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APPROXIMATION OF FUNCTIONS OF TWO VARIABLES AT LYAPUNOV SCELETONS IN GENERALIZED HÖLDER SPACES

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The approximative properties of interpolational Lagrange polynomials, Faber – Laurent series of functions of two variables which are defined at an arbitrary closed Lyapunov sceleton, is stated. For functions from generalized Hölder spaces H_ω the estimations are conducted for norms of $C, H_\omega, L_p, p>1$ spaces.

This work is a continuation of [1]. Here we generalize the results obtained in [1]. The statements proved earlier for the case of a unit bicircle are extended in this paper to an arbitrary closed Lyapunov skeleton.

Let Γ be an arbitrary simple closed Lyapunov contour which confines the one-connected domain F^+ (origin is contained inside F^+). Let F^- denotes the complement of $F^+ \cup \Gamma$ to the whole complex plain. Let function $\psi(w) = cw + c_0 + \frac{c_1}{w} + \cdots$ performs the conform transformation of the exteriority of the unit circle with center at 0 on the domain F^- such that $\psi(\infty) = \infty$, $\psi'(\infty) = c > 0$ (without loss of generality we can assume that $c = 1$). In this case the unit circle $\Gamma_0 = \{w : |w| = 1\}$ is transformed on Γ continuously and one-to-one-nessly. Let $w_j = exp\left\{\frac{2\pi i}{2n+1}(j-n)\right\}$, $i^2 = -1$, $j = 0,\dots,2n$ be the system of equidistant points at the unit circle Γ_0 and $t_j = \psi(w_j)$, $j = 0, \dots, 2n$, is the system of its images at the contour Γ (so-called Fejér points).

Let $\gamma = \gamma_1 \times \gamma_2$ is sceleton which is formed by arbitrary simple closed Lyapunov contours γ_1 and γ_2 (origin is contained inside them). Let $\omega(\delta_1, \delta_2)$ be the some modulus of continuity, and $\Omega_1(\delta)$, $\Omega_2(\delta)$ are the simple moduli of continuity, which respect to that one. Let H_ω denotes the space of continuous functions $x(t,\tau)$ at γ, which satisfy the conditions:

$$H(x;\omega) = \sup_{\delta_1^2+\delta_2^2 \neq 0} \frac{\omega(\delta_1,\delta_2;x)}{\omega(\delta_1,\delta_2)} \leq c_1,$$

$$H^{t\tau}(x;\omega) = \sup_{\delta_1^2+\delta_2^2\neq 0} \frac{\omega_{1,1}(\delta_1,\delta_2;x)}{\Omega_1(\delta_1)\Omega_2(\delta_2)} \leq c_2 .$$

Here c_1 and c_2 are the some constants; $\omega_{1,1}$ denotes the mixed modulus of continuity of the second order [2]. In the space H_ω the norm is defined as:

$$\|x(t,\tau)\|_{H_\omega} = \|x(t,\tau)\|_C + H(x;\omega) + H^{t\tau}(x;\omega).$$

Besides, we'll consider the other kinds of moduli of continuity (such as full, mixed and partial ones) and constants $H^t(x;\Omega_1) = \sup_{\delta_1>0} \frac{\omega(\delta_1,0;x)}{\Omega_1(\delta_1)}$, $H^\tau(x;\Omega_2) = \sup_{\delta_2>0} \frac{\omega(0,\delta_2;x)}{\Omega_2(\delta_2)}$, which are defined in [3].

Let

$$(L_{mn}g)(t,\tau) = \sum_{p=0}^{2m}\sum_{q=0}^{2n} g(t_p,\tau_q) l_p(t) l_q(\tau) = \sum_{k=-m}^{m}\sum_{l=-n}^{n} \lambda_{kl} t^k \tau^l ,$$

$$l_p(t) = \prod_{\substack{k=0\\ k\neq p}}^{2m} \frac{t-t_k}{t_p-t_k}\left(\frac{t_p}{t}\right)^m = \sum_{k=-m}^{m} \lambda_{1,k}^{(p)} t^k ,$$

$$l_q(\tau) = \prod_{\substack{j=0\\ j\neq q}}^{2n} \frac{\tau-\tau_j}{\tau_q-\tau_j}\left(\frac{\tau_q}{\tau}\right)^n = \sum_{j=-n}^{n} \lambda_{2,j}^{(q)} \tau^j ,$$

denotes interpolational Lagrange polynomial of the function $g(t,\tau)$ with respect to the system of Fejér points $(t_p;\ \tau_q)$, $p = 0,\ldots,2m$, $q = 0,\ldots,2n$ at the sceleton γ.

Theorem 1. *Let $x(t,\tau) \in H_\omega$, the points $\{t_p\}_{p=0}^{2m}$, $\{\tau_q\}_{q=0}^{2n}$ form the system of Fejér points at the sceleton γ and $(L_{mn}x)(t,\tau)$ is the interpolational Lagrange polynomial which is formed with respect to this system of points. Then the estimation is true:*

$$\|x - L_{mn}x\|_C \leq c\ln m \ln n H(x;\omega)\omega\left(\frac{1}{m},\frac{1}{n}\right).$$

Let us denote by F_{mn} the operator which assignes to any twice continuously differentiable function $x(t,\tau)$ at sceleton γ (whose partial derivatives $\frac{\partial x}{\partial t}$, $\frac{\partial x}{\partial \tau}$,

$\frac{\partial^2 x}{\partial t \partial \tau}$ belong to the space H_ω) the mn-th partial sum of its double Faber – Laurent series.

Theorem 2. *For any continuously differentiable function* $x(t,\tau)$, *the partial derivatievs of which* $\frac{\partial x}{\partial t}$, $\frac{\partial x}{\partial \tau}$, $\frac{\partial^2 x}{\partial t \partial \tau}$ *belong to the space* H_ω, *and for arbitrary natural* m, n *the unequality is fulfilled:*

$$\|x - F_{mn}x\|_C \leq c(lnm + lnn)H(x;\omega)\omega\left(\frac{1}{m},\frac{1}{n}\right).$$

In stating approximative properties of the concrete approximative apparatus at Lyapunov sceleton γ the analogue of Bernstein unequality is used essentially to estimate derivatives of polynomials.

Note also that for arbitrary Lyapunov contour Γ the constant d exists which depends on Γ only such that $s(t_1,t_2) \leq d|t_1 - t_2|$, where t_1, t_2 are arbitrary points at contour Γ, and $s(t_1,t_2)$ is a length of the arc $t_1 \cup t_2$.

With regard to the mentioned remarks all proofs of statements which are formulated in [1] remain true for arbitrary Lyapunov sceleton γ .

Theorem 3. *Let* $x(t,\tau) \in H_{\omega^{(1)}}$; $\omega^{(2)}(\delta_1,\delta_2)$ *is such that* $H_{\omega^{(1)}} \subset H_{\omega^{(2)}}$ *and* $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ *are increasing functions. Then the estimation is true:*

$$\|x - L_{mn}x\|_{H_{\omega^{(2)}}} \leq clnmlnn \frac{\omega^{(1)}\left(\frac{1}{m},\frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)}\|x\|_{H_{\omega^{(1)}}}.$$

Theorem 4. *Let function* $x(t,\tau)$ *be continuously differentiable at* γ *and its partial derivatives* $\frac{\partial x}{\partial t}$, $\frac{\partial x}{\partial \tau}$, $\frac{\partial^2 x}{\partial t \partial \tau}$ *belong to hte space* $H_{\omega^{(1)}}$; $\omega^{(2)}(\delta_1,\delta_2)$ *is such that* $H_{\omega^{(1)}} \subset H_{\omega^{(2)}}$ *and* $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ *are increasing functions. Then for Faber – Laurent operator the estimate is true:*

$$\|x - F_{mn}x\|_{H_{\omega^{(2)}}} \leq clnmn \frac{\omega^{(1)}\left(\frac{1}{m},\frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)}\|x\|_{H_{\omega^{(1)}}}.$$

Les us state now the estimations of approximations of the functions from the space H_ω at γ in the space L_p, $p > 1$.

Say to the smooth simple closed contour Γ then Γ belongs to the class $C_{(r,\Omega)}$ (where r is an integer non-negative number and $\Omega(\delta)$ some modulus of continuity) if the Riemann function $\psi(w)$ is continuously differentiable r times in the domain $\{w : |w| > 1\}$, such that $\psi^{(r)}(w)$ satisfies the condition

$$\left|\psi^{(r)}(w') - \psi^{(r)}(w'')\right| \le c\Omega(|w' - w''|)$$

for arbitrary $w', w'' \in \gamma_0$, that is $\psi(w) \in H_\Omega^{(r)}$ at Γ.

Let $C_{(r,\omega)}$ denotes the class of sceletons γ such that $\gamma = \gamma_1 \times \gamma_2$ and $\gamma_1 \in C_{(r,\Omega_1)}$, $\gamma_2 \in C_{(r,\Omega_2)}$, where $\Omega_1(\delta)$, $\Omega_2(\delta)$ are the simple moduli of continuity respecting to the modulus of continuity $\omega(\delta_1, \delta_2)$. Evidently, class of sceletons $C_{(r,\omega)}$ is contained in the class of Lyapunov sceletons.

Theorem 5. *Let* $\gamma \in C_{(2,\omega)}$ *. For the Lagrange operator set* $L_{mn} : C \to L_p$, $p > 1$*. Then the estimation is fulfilled:*

$$\| L_{mn} \|_{C \to L_p} \le c(p),$$

where $c(p)$ *is the completely defined constant which depends on* p *only.*

Theorem 6. *Let* $\gamma \in C_{(2,\omega)}$*, the points* $\{t_p\}_{p=0}^{2m}$, $\{\tau_q\}_{q=0}^{2n}$ *form the system of Fejér points at the sceleton* γ*. Then for any function* $g(t,\tau) \in H_{\omega^{(1)}}$ *at* γ *the estimation is true:*

$$\| g - L_{mn} g \|_{L_p} \le (1 + c(p)) H(g; \omega^{(1)}) \omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right).$$

Proof. Let $P_{mn}(t,\tau)$ be the polynomial of the best uniform approximation of the function $g(t,\tau)$. When it is remembered that L_{mn} is projector, estimation $\|\bullet\|_{L_p} \le \|\bullet\|_C$ is fulfilled and the result from Theorem 5 are stated, then we have following unequalities:

$$\| g - L_{mn} g \|_{L_p} \le \| g - P_{mn} \|_{L_p} + \| L_{mn}(g - P_{mn}) \|_{L_p} \le \left(1 + \| L_{mn} \|_{C \to L_p}\right) \| g - P_{mn} \|_C =$$

$$= \left(1 + \| L_{mn} \|_{C \to L_p}\right) E_{mn}(g) \le (1 + c(p)) H(g; \omega^{(1)}) \omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right).$$

From this unequality proof of Theorem 6 follows.

Remark 1. If under the conditions of Theorems 1 - 4, 6 the partial derivatives of the approximated functions of orders p and q with respest to t and τ belong to the space H_ω (it means $x(t,\tau) \in H_\omega^{(p,q)}$), that the estimations in the mentioned theorems can be considerably improved, namely by value $\frac{1}{m^p} + \frac{1}{n^q}$. This conclusion follows directly from the unequalities [1, Remark 1]. For example, in this case the theorem which respects to Theorem 3, can be formulated by the way.

Theorem 7. *Let* $x(t,\tau) \in H_{\omega^{(1)}}^{(p,q)}$; $\omega^{(2)}(\delta_1, \delta_2)$ *is such that* $H_{\omega^{(1)}} \subset H_{\omega^{(2)}}$ *and* $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ *are increasing functions.Then the estimation is true:*

$$\|x - L_{mn}x\|_{H_{\omega^{(2)}}} \le c \ln m \ln n \left(\frac{1}{m^p} + \frac{1}{n^q} \right) \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right) \Omega_2^{(2)}\left(\frac{1}{n}\right)} \left\| x^{(p,q)} \right\|_{H_{\omega^{(1)}}},$$

where the interpolational Lagrange polynomial is constructed with respect to the system of Fejér points at sceleton γ *.*

References

1. Snizhko, N. Approximation of functions of two variables on the unit bicircle in generalized Hölder spaces [Електронний ресурс] / N. Snizhko // Modern problems in science. Abstracts of VIII International Scientific and Practical Conference. Prague, Czech Republic, 2020. – 577–582 p.p. – Available at: DOI: 10.46299/ISG.2020.II.VIII – URL: https://isg-konf.com
2. Тиман А.Ф. Теория приближения функций действительного переменного. – М.: ГИФМЛ, 1965. – 624 с.
3. Сніжко Н.В. Класифікація узагальнених просторів Гельдера функцій двох змінних//Вісник Київського університету. Сер. фіз.-мат. науки. – 1999. – Вип № 3. – С. 124 – 128.