

2. Screw the lamp into the holder and set the first voltage value on the SSCV. Close the switch and measure the lamp's lifetime. Repeat the measurement for other voltage values. Record the results in Table LM.20.

3. Plot a graph of the relationship $\tau(U)$ on a graded paper in coordinates $x = U$ and $y = \tau$. Did you get a linear relationship?

4. Calculate the values of the straightening coordinates X and Y and enter their values in the last two columns of Table LM.20.

Table LM.20 — Data

i	U, V	τ, s	$X = 1/\sqrt{U}$	$Y = \ln\tau$
1				
…				
10				

5. Plot the experimental data on graph paper in coordinates (X, Y). Are the data homogeneous, or should they be divided into two clusters, each potentially following its own law for lamp lifetime? (The presence of such clusters indicates different mechanisms of spiral destruction.)

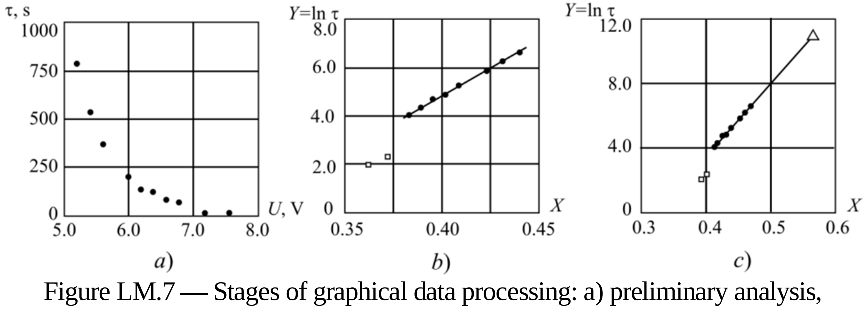


Figure LM.7 — Stages of graphical data processing: a) preliminary analysis, b) straitening, c) hypothesis testing

6. Select the cluster more relevant to our study and draw the experimental line for it. Did you get a linear relationship?

7. Using the extrapolation method, find the value of $\bar{Y}$ corresponding to the operating voltage $U_{\text{oper}} = 3.5$ V. Record the result in Table LM.21.

8. Determine $2\delta Y$ using the "swinging line" method (Fig. LM.7c). Based on the obtained value, calculate the absolute uncertainty ΔY and the relative uncertainty ε_Y. Enter the results in Table LM.21.

9. Using the data obtained for the auxiliary quantity Y, construct a confidence interval for the lamp lifetime τ.

Note that $\varepsilon_f = \left|\frac{d\ln f}{d\ln x}\right| \varepsilon_x$ (see Session 6), and for the dependence $\tau = \exp(Y)$, the equation $\varepsilon_\tau = \bar{Y}\varepsilon_Y$ applies. Here again, we observe the phenomenon of a significant increase in relative uncertainty while using logarithmic coordinates.

Table LM.21 — Data Processing

<table>
<tr><td>U_{oper}, V</td><td>X_{oper}</td><td>$\bar{Y}$</td><td>$2\delta Y$</td><td>δY</td><td>Number of points in the cluster, n_{cl}</td><td>$\Delta Y = \delta Y / \sqrt{n_{\text{cl}}}$</td><td>$\varepsilon_Y = \Delta Y / \bar{Y}$</td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td colspan="3">$\bar{\tau} = \exp\bar{Y}$, s</td><td colspan="3">$\varepsilon_\tau = \bar{Y}\,\varepsilon_Y$</td><td colspan="2">$\Delta\tau = \bar{\tau}\,\varepsilon_\tau$</td></tr>
<tr><td colspan="3"></td><td colspan="3"></td><td colspan="2"></td></tr>
<tr><td colspan="6">"Raw" result of the laboratory work</td><td colspan="2">$\tau =$ _______±_______ s</td></tr>
<tr><td colspan="6">Final result of the laboratory work</td><td colspan="2">$\tau =$ _______±_______ hours</td></tr>
<tr><td colspan="6">Relative uncertainty $\varepsilon_\tau = \Delta\tau / \bar{\tau}$</td><td colspan="2">$\varepsilon_\tau =$____%</td></tr>
<tr><td colspan="6">Nominal lifetime according to technical reference books</td><td colspan="2">$\tau_{\text{nominal}} =$ _____</td></tr>
</table>

10. Compare the obtained result with the lamp's lifetime according to technical reference books.

11. Formulate a conclusion for the laboratory work.

Laboratory Work 10 "Temperature Dependence of Semiconductor Resistivity"

Purpose: to test the proposed theoretical hypothesis experimentally.

Equipment: semiconductor thermistor MMT-18kΩ, digital ohmmeter DT830V, thermocouple with digital indicator, oven for heating the sample, video camera.

Brief Theoretical Information

Propose a hypothetical formula for the semiconductor resistance dependence on the temperature $R(T)$, which could explain the radicaldecrease in semiconductor resistance during heating. Write down your formula and the corresponding straightening coordinates:

$$\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_. \qquad (1)$$

Straightening coordinates: X = _________ and Y = ________________.

Obtain permission to perform measurements from an instructor.

Description of the Experimental Setup

The scheme of the experimental setup is shown in Fig. LM.8. The semiconductor sample (1) is located inside a miniature oven (2), which heats it. The resistance of the semiconductor is measured by a digital ohmmeter (3), the temperature inside the oven is measured with a thermocouple (4) connected to a digital indicator (5).

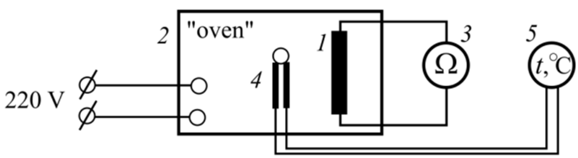


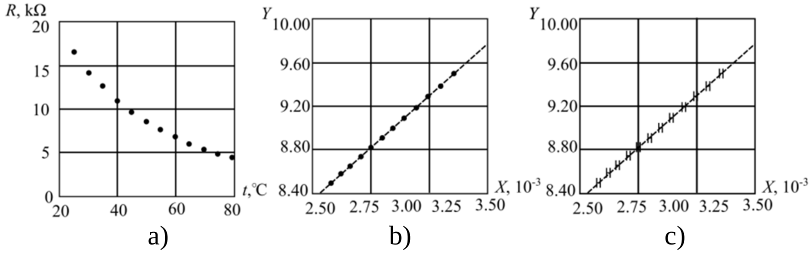
Figure LM.8 — The experimental setup

At the beginning of the experiment, the oven is switched on, and students wait until its temperature reaches 80°C. After that, the oven is switched off. During the cooling phase, students record its resistance and temperature at several times intervals. As the cooling process is fast enough, it is recommended to record the process by a phone video camera and extract data later from the footage.

Let's make a methodical remark: In this laboratory work, the resistance of the semiconductor was measured quite accurately (δ_R = 0.005 kΩ, $\varepsilon_R \approx$ 0.05%). Therefore, we will consider resistance as an accurately measured value and take into account only the instrumental uncertainty of temperature measuring δ_T = 0.5 K.

Measurements and Calculations

1. Connect the devices according to Fig. LM.8. Place measuring instruments, the digital ohmmeter and the thermocouple indicator on the table so that the readings of both devices can be recorded by a phone video camera simultaneously.



a — preliminary analysis; b — straitening; c — hypothesis testing

Figure LM.9 — Stages of graphical data processing

2. Switch on the oven and wait until the digital thermocouple indicator shows 80° C. Then switch off the oven.

Do not overheat the semiconductor sample!

3. Record a video of changes in the device's readings while cooling the sample to room temperature. Then, while watching the video, fill in the third and fourth columns of Table LM.22.

Table LM.22 — Data

i	t, °C	T, K	R, Ω	X = ______	Y = ______
1	80				
2	75				
…					
12	25				

4. Plot the relationship $R(t)$ on graph paper (see Fig. LM.9). Have you obtained a linear relationship?

5. Calculate values of the straightening coordinates and enter them in the last two columns of Table LM.22.

6. Plot the experimental line $Y(X)$ on graph paper. Did you get a linear relationship?

7. Experimental substantiation of the proposed hypothesis using the geometrical method. We will apply the following criterion: if the deviation of the experimental points from the best-fit line can be explained by instrumental uncertainty, the hypothesis is accepted. To apply this criterion, we need to replace the experimental points with experimental segments that reflect the fact the instrumental uncertainty of temperature measurement is equal to $\delta_T = 0.5$ K. Then we check whether the experimental line passes through all experimental segments (see Session 2).

Plot the experimental points as horizontal segments of width $2\delta_X = 2\delta_T/T^2$ (why?) in the coordinates (X, Y). Does the experimental line pass through all these segments? Record the result in Table LM.23.

Table LM.23 — Data Processing

1	Experimental line… through all experimental segments	*passes* *or* *does not pass*	the hypothesis is accepted	+/–
2			the hypothesis is not accepted	+/–

We will give you practical advice: to simplify plotting the experimental segments, consider if X and T are connected as $X = 1/T$, then the following equation is true with great accuracy.

$$\frac{2\delta_X}{(X_{\max} - X_{\min})} = \frac{2\delta_T}{(T_{\max} - T_{\min})}.$$

For the temperature $2\delta_T = 1$ K, $(T_{\max} - T_{\min}) = 50$ K, $2\delta_T/(T_{\max} - T_{\min}) = 1/50$. Therefore, the length of the experimental segments on the graph in straightening coordinates (X, Y) should also be $1/50$ of the graph's horizontal field length.

8. Formulate a conclusion for the laboratory work.

Laboratory Work 11 "Dulong – Petit Law"

Purpose: 1) to get acquainted with the basic concepts of data mining; 2) to derive the Dulong – Petit law from the proposed database.

Equipment: data table, computer, Excel.

Brief Theoretical Information

The thermal properties of substances in physics are described using the concept of specific heat capacity, denoted as c_{spec}. The *specific heat capacity* is numerically equal to the amount of heat required to raise the temperature of 1 kg of a substance by 1°C. In addition, the concept of molar heat capacity c_{mol} is also used in physics. The *molar heat capacity* is numerically equal to the amount of heat needed to increase the temperature of 1 mol of a substance by 1°C. Both quantities are related by the equation $c_{\text{mol}} = \mu c_{\text{spec}}$.

Description of the Data

You are provided with a database (Table LM.24) containing the specific heat capacity c_{spec}, density ρ, and molar mass μ for fourteen elements. Your task is to analyze this data and identify the law!

Measurements and Calculations

1. Create an Excel workbook. Enter the given data for i =1-12.

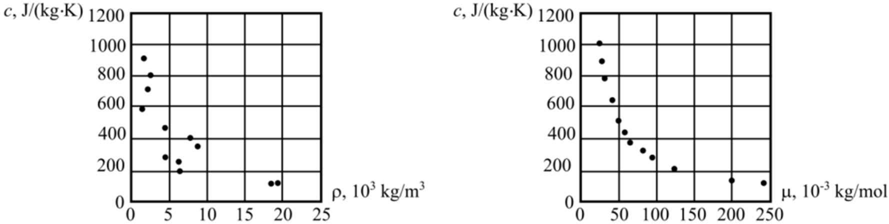
2. Plot two trends: the trend *density – specific heat* and the trend *molar mass – specific heat*. Use natural coordinates:
in the first case ($x = \rho$, $y = c_{\text{spec}}$), in the second case ($x = \mu$, $y = c_{\text{spec}}$).

Table LM.24 — Data

i	Symbol	Name	ρ, 10^3 kg/m^3	μ, 10^{-3} kg/mol	c_{spec}, J/(kg·K)
1	Al	Aluminum	2.7	26.98	903
2	Au	Gold	19.3	196.97	128
3	Ca	Calcium	1.55	40.08	656
4	Cu	Copper	8.93	63.57	386
5	Fe	Iron	7.87	55.85	448
6	Mg	Magnesium	1.74	24.31	1024
7	P	Phosphorus	2.34	30,70	797
8	Sb	Trumpet	6.62	121.75	207
9	Se	Selenium	4.50	78.96	321
10	Ti	Titanium	4.50	47.90	522
11	U	Uranus	18.7	238.03	117
12	Zr	Zirconium	6.44	91.22	276
13*	Li	Lithium	0.534	6.94	3552
14*	C	Diamond	3.52	12,01	510

**Elements marked with an asterisk are used to verify the obtained law*

3. Determine which of the trends you will continue to explore for obtaining the dependence (see Fig. LM.10). Get permission to conduct further research from an instructor.



a) b)

a — the trend *density – specific heat*; b — the trend *molar mass – specific heat*

Figure LM.10 — Selecting the best trend for research

4. To select the type of power coordinates, set the values of μ and c_{spec} in logarithmic coordinates (X_{ln} = lnμ, Y_{ln} = lnc_{spec}). Did you get a linear relationship? Use the built-in Excel function to determine the slope k_{ln} for this relationship. Propose a hypothesis about the type of the power dependence that connects μ and c_{spec}. Suggest power straightening coordinates for this dependence (see Table LM.25). After that, get permission for further research from your instructor.

Table LM.25 — Preliminary Analysis

The Value of the Slope in Logarithmic Coordinates	Hypothesis	Power Straightening Coordinates
k_{ln} = _____	n = ___, c_{spec} = _____	X =______, Y =______

5. Using Excel, plot the relationship $c_{\text{spec}}(\mu)$ in power straightening coordinates (see Fig. LM.11). Note that this relationship has an a priori zero (why?). Did you get a linear relationship?

6. Formulation of the Dulong – Petit Law.

6.1. Construct a confidence interval for the slope of the obtained linear relationship using an analytical method. We characterize this case as a case of linear relationship with an a priori zero (model **A**). In this case, calculations of the mean and the half-width of the confidence interval can be determined by the formulae:

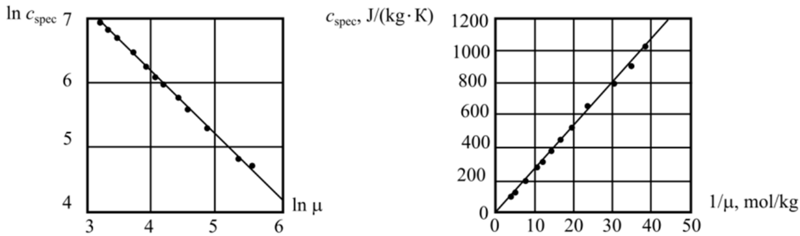
$$\bar{k} = \frac{1}{I_0}\sum_{i=1}^{n} X_i Y_i, \;\; \Delta k = \sqrt{\frac{1}{(n-1)}\frac{1}{I_0}\sum_{i=1}^{n} X_i^2\left(k_i - \bar{k}\right)^2},$$

where $I_0 = \sum_{i=1}^{n} X_i^2$, $k_i = Y_i / X_i$. Perform the calculations using Excel and record the results in Table LM.26.

Table LM.26 — Data Processing

$\bar{k}$, J/(mol·K)	Δk, J/(mol·K)	Confidence Interval	ε_k	Substitution

6.2. Find out which of the values R, $2R$, $3R$, …, where $R = 8.31$ J/(mol·K) is the universal gas constant, get caught into the constructed interval and replace the slope with this value.

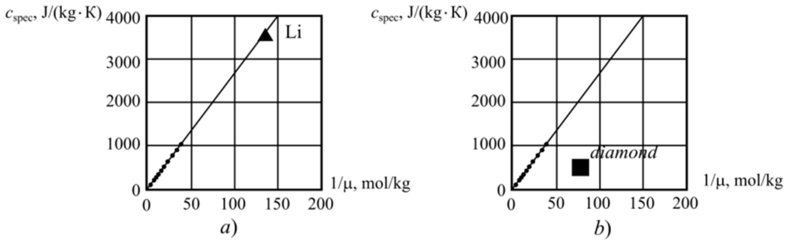


a) b)

a — preliminary analysis; b — straitening; c — hypothesis testing

Figure LM.11 — Straitening in different coordinates

6.3. Formulate the Dulong – Petit law using the concept of molar heat capacity.



a) b)

a — preliminary analysis; b — straitening; c — hypothesis testing

Figure LM.12 — Arguments “for” (lithium) and “against” (diamond)

7. Check the law you received in two separate cases (see Fig. LM.12). Does a) lithium; b) **diamond** lie on the obtained experimental line?

8. Formulate a conclusion for the laboratory work.

Laboratory Work 12 "Electrostatic Field Simulation by Using a Conductive Paper"

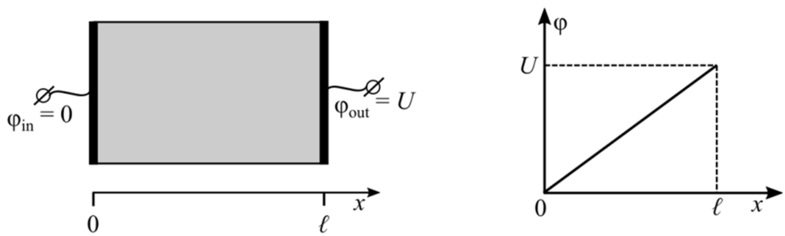
Purpose: 1) to learn how to interpret "strange" facts; 2) to find hidden parameters and build "anti-trend" coordinates; 3) to perform the interpolation (construct a confidence interval for the electric potential of a "black" point).

Equipment: three-layer plate, power supply, probe, voltmeter, connecting wires.

Brief Theoretical Information

In this laboratory work, we will study the distribution of electric potential in a rectangular electrically conductive paper of 16×20 cm in size (Fig. LM.13). The theory says that the distribution is a linear relationship with an a priori zero and could be described by a simple formula: $\varphi(x) = U\frac{x}{\ell}$, where U is the applied voltage, $\ell = 20$ cm is the distance between the electrodes.

However, everything is simple only in theory (for ideal objects). Real objects often produce something we don't expect. In such cases, we need to look for an explanation, that is, to give an *interpretation* of what is happening. And we already know that the first thing is to find out whether the observed deviations can be explained by instrumental uncertainty.



a) b)

a — electrically conductive paper;
b — theoretical potential distribution along the length of the paper

Figure LM.13 — The setup and the theoretical plot

If possible, we have only one thing left to do: replace the measuring instruments. If not, we get unlimited opportunities for creative search. We have to admit that the system we are studying is more complex than we expected, and its behavior depends on hidden parameters unknown to us.

We can put forward and test a variety of hypotheses to uncover these hidden parameters.

In this laboratory work, we need to guess which hidden parameter distorts the results and introduce a new coordinate that transforms a trend into a dependency. Such coordinates, which convert a trend into a dependency, are referred to as anti-trend coordinates.

Description of the Experimental Setup

The core component of the laboratory setup is a 16×20 cm sheet of electrically conductive paper with metal electrodes attached along its sides (Fig. LM.13a). This sheet is mounted on a plywood stand and covered by an opaque screen (Fig. LM.14a). The screen has a Cartesian grid with 4×4 cm cells and contains 15 holes, through which the probe can touch the conductive paper and measure the point potential. In the laboratory work, the potentials of fourteen points will be measured, and the potential of the fifteenth ("black") point will be obtained by analyzing the experimental data.

The unit is powered by a DC voltage source (1). The potential difference is measured with a voltmeter (2). One of its terminals is connected to point *O* and the other to the probe (3) (Fig. LM.14b). It should be noted that a contact difference between the metal electrode and the paper of approximately 2 V exists in such experiments.

Therefore, point *O* is not the electrode itself but the point of the paper near the electrode. The instrumental uncertainty of the potential measurement is equal to $\delta_\varphi = 0.2$ V.

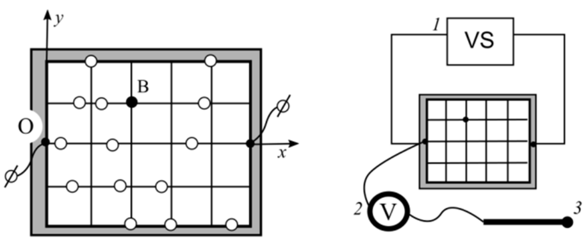


Figure LM.14 — The experimental setup

For instructors: The conductive paper is rotated relative to the drawn Cartesian grid by an angle $\alpha = \mathrm{arctg}(1/4) = 0.25$ rad = 14° about point *O*. This angle is a hidden parameter of this system. You can organize the laboratory work in different ways: you may either provide students with the value of this parameter or ask them to draw real equipotential surfaces based on the obtained data. Then, the turn will become obvious.

Measurements and Calculations

1. Connect the plate to a power source. Set the voltage that an instructor will tell you (usually 16 V). Connect one voltmeter terminal to the plate at point O.

2. Measure the potential at fourteen points on the plate. The points coordinates are indicated in the first two columns. Record the results in Table LM.27.

Table LM.27 — Data

i	x, cm	y, cm	φ, V	X_a, cm
1	4	8		
2	3	4		
3	1	0		
4	2	– 4		
5	5	4		
6	6	0		
7	7	– 4		
8	8	– 8		
9	11	– 4		
10	12	– 8		
11	16	8		
12	15	4		
13	14	0		
14	18	– 8		

3. Plot the relationship $\varphi(x)$ on graph paper (see Fig. LM.15).

4. Carry out a preliminary analysis. Record the results of the analysis in Table LM.28.

4.1. Is there an outside cluster in your data? Are there any points that clearly do not fit the observed relationship and should be excluded from further consideration? (Note that some points are outside the conductive paper!)

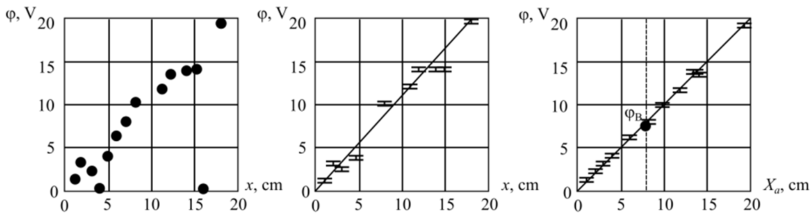
4.2. Replace the experimental points with experimental segments, taking into account that δ_φ = 0.2 V. Is it possible to draw an experimental line through all segments? How would you describe the datayou got: as a trend or a dependence?

4.3. Give your assumption why the real data differ from the expected ones. To solve the puzzle, you have a key: three points share the same potential!

Table LM.28 — Preliminary Analysis

The presence of clusters	Main cluster	Points …
	Outside cluster	Points …
Type of data	Trend	+/–
	Dependence	+/–

4.4. Suggest an anti-trend coordinate for your hypothesis and get permission to continue working from your instructor.



a) b) c)

a — finding clusters; b — proving the presence of hidden parameters; c — interpolation

Figure LM.15 — Stages of the study

5. Calculate the values of the anti-trend coordinate X_a for the main cluster points and plot them in the coordinates (X_a, φ). Did you get a linear relationship?

6. Perform interpolation using the geometrical method, construct a confidence interval for the “black” point B. Record the results in Table LM.29.

6.1. Calculate the value of the anti-trend coordinate X_a for point B.

6.2. Draw an experimental line and determine the mean $\bar{\varphi}$ of the potential at the “black” point.

Table LM.29 — Data Processing

x, cm	y, cm	X_a, cm	$\bar{\varphi}$, V	$2\delta\varphi$, V	$\Delta\varphi$, V	Confidence Interval	Measured Value φ_B, V

6.3. Determine $2\delta\varphi$ using the line-swinging method, then calculate the half-width of the confidence interval according to the formula $\Delta\varphi = \frac{\delta\varphi}{\sqrt{n_{\text{cl}}}}$, where n_{cl} is the number of points in the cluster. Express your result for the potential at the “black” point in the form of a confidence interval.

7. Measure the potential at point *B*. Does the measured value get into the interval you constructed?

8. Formulate a conclusion for the laboratory work.

ANSWERS AND SOLUTIONS

Session 1. *Additional Task 1.*

For $\nu = 2$ and $\alpha = 0.99$ Student's coefficient is equal $t_\nu^\alpha = 9.925$.

Session 2. *Additional Task 2.*

a) It is possible to draw the straight line $y = b$ through all error bars only when the following system of inequalities holds:

$$\{y_i - \delta_y \le b \le y_i + \delta_y\},\text{ where } i = 1, 2, \dots, n.$$

And this system of inequalities holds only when $\max(y_i) - \min(y_i) \le 2\delta_y$. This condition you have to check.

b) For the straight line $y = kx$ the first system of inequalities has the form

$$\{y_i - \delta_y \le kx \le y_i + \delta_y\},\text{ where } i = 1, 2, \dots, n.$$

The condition you have to check is $\max\left(\frac{y_i - \delta_y}{x_i}\right) \le \min\left(\frac{y_i + \delta_y}{x_i}\right)$.

Session 11. *Additional Task 2.*

a) For all substances, the deviations δY from the experimental straight line are much larger than $\delta_c = 0.5$ J/(kg·K). Thus, for calcium $\frac{|\delta Y|}{\delta_c} = 71$, for aluminum $\frac{|\delta Y|}{\delta_c} = 38$, for phosphorus $\frac{|\delta Y|}{\delta_c} = 26$, for uranium $\frac{|\delta Y|}{\delta_c} = 25$. Such deviations cannot be explained by instrumental uncertainty.

b) $\ell^2_{\mathrm{real}} = \sum_{i=1}^{n} \delta Y_i^2 = 2042$ is much greater than the value that could be observed in the case when there was only instrumental uncertainty, $\langle \ell^2_{\mathrm{ins}} \rangle = (0.8 \pm 0.3)$. Such a deviation has virtually zero probability. Despite the excellent agreement between $\bar{k}$ and the classical theory value $3R$ ($\frac{\bar{k}}{3R} = 0.998$), the hypothesis is rejected. It is interesting to note that for this data set the hypothesis could have been accepted at a stage when the relative accuracy of the specific heat measurements was 10%.

c) From the formula $\Delta x^2 = \Delta x^2_{\mathrm{ran}} + \Delta x^2_{\mathrm{inst}}$ from Session 2 follows that in this case the contribution of the instrumental uncertainty to the total uncertainty is only 0.04%.

RECOMMENDED READING

1. Feynman R.P., Leighton R.B., Sands M. The Feynman Lectures on Physics. Mainly Electromagnetism and Matter. Vol. II. The New Millennium Edition. New York: Basic Books; 2010. P. 566.

2. Probability and Statistics for Engineers and Scientists / R.E. Walpole, R.H. Myers, Sh.L. Myers (6th Edition). Prentice Hall, 1998, 739 pp.

3. Serway R.A. and Jewett J.W. Physics for Scientists and Engineers with Modern Physics, 9th ed., Brooks Cole, 2014. 1622 pages.

4. Soong T.T. Fundamentals of Probability and Statistics for Engineers. John Wiley and Sons, 2004, 391 pp.

5. Taylor J. R. An Introduction to Error Analysis (Second Edition). Sausalito, California: University Science Books, 1997, 327 pp.

APPENDIX A
Basic Formulae for Finding Parameters of an Experimental Straight Line

1. For a case of a constant value $y = b$:

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i, \qquad \Delta y = \frac{1}{\sqrt{n}}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2}.$$

2. For a linear relationship with a priori zero $y = kx$ $(b = 0)$:

$$\bar{k} = \frac{1}{I_0}\sum_{i=1}^{n} x_i y_i, \qquad \Delta k = \frac{1}{\sqrt{I_0}}\sqrt{\frac{1}{(n-1)}\sum_{i=1}^{n}\left(y_i - x_i\bar{k}\right)^2},$$

where $I_0 = \sum_{i=1}^{n} x_i^2$ is the "moment of inertia" of the experimental points relative to zero.

3. For a general case of linear relationship $y = kx + b$:

$$\bar{k} = \frac{1}{I_c}\sum_{i=1}^{n}(x_i - x_c)\,(y_i - \bar{y}), \quad \Delta k = \frac{1}{\sqrt{I_c}}\sqrt{\frac{1}{(n-2)}\sum_{i=1}^{n}\left(y_i - \bar{b} - x_i\bar{k}\right)^2},$$

$$\bar{b} = \bar{y} - \bar{k}x_c, \qquad \Delta b = \sqrt{\frac{I_0}{nI_c}}\sqrt{\frac{1}{(n-2)}\sum_{i=1}^{n}\left(y_i - \bar{b} - x_i\bar{k}\right)^2},$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, $x_c = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the *x*-coordinate of the "center of mass" of the system of experimental points, $I_c = \sum_{i=1}^{n}(x_i - x_c)^2$ is the "moment of inertia" of the experimental points relative to its "center of mass".

4. For a general case of a linear relationship $y = y_m + k(x - x_m)$ taking into account the "masses" of experimental points:

$$\bar{y}_m = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \quad \Delta y = \frac{1}{\sqrt{M}}\sqrt{\frac{1}{\nu}\sum_{i=1}^{n} m_i\left(y_i - \bar{y}_m - (x_i - x_m)\bar{k}\right)^2},$$

$$\bar{k} = \frac{1}{I_m}\sum_{i=1}^{n} m_i(x_i - x_m)y_i\,, \ \Delta k =$$

$$\frac{1}{\sqrt{I_m}}\sqrt{\frac{1}{\nu}\sum_{i=1}^{n} m_i\left(y_i - \bar{y}_m - (x_i - x_m)\bar{k}\right)^2},$$

where $m_i = 1/\Delta y_i^2$ are the "masses" of the experimental points on the (X, Y)-plane, $M = \sum_{i=1}^{N} m_i$, $x_m = \frac{1}{M}\sum_{i=1}^{N} m_i x_i$, $I_m = \sum_{i=1}^{N} m_i(x_i - x_m)^2$, and $\nu = n - 2$.

If you assume that the absolute uncertainty Δy_i in all experimental points is equal, then $m_i = 1$. If this condition holds for directly measured quantities but you use straightening coordinates $Y(y)$ then $m_i = \frac{1}{\left(\frac{\partial Y}{\partial y}\right)_i^2}$.

APPENDIX B
Parameters of a Linear Relationship and Their Uncertainties in Excel

Suppose that you received experimental data, entered them into a table, built a dot plot and decided that the corresponding quantities are related by a linear relationship (see Fig. A.1).

X	Y
37.06449	903
5.076915	128
24.9501	656
15.73069	386
17.9051	448
41.13534	1024
32.57329	797
8.213552	207
12.66464	321
20.87683	522
4.201151	117
10.96251	276

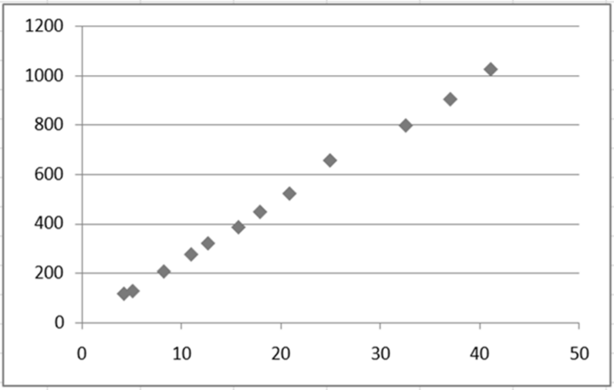


Figure A.1 — Experimental data and the dot plot

Case 1. If you have reason to believe that the linear relationship has the form $y = kx$, then to obtain the slope k and its uncertainty Δk, you need to do the following:

1. Select an area of 5 rows × 2 columns.
2. Enter the function LINEST(Yvalue; Xvalue; 0; 1).
3. Press Ctrl+Shift+Enter.

k →	24.87383	0
Δk →	0.173829	#Н/Д
R^2 →	0.999463	13.62618
	20475.87	11
	3801811	2042.4

Figure A.2 — Results of calculations in Case 1

Statistical data will appear in the selected area (see Fig. A.2). You will be interested only in the first three lines of the first column:

$$k = 24.87383,$$
$$\Delta k = 0.173829.$$

The third number 0.999463 is called the *correlation coefficient* (denoted R^2). If its value lies between 0.8 and 1, then it means that your assumption about the linearity of the relation is reasonable.

Case 2. If you have reason to believe that the linear relationship has the form $y = kx + b$, then to obtain the slope k and its uncertainty Δk, the y-intercept b and its uncertainty Δb, you need to do the following:

1. Select an area of 5 rows × 2 columns.
2. Enter the function LINEST(Yvalue; Xvalue; 1; 1).
3. Press Ctrl+Shift+Enter.

Statistical data will appear in the selected area (see Fig. A.3).

$\boldsymbol{k} \rightarrow$	24.52671	9.219418	$\leftarrow \boldsymbol{b}$
$\boldsymbol{\Delta k} \rightarrow$	0.323509	7.320635	$\leftarrow \boldsymbol{\Delta b}$
$\mathbf{R}^2 \rightarrow$	0.998263	13.2771	
	5747.846	10	
	1013238	1762.814	

Figure A.3 — Results of calculations in Case 2

Since in this case R^2 is smaller than in the first case, it can be concluded that these results are better described by the relationship $y = kx$.

To learn more about the meaning of the other numbers in LINEST function output see statistics textbooks.

APPENDIX C
Propagation of Uncertainties

If $f(x, y, z, \dots)$ is any function of $x, y, z, \dots$, then

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x}\Delta x\right)^2 + \left(\frac{\partial f}{\partial y}\Delta y\right)^2 + \left(\frac{\partial f}{\partial z}\Delta z\right)^2 + \dots}$$

(provided all uncertainties are independent and random)

and

$$\Delta f \le \left|\frac{\partial f}{\partial x}\right|\Delta x + \left|\frac{\partial f}{\partial y}\right|\Delta y + \left|\frac{\partial f}{\partial z}\right|\Delta z + \cdots$$

Example. You want to measure the free fall acceleration g. You know that $g = \frac{2h}{t^2}$, so you measured the time it takes for a ball to fall a certain distance, and you got

$$h = (2.124 \pm 0.003)\ \mathrm{m},\ t = (0.651 \pm 0.015)\ \mathrm{s}.$$

You have to find the mean value of g and the uncertainty in this value.

Step 1. The mean value

$$\bar{g} = \frac{2\bar{h}}{\bar{t}^2} = \frac{2 \cdot 2.124\ \mathrm{m}}{(0.651\ \mathrm{s})^2} = 10.024\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

Step 2. Partial derivatives

$$\frac{\partial g}{\partial h} = \frac{2}{t^2} = \frac{2}{(0.651\ \mathrm{s})^2} = 4.719\,\frac{1}{\mathrm{s}^2},$$

$$\frac{\partial g}{\partial t} = -\frac{4h}{t^3} = -\frac{4\,(2.124\ \mathrm{m})}{(0.651\ \mathrm{s})^3} = -30.794\,\frac{\mathrm{m}}{\mathrm{s}^3}.$$

Step 3. Uncertainty

$$\Delta g = \sqrt{\left(\frac{\partial g}{\partial h}\cdot\Delta h\right)^2 + \left(\frac{\partial g}{\partial t}\cdot\Delta t\right)^2} =$$

$$= \sqrt{\left(4.719\,\frac{1}{\mathrm{s}^2}\cdot 0.003\ \mathrm{m}\right)^2 + \left(-30.794\,\frac{\mathrm{m}}{\mathrm{s}^3}\cdot 0.015\ \mathrm{s}\right)^2} = 0.462\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

Step 4. Confidence interval

$$g = (10.024 \pm 0.462)\,\frac{\mathrm{m}}{\mathrm{s}^2} \approx (10.0 \pm 0.5)\,\frac{\mathrm{m}}{\mathrm{s}^2}.$$

Навчальне видання

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SEARCH FOR PHYSICS LAWS

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