Ministry of Education and Science of Ukraine

National University “Zaporizhzhia Polytechnic”

Methodical instructions
on performance of calculational tasks
on the course “Applied mechanics”
for students of specialty
“Electrical power engineering, electrical engineering
and electromechanics”
of all forms of learning

2023

Methodical instructions on performance of calculational tasks on the course “Applied mechanics” for students of specialty “Electrical power engineering, electrical engineering and electromechanics” of all forms of learning / V.G.Shevchenko, S.L.Ryagin, S.O.Shumykin. – Zaporizhzhya: National University “Zaporizhzhia Polytechnic”, 2023. – 18 p.

Authors: V.G.Shevchenko, associate professor, cand. sc.
S.L.Ryagin, associate professor, cand. sc.
S.O.Shumykin, associate professor, cand. sc.

Reviewer: A.D.Fursina, associate professor, cand. sc.

Responsible for issue: V.G.Shevchenko,
chief of the chair of mechanics,
associate professor, cand. sc.

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CONTENTS

GENERAL INFORMATION

Variant number is appointed to each student by the teacher. A student chooses from the respective tables initial data to each calculational task in accordance with his variant number. First three letters of the alphabet should be placed under figures of variant number, for example:

Variant number – *0 7 2*;

Letters – *a b c*.

From each vertical column of the table only one value located on the horizontal line which number corresponds to the letter under the column should be chosen.

For example, from table 1.1 for the given variant number *072* a student has to take value from columns marked "c" from line 2: scheme II, q=4 kN/m, c=1 m; from columns marked "b" from line 7: P=2 kN, a=3 m; and from columns marked "*a*" from line 0: M=5 kN·m, b=2 m.

Calculational tasks should be performed in accordance with the requirements of the correspondent standards.

Calculational scheme and table with initial data should be placed at the beginning of calculational task. Record of calculations should be detailed enough for its check. After check student has to correct all mistakes and to prepare calculational task for re-check. If there are no mistakes, student has to support the calculational task.

CALCULATIONAL TASKS

1 TASK 1 DETERMINATION OF SUPPORT REACTIONS OF RIGID BODY, LOADED BY PLANE FORCE SYSTEM

It is necessary to calculate support reactions, if plane system is loaded by:

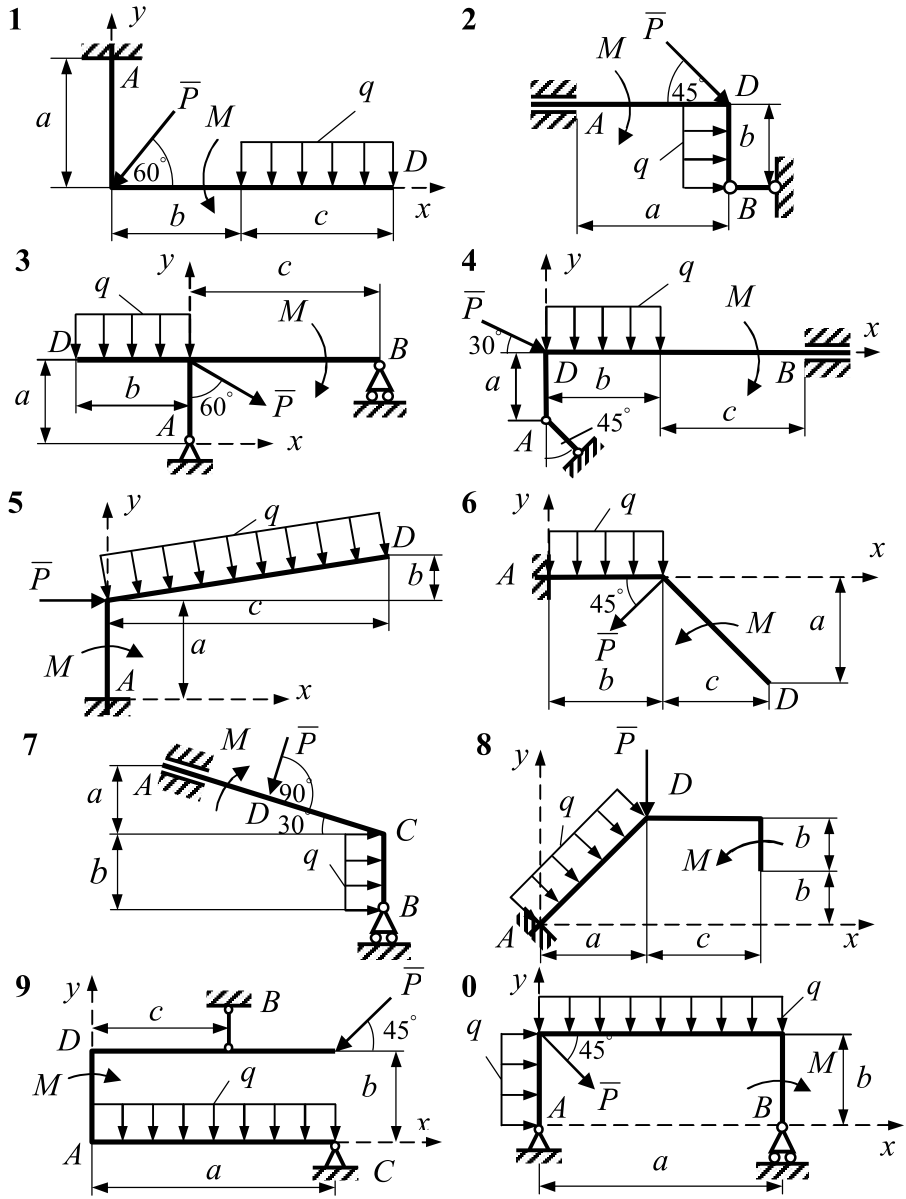
– concentrated force P;

– concentrated moment M;

– uniformly distributed force q.

Answers have to be verified.

Load schemes are given in Picture 1.1; numerical values are given in Table 1.1.


1
y
A
a
P
60°
M
q
D
b
c
x
2
P
M
45°
D
A
b
q
B
a
3
y
c
q
M
D
B
a
b
60°
P
A
x
4
y
q
P
M
x
30°
D
b
B
a
c
A
45°
5
y
q
D
P
b
c
a
M
A
x
6
y
q
x
A
45°
P
M
a
b
c
D
7
M
P
a
A
90°
D
30°
C
b
q
B
8
P
y
D
q
b
M
b
A
a
c
x
9
P
y
B
c
45°
D
M
q
b
x
A
a
C
0
y
q
q
45°
M
b
A
P
B
x
a

Picture 1.1

Table 1.1

Scheme	P, kN	M, kN·m	q, kN/m	Dimensions, m		
				a	b	c
1	10	6	2	3	2	4
2	20	5	4	3	2	1
3	15	8	1	1	2	3
4	5	2	1	2	4	2
5	20	12	2	2	3	4
6	6	2	1	3	2	3
7	2	4	2	3	2	1
8	20	10	4	3	2	4
9	10	4	3	2	3	4
0	10	5	2	4	2	2
c	***b***	***a***	***c***	***b***	***a***	***c***

Example

Load scheme is given in pic. 1.2.a; numerical values are the following: P=4 kN; M=6 kN·m; q=5 kN/m; a=2 m; b=2 m; c=2 m.

Supports have to be replaced by reactions according to table C.1. Uniformly distributed force q can be replaced by concentrated resultant force $Q=q\cdot c=5\cdot 2=10$ kN, which acts from geometrical centre of distributed force q. Correspondent calculational scheme is given in pic. 1.2.b.

The system is a rigid body in equilibrium state under the action of active forces and support reactions. We should calculate three support reactions Y_A, M_A, R_B from three equations of equilibrium on plane. Hence the system is statically determinable.

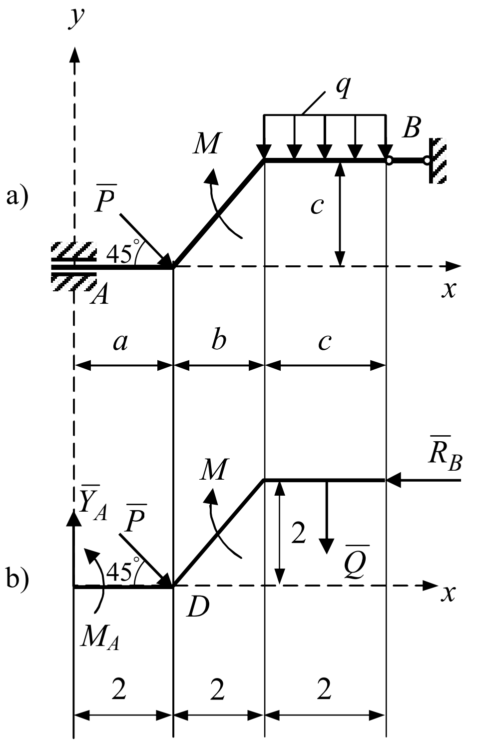
It is convenient to write the following equations of equilibrium:

$$\sum F_{kx} = 0\,; \quad -R_B + P\cos 45^\circ = 0\,; \tag{1.1}$$

$$\sum F_{ky}=0\,;\quad Y_A - P\sin 45^\circ - Q = 0\,; \tag{1.2}$$

$$\sum M_A\left(\overline{F}_k\right)=0\,;$$

$$M_A - M - P\sin 45^\circ\cdot a - Q\cdot(a+b+c/2)+R_B\cdot c=0\,. \tag{1.3}$$



Picture 1.2

From equation (1.1):

$R_B=P\cdot cos45°;$

$R_B=4\cdot 0.707=2.83$ kN.

From equation (1.2):

$Y_A=P\cdot sin45°+Q;$

$Y_A=4\cdot 0.707+10=12.83$ kN.

From equation (1.3):

$M_A=M+P\cdot sin45°\cdot a+$
$+Q(a+b+c/2)-R_B\cdot c;$

$M_A=6+4\cdot 0.707\cdot 2\cdot 10\cdot 5-$
$-2.83\cdot 2=56$ kN.

To verify these results the following additional equation of equilibrium can be written:

$$\sum M_D\left(\overline{F}_k\right)=0\,;$$

$-Y_A\cdot a+M_A-M-Q\cdot(b+c/2)+R_B\cdot c=0;$

$-12.83\cdot 2+56-6-10\cdot(2+2/2)+$
$+2.83\cdot 2=0;$

$0=0.$

Hence the results obtained from equations (1.1), (1.2), (1.3) are true.

Answer: $Y_A=12.83$ kN;
$M_A=56.0$ kN·m;
$R_B=2.83$ kN.

2 TASK 2. CONSTRUCTION OF THE DIAGRAMS OF LONGITUDINAL INTERNAL FORCE AND DEFORMATION OF A STEP ROD UNDER THE TENSION CONSIDERING WEIGHT

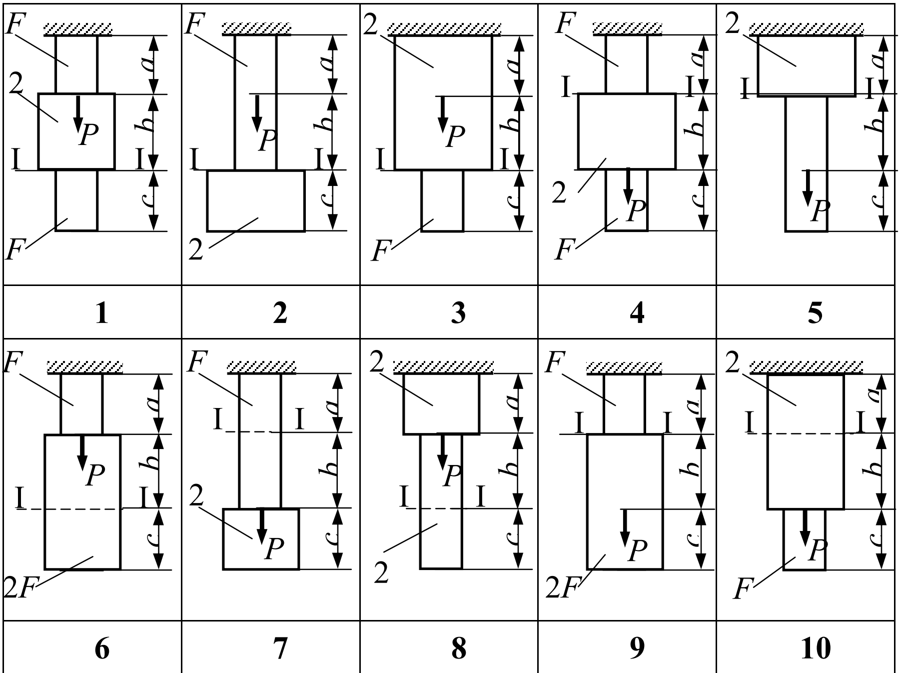
A longitudinal force P and weight act on a rod from steel. Young modulus of rod material is E=2·10^5 MPa, relative weight is γ=78 kN/m^3.

It is necessary to fulfill the task:

– to construct the diagram of longitudinal internal force N;

– to construct the diagram of deformation Δ.

Design schemes and initial data are given in Picture 2.1 and Table 2.1.



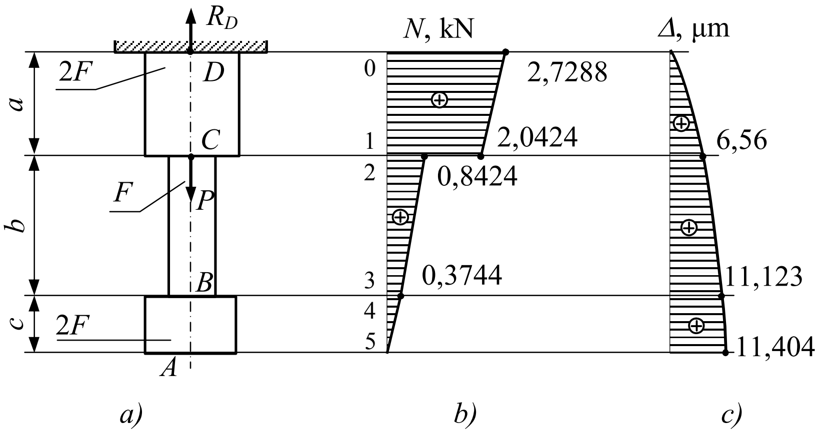
Picture 2.1

Table 2.1

Scheme	Area, F, cm^2	Interval length, m			Concentrated force P, kN
		a	b	c	
1	11	2,1	2,1	1,1	1,1
2	12	2,2	2,2	1,2	1,2
3	13	2,3	2,3	1,3	1,3
4	14	2,4	2,4	1,4	1,4
5	15	2,5	2,5	1,5	1,5
6	16	2,6	2,6	1,6	1,6
7	17	2,7	2,7	1,7	1,7
8	18	2,8	2,8	1,8	1,8
9	19	2,9	2,9	1,9	1,9
10	20	3,0	3,0	2,0	2,0
c	***b***	***a***	***b***	***c***	***a***

Example

Design scheme is given in Picture 2.2.a; numerical values are the following: P=1,2 kN; F=20 cm^2; a=2,2 m; b=3 m; c=1,2 m.



Picture 2.2 − Design scheme and the diagrams

A weight creates additional distributed force, proportional to area of a cross-section, at all intervals. This distributed force is equal to area of cross-section, multiplyed by relative weight.

The diagrams of longitudinal internal force N and of deformation Δ will be constructed by the rules (see Appendix D) in this example.

$$N_A = N_5 = 0; \tag{2.1}$$

$$N_B = N_{3,4} = N_A + \gamma \cdot 2F \cdot c;$$
$$N_B = N_{3,4} = 0 + 78 \cdot 10^3 \cdot 2 \cdot 20 \cdot 10^{-4} \cdot 1{,}2 = 374{,}4 \text{ N}; \tag{2.2}$$

$$N_C = N_2 = N_B + \gamma \cdot F \cdot b;$$
$$N_C = N_2 = 374{,}4 + 78 \cdot 10^3 \cdot 20 \cdot 10^{-4} \cdot 3 = 842{,}4 \text{ N}; \tag{2.3}$$

$$N_C' = N_1 = N_C + P;$$
$$N_C' = N_1 = 842{,}4 + 1200 = 2042{,}4 \text{ N}; \tag{2.4}$$

$$N_D = N_0 = N_C' + \gamma \cdot 2F \cdot a;$$
$$N_D = N_0 = 2042{,}4 + 78 \cdot 10^3 \cdot 2 \cdot 20 \cdot 10^{-4} \cdot 2{,}2 = 2728{,}8 \text{ N}; \tag{2.5}$$

$$\Delta_D = \Delta_0 = 0; \tag{2.6}$$

$$\Delta_C = \Delta_{1,2} = \Delta_D + \frac{0{,}5 \cdot (N_D + N_C') \cdot a}{E \cdot 2F};$$
$$\Delta_C = \Delta_{1,2} = 0 + \frac{0{,}5 \cdot (2728{,}8 + 2042{,}4) \cdot 2{,}2}{2 \cdot 10^{11} \cdot 2 \cdot 20 \cdot 10^{-4}} = 6{,}56 \cdot 10^{-6} \text{ m}; \tag{2.7}$$

$$\Delta_B = \Delta_{3,4} = \Delta_C + \frac{0{,}5 \cdot (N_C + N_B) \cdot b}{E \cdot F};$$
$$\Delta_B = \Delta_{3,4} = 6{,}56 \cdot 10^{-6} + \frac{0{,}5 \cdot (842{,}4 + 374{,}4) \cdot 3}{2 \cdot 10^{11} \cdot 20 \cdot 10^{-4}} = 11{,}123 \cdot 10^{-6} \text{ m}; \tag{2.8}$$

$$\Delta_A = \Delta_5 = \Delta_B + \frac{0{,}5 \cdot (N_B + N_A) \cdot c}{E \cdot 2F};$$
$$\Delta_A = \Delta_5 = 11{,}123 \cdot 10^{-6} + \frac{0{,}5 \cdot (374{,}4 + 0) \cdot 1{,}2}{2 \cdot 10^{11} \cdot 2 \cdot 20 \cdot 10^{-4}} = 11{,}404 \cdot 10^{-6} \text{ m}. \tag{2.9}$$

Correspondent diagrams are placed in Picture 2.2.b,c.

LITERATURE

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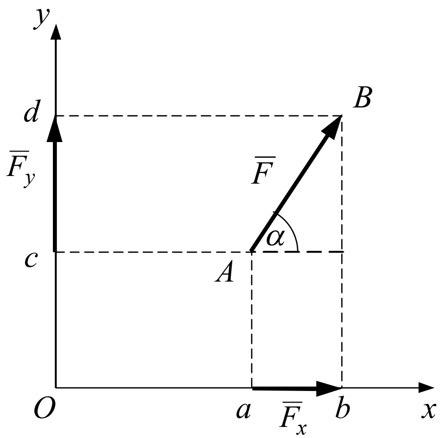
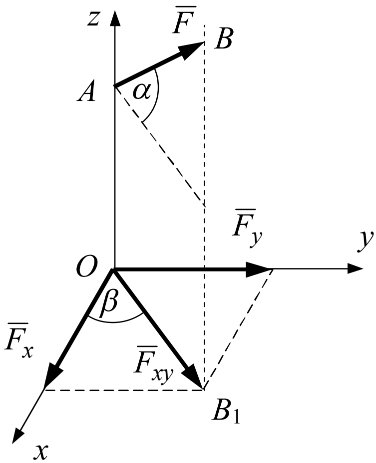
3. Feodosiev, V.I. Advanced Stress and Stability Analysis Worked Examples. – Springer, 2005. – 427 p.

4. Methodical instructions on performance of calculational tasks on the course “Theoretical mechanics. Statics” for students of specialty “Electric machines and devices” of all forms of learning / V.G.Shevchenko, P.K.Shtanko, A.M.Polyakov, I.A.Petrik, S.L.Ryagin – Zaporizhzhya: ZNTU, 2014. – 26 p.

5. Методичні вказівки до виконання розрахунково-графічних робіт з курсу “Опір матеріалів” для студентів галузі знань “Механічна інженерія” усіх форм навчання. / Укл.: В.Г. Шевченко, С.Л. Рягін, О.Г. Попович, О.С. Омельченко, А.А. Скребцов. – Запоріжжя: НУ „Запорізька політехніка”, 2023. − 58с.

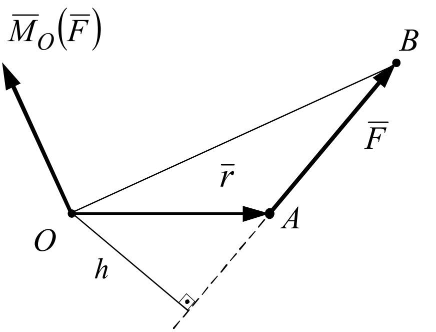
Appendix A
Projection of a Force

Table A.1

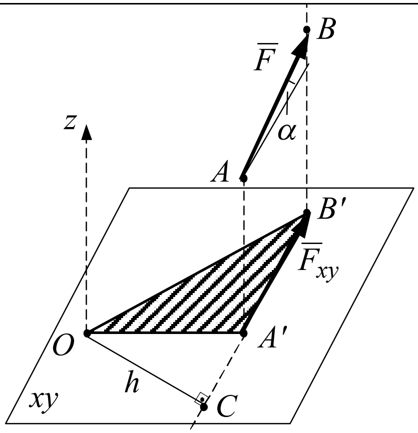
1	**Projection of a force to an axis**
	$F_x=F\cdot cos\alpha$; $F_y=F\cdot sin\alpha$
2	**Projection of a force to a plane**
	$F_x=F_{xy}\cdot cos\beta=F\cdot cos\alpha\cdot cos\beta$; $F_y=F_{xy}\cdot sin\beta=F\cdot cos\alpha\cdot sin\beta$;

Appendix B
Moment of a Force

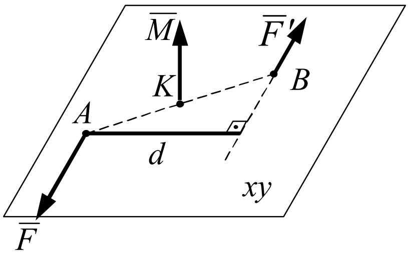
Table B.1

<table>
<tr><th colspan="2">Moment of a force about a point</th></tr>
<tr>
<td></td>
<td>1 Vector moment of force $\overline{F}$ about (with respect to) point O

$$\overline{M}_O(\overline{F}) = \overline{r} \times \overline{F}$$
Vector moment $\overline{M}_O(\overline{F})$ is applied to the point O and directed perpendicularly to the plane OAB by right-hand rule.

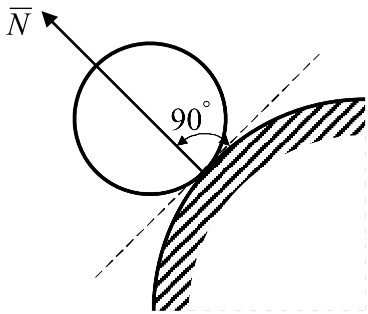
2 Scalar moment of a force $\overline{F}$ about point O
$$M_O(\overline{F}) = \pm F \cdot h,$$
where h is a moment arm (shortest distance from the point to the force line).</td>
</tr>
<tr><th colspan="2">Moment of a force about an axis</th></tr>
<tr>
<td></td>
<td>Moment of a force $\overline{F}$ about an axis z is equal to a moment of the force projection to a plane xy, perpendicular to this axis, about the intersection point of plane xy and axis z
$$M_z(\overline{F}) = M_O(F_{xy}) = \pm F_{xy} \cdot h =$$
$$= \pm F \cdot cos\,\alpha \cdot h$$
Moment of a force about an axis is equal to zero in two cases:
– force line intersects the axis;
– force line is parallel to the axis.</td>
</tr>
</table>

Continuation of the table B.1

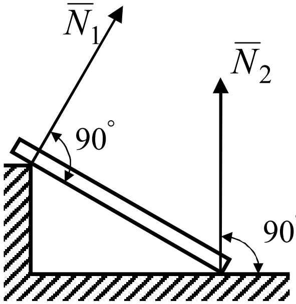
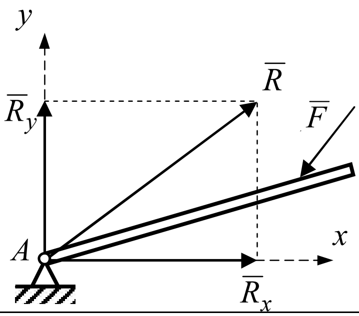
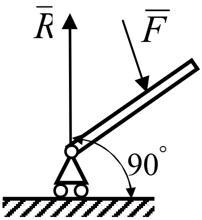
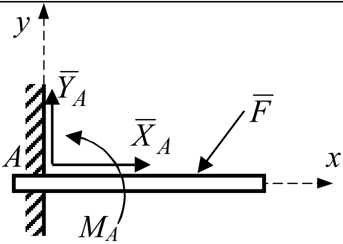
<table>
<tr><th colspan="2">Force couple and its moment</th></tr>
<tr><td rowspan="5">
</td><td>1 Vector moment of force couple</td></tr>
<tr><td>$\overline{M} = \overline{M}(\overline{F}, \overline{F}') = \overline{M}_A(\overline{F}') =$
$= \overline{M}_B(\overline{F}) = \overline{AB} \times \overline{F}' = \overline{BA} \times \overline{F}$
Vector moment $\overline{M}$ of force couple is directed perpendicularly to its plane of action by right-hand rule. Vector $\overline{M}$ is a free vector.</td></tr>
<tr><td>2 Scalar moment of force couple</td></tr>
<tr><td>$M = \pm F \cdot d$</td></tr>
</table>

Appendix C
Supports and their Reactions

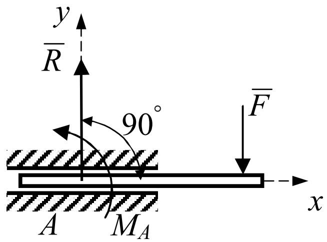
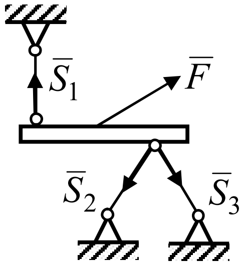
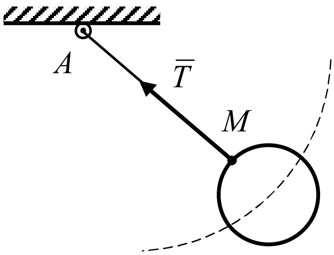
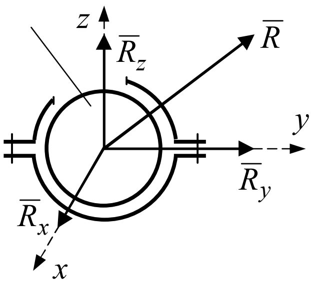
Table C.1

№	Type of the support	Name	Direction of the reaction
1	2	3	4
1		Smooth surface (without friction)	Perpendicular to general tangent surface at a point of contact of bodies

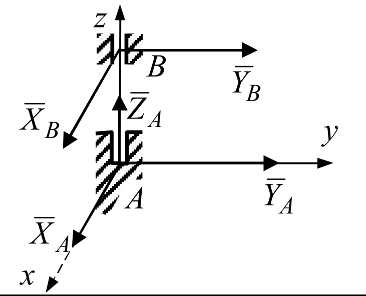
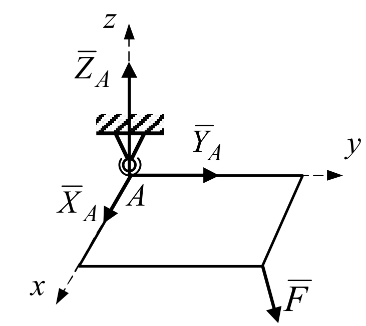
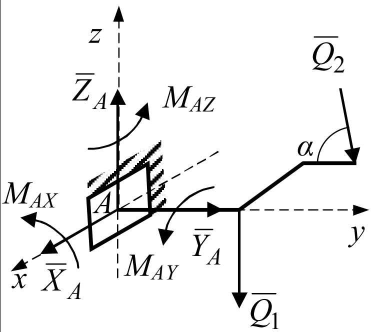
Continuation of the table C.1

1	2	3	4
2		Contact of bodies at the point (without friction)	Perpendicular to the surface
3		Cylindrical hinge (immovable)	General reaction $\overline{R}$ is divided into two projections
4		Cylindrical hinge (movable)	Perpendicular to the surface
5		Built-in support	Reactive force is divided into two projections $\overline{X_A}$ and $\overline{Y_A}$. Reactive moment M_A acts at the plane xAy

Continuation of the table C.1

1	2	3	4
6		Sliding support	Perpendicularly to support surface. Reactive moment M_A acts at the plane xAy
7		Weightless rod	Along rod axis
8		Flexible unstrainable string	Along the string to suspension point
9		Spherical hinge	General reaction $\overline{R}$ is divided into three projections

Continuation of the table C.1

1	2	3	4
10		Thrust-bearing *A* and bearing *B*	General reactions are divided into components X_A, Y_A, Z_A and X_B, Y_B correspondently
11		Spherical hinge (immovable)	General reaction is divided into three projections $\overline{X}_A$, $\overline{Y}_A$, $\overline{Z}_A$
12		Three-dimensional built-in support	Reactive force is divided into three projections. Reactive moment is divided into three projections

Appendix D
The rules of construction of the diagrams of a longitudinal internal force and of a deformation at tension

The diagram parametre can be a stress, or an internal force factor, or a deformation. The diagrams have to be constructed by intervals. Interval limits are the points of load action, or the points where cross-section dimension changes.

The diagrams have to be constructed with respect to a base line, which is parallel to a rod axis. The parametre value have to be put perpendicularly to base line in scale considering sign. Positive sign corresponds tension, negative – compression. The signs have to be put at all intervals. Numerical values have to be put at all characteristic points. A hatch have to be perpendicular to a base line.

A shape of the N-diagram is horizontal straight line, and a shape of the Δ-diagram is inclined straight line, if there are no distributed force at the interval. A shape of the N-diagram is inclined straight line, and a shape of the Δ-diagram is parabolic curve, if uniform distributed force acts at the interval. N changes at the point of concentrated force action on this force value considering sign, at Δ-diagram changes tangent direction.

Change in N value is equal to distributed force area at the interval considering sign. Change in Δ value is equal to N-diagram area at the interval, divided by cross-section rigidity, considering sign.

Tangent inclination angle at the Δ-diagram is proportional to N value at the cross-section.