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THE APPLICATION OF LAPLACE-TRANSFORM METHOD TO FINDING CIRCUIT SOLUTIONS

The classical method of transient analysis requires, in the general case, the repeated manipulation of algebraic equations in order to elucidate constants of integration from initial conditions and the initial values of a function and its derivatives. This is the major drawback of the classical method.

As the differential equations describing the transient taking place in lumped-parameter linear circuits are linear equations with constant coefficients, they may be integrated by the operational method based on Laplace transform.

At a transient analysis by operational method it is convenient to write down the equations by any known method which is based on Kirchhoff's equations. Each of these combined equations can be written having made equivalent operational circuit for the rated electrical circuit. The definition of originals of required currents or voltages is used by going from the image domain to the time domain through the performing the inverse Laplace transformation.

Let's consider step response of second-order system (RLC circuit). Let it be desired to find the complete current response in the circuit of Fig. 1 after the switch S is closed. For simplification of the problem we consider that the capacitor has no initial charge before switching.

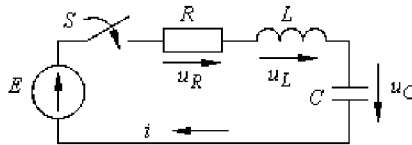


Figure 1 - RLC series circuit

Because the switch in Fig. 1 is initially open, it follows that $u_C(0) = 0$. Since we have a zero initial condition here the internal e.m.f. produced by the voltage across the capacitor is also equal to zero. Then we have the following operational circuit described in Fig. 2.

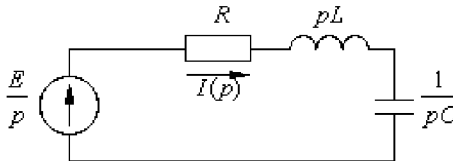


Figure 2 - Operational RLC circuit.

The governing differential equation for the circuit is found upon applying Kirchhoff's voltage law. Thus

$$E = Ri + L \frac{di}{dt} + \frac{1}{C} \int i dt . \quad (1)$$

Equation (1) is a second-order nonhomogeneous differential equation because it involves a derivative as well as an integral term. Laplace transforming each term of the equation yields

$$\frac{E}{p} = I(p) \cdot \left(R + pL + \frac{1}{pC} \right). \quad (2)$$

The function in brackets is the operational impedance of the series RLC circuit. The operational current in the p domain then readily becomes

$$I(p) = \frac{CE}{p^2LC + pRC + 1} = \frac{E/L}{\left(p^2 + p \frac{R}{L} + \frac{1}{LC} \right)}. \quad (3)$$

A glance at Eq. (3) shows that the denominator expression does not include an p factor standing alone. This means that a constant term in the steady-state solution does not exist. Accordingly, it involves each of the three circuit parameters. Moreover, the expression is quadratic because of the presence of two energy-storing elements. From the experience we have gained so far in evaluating the inverse Laplace transform using the Table of Laplace transform into a time function.

Before we can proceed further in the solution procedure for $i(t)$ in the situation now under consideration, we must first find the specific roots of the quadratic expression. That is, we must determine those values of p which satisfy the equation

$$R + pL + \frac{1}{pC} = 0. \quad (4)$$

It is called the *characteristic equation* of the second-order system. The roots of this equation, which are also the poles of $I(p)$ in this instance, depend solely upon the circuit parameters. If p_1 and p_2 are the roots of the characteristic equation the formal expression for the time solution becomes

$$i(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t}. \quad (5)$$

Thus the characteristic manner in which the transient terms decay is solely dependent on the roots p_1 and p_2 . The total solution in this case involves only a transient term because the forced solution is zero because it is always equal to zero in dc series RLC branches.

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