MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
National University Zaporizhzhia Polytechnic

CALCULATION TASKS ON HIGHER MATHEMATICS (2nd module)

for students majoring in
G3 Electrical Engineering

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INTRODUCTION

One of the types of unaided work of students is the execution of calculation tasks during the semester. Their purpose is to develop practical skills of students. They are designed to help students capture theoretical material more deeply and learn how to apply the acquired knowledge to solve practical problems. The offered typical calculation tasks correspond to the course "Higher Mathematics" taught to students majoring in G3 Electrical Engineering full-time in the second module of the two-semester course.

This task book covers virtually all major sections of the second module of the course on higher mathematics; it contains assignments for all the topics of the course. A list of recommended literature is also provided.

Calculation tasks are executed during the semester, which provides students with a systematic study of the course. While carrying out these tasks, students work with recommended textbooks and manuals, independently search for necessary literary sources and materials, analyze them and summarize, independently research and make written presentation of practical assignments.

The student chooses the option (the number of the variant) according to his number in the register list. The work is done in English in writing, preferably in a notebook in a cell. Note fields must be left blank. The name of the subject, major, group and course, surname, first name and patronymic of the student, name of the teacher who accepts the work should be indicated on the title page of the work.

While performing the work, the student should solve the offered tasks by the methods specified in the tasks, as well as make all the necessary drawings (graphical solutions). The student must show the acquired theoretical knowledge of the course.

When doing the work, the student can use both the lecture and practical material, as well as the supporting literature listed at the end of this manual. Before starting the work, it is recommended to study the relevant theoretical material, then to understand the solutions of the tasks that were performed in the practical classes, and only after that to start the actual calculation work.

When evaluating a work, the indicator of its quality is, first of all, how the student independently and correctly solved the tasks and understood the content of the obtained solutions. That is why additional

questions may be asked to protect the student's work, including the theoretical material presented for the exam.

Completed work is submitted to the teacher for verification and subsequent protection in the form of an interview (usually during modular control). The student must be able to:

– present the content of the tasks and to prove their solutions;
– answer questions about the content of the solutions obtained;
– answer additional questions.

If the work is successfully protected, the student receives a certain number of rating points. If the specified requirements are not fulfilled, then the work is returned to the student for completion, indicating the term of re-protection.

1 INTEGRATION OF FUNCTIONS OF A SINGLE VARIABLE

Task 1.1. Evaluate the following indefinite integrals.

1. a) $\int \sin(5x-4)dx$;

   b) $\int \frac{xdx}{2x^2-7}$;

   c) $\int \frac{2x-1}{x^2-4x-12}dx$;

   d) $\int (6x+5)e^{2x}dx$;

   e) $\int \ln(3x)dx$;

   f) $\int \frac{x^2-3x-4}{x(x^2+4x+4)}dx$;

   g) $\int \frac{\sqrt{x}}{x-2\sqrt{x}-3}dx$;

   h) $\int \frac{dx}{4+5\cos x}$;

   i) $\int \sin^3 2x dx$.

2. a) $\int e^{2-3x}dx$;

   b) $\int \frac{\tan^3 x dx}{\cos^2 x}$;

   c) $\int \frac{x-6}{\sqrt{8-2x-x^2}}dx$;

   d) $\int (9x-4)\sin 3x dx$;

   e) $\int \arctan 2x dx$;

   f) $\int \frac{9x-28}{(x-4)(x^2-16)}dx$;

   g) $\int \frac{\sqrt[4]{x+1}}{\sqrt{x+1}+2}dx$;

   h) $\int \frac{\sin x}{1+\sin x}dx$;

   i) $\int \sin 3x \cdot \cos 5x\, dx$.

3. a) $\int 5^{7x+5}dx$;

   b) $\int x^2 e^{x^3}dx$;

   c) $\int \frac{3x-8}{x^2+6x+25}dx$;

   d) $\int (4x+7)\cos 4x dx$;

   e) $\int \arcsin 5x dx$;

   f) $\int \frac{2x^2-57}{(x+2)(x^2-10x+25)}dx$;

   g) $\int \frac{1}{x-2\sqrt{x}+1}dx$;

   h) $\int \frac{dx}{4-5\sin x}$;

   i) $\int \sin^2 3x\, dx$.

4. a) $\int \cos(2x+7)dx$;

b) $\int \frac{dx}{(x-3)\ln^2(x-3)}$;

c) $\int \frac{2x+3}{\sqrt{x^2-4x+13}}dx$;

d) $\int (7x-4)e^{5x}dx$;

e) $\int \arccos 4x dx$;

f) $\int \frac{x^2+13x+24}{(x+3)(x^2-9)}dx$;

g) $\int \frac{\sqrt[4]{x+2}}{\sqrt{x+2}+2\sqrt[4]{x+2}}dx$;

h) $\int \frac{dx}{3\cos x+4\sin x}$;

i) $\int \cos^3 2x\,dx$.

5. a) $\int \frac{dx}{4x-5}$;

b) $\int \frac{dx}{\sin^2 x\cot^4 x}$;

c) $\int \frac{3x-4}{x^2+10x-11}dx$;

d) $\int (4x+5)\sin 2x dx$;

e) $\int \arctan 3x dx$;

f) $\int \frac{x^2-3x+1}{(x-2)(x^2-2x+1)}dx$;

g) $\int \frac{\sqrt{x+3}}{\sqrt{x+3}+2}dx$;

h) $\int \frac{dx}{3\cos x+5}$;

i) $\int \sin 4x\cdot\sin 2x\,dx$.

6. a) $\int \frac{dx}{\sqrt{7x+3}}$;

b) $\int \frac{xdx}{\sqrt{3x^2+5}}$;

c) $\int \frac{2x+1}{\sqrt{x^2-2x-15}}dx$;

d) $\int (2x-3)\cos 3x dx$;

e) $\int \arcsin 2x dx$;

f) $\int \frac{x^2+5x-10}{(x-2)(x^2-4)}dx$;

g) $\int \frac{\sqrt{x}}{x-5\sqrt{x}+6}dx$;

h) $\int \frac{dx}{5\cos x-12\sin x}$;

i) $\int \cos^2 3x\,dx$.

7. a) $\int \frac{dx}{\cos^2 4x}$;

f) $\int \frac{7x+39}{(x+1)(x^2+10x+25)}dx$;

b) $\int \frac{\arctan^3 x dx}{x^2+1}$;

c) $\int \frac{3x+1}{x^2-6x+5} dx$;

d) $\int (3x+5)e^{4x} dx$;

e) $\int \arccos 5x dx$;

g) $\int \frac{1}{x-2\sqrt{x}} dx$;

h) $\int \frac{dx}{5-3\cos x}$;

i) $\int \sin^3 2x \cdot \cos^2 2x\, dx$.

8. a) $\int \sin(8x+5) dx$;

b) $\int \frac{\cos x dx}{\sin^4 x}$;

c) $\int \frac{2x+5}{\sqrt{7-6x-x^2}} dx$;

d) $\int (8x-3)\sin 6x dx$;

e) $\int \arctan 7x dx$;

f) $\int \frac{5x+3}{(x+1)(x^2-1)} dx$;

g) $\int \frac{\sqrt{x-5}}{\sqrt{x-5}+3} dx$;

h) $\int \frac{dx}{4\cos x - 3\sin x}$;

i) $\int \cos 2x \cdot \cos 3x\, dx$.

9. a) $\int e^{4x+9} dx$;

b) $\int \frac{dx}{\sqrt{1-x^2}\sqrt{\arcsin x}}$;

c) $\int \frac{3x-2}{x^2+8x+25} dx$;

d) $\int (5x+1)\cos 7x dx$;

e) $\int \arcsin 6x dx$;

f) $\int \frac{x^2-4x+5}{(x-1)(x^2-4x+4)} dx$;

g) $\int \frac{\sqrt{x}}{x-3\sqrt{x}-4} dx$;

h) $\int \frac{dx}{5+4\cos x}$;

i) $\int \sin^2 4x\, dx$.

10. a) $\int 4^{5-6x} dx$;

b) $\int x e^{3x^2+5} dx$;

f) $\int \frac{2x^2+9x+5}{(x+5)(x^2-25)} dx$;

g) $\int \frac{\sqrt[4]{x+3}}{\sqrt{x+3}-4} dx$;

c) $\int \frac{2x-1}{\sqrt{x^2+6x+25}}\,dx$;

d) $\int (4x-7)e^{6x}dx$;

e) $\int \ln(2x)dx$;

h) $\int \frac{dx}{5\cos x+12\sin x}$;

i) $\int \cos^3 3x\cdot\sin^2 3x\,dx$;

11. a) $\int \cos(9x-4)dx$;

b) $\int \frac{xdx}{3x^2-7}$;

c) $\int \frac{x+3}{x^2-10x+21}\,dx$;

d) $\int (2x+1)\sin 8xdx$;

e) $\int \arctan 9xdx$;

f) $\int \frac{x^2+4x+2}{x(x^2+2x+1)}dx$;

g) $\int \frac{\sqrt{x+4}}{\sqrt{x+4}-3}dx$;

h) $\int \frac{dx}{4+5\sin x}$;

i) $\int \sin^4 3x\,dx$.

12. a) $\int \frac{dx}{3-5x}$;

b) $\int \cos^5 x\sin x dx$;

c) $\int \frac{2x-3}{\sqrt{21+4x-x^2}}\,dx$;

d) $\int (3x-4)\cos 9xdx$;

e) $\int \arcsin 8xdx$;

f) $\int \frac{x^2+7x-24}{(x^2-9)(x-3)}dx$;

g) $\int \frac{\sqrt{x}}{x-7\sqrt{x}+12}dx$;

h) $\int \frac{dx}{4\sin x-3\cos x}$;

i) $\int \cos 2x\cdot\sin 2x\,dx$.

13. a) $\int \frac{dx}{\sqrt{3x-7}}$;

b) $\int \frac{dx}{(x^2+1)\arctan^5 x}$;

c) $\int \frac{3x-5}{x^2-12x-13}\,dx$;

f) $\int \frac{x^2+7x+4}{(x+3)(x^2+4x+4)}dx$;

g) $\int \frac{\sqrt[4]{x-1}}{\sqrt{x-1}+4}dx$;

h) $\int \frac{dx}{12\cos x+5\sin x}$;

d) $\int (2x+7)e^{8x}dx;$ i) $\int \cos^2 \frac{3x}{2}dx.$

e) $\int \arccos 9x dx;$

14. a) $\int \frac{dx}{\sin^2 7x};$ f) $\int \frac{x^2+x-4}{(x-1)(x^2-1)}dx;$

b) $\int \frac{dx}{(x+5)\ln^2(x+5)};$ g) $\int \frac{\sqrt{x+5}}{\sqrt{x+5}-7}dx;$

c) $\int \frac{5x+3}{\sqrt{x^2-4x-12}}dx;$ h) $\int \frac{dx}{4-5\cos x};$

d) $\int (5x-2)\sin 4x dx;$ i) $\int \cos^4 \frac{x}{2}dx.$

e) $\int \arctan 5x dx;$

15. a) $\int e^{2x-5}dx;$ f) $\int \frac{2x^2+9x+10}{(x-2)(x^2+8x+16)}dx;$

b) $\int \frac{xdx}{\sqrt{5x^2+8}};$ g) $\int \frac{\sqrt{x}}{x+5\sqrt{x}-6}dx;$

c) $\int \frac{2x+7}{x^2-2x-15}dx;$ h) $\int \frac{dx}{4\sin x+3\cos x};$

d) $\int (4x-7)\cos 5x dx;$ i) $\int \sin 3x \cdot \sin 5x\, dx.$

e) $\int \ln 4x dx;$

Task 1.2. Evaluate the definite integral.

1. $\int_0^4 \frac{xdx}{\sqrt{x^2+9}}.$ 2. $\int_0^{\frac{\pi}{2}} 4\sin^3 x \cos x\, dx.$

3. $\int_1^e \frac{dx}{x(1+\ln^2 x)}.$ 4. $\int_1^3 \frac{(x+6)dx}{\sqrt{x^2+12x-9}}.$

5. $\int\limits_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\tan^3 x dx}{\cos^2 x}$.

6. $\int\limits_0^{\ln\sqrt[4]{3}} \frac{e^{2x}dx}{1+e^{4x}}$.

7. $\int\limits_{e^{\frac{\pi}{2}}}^{1} \frac{\sin(\ln x)dx}{x}$.

8. $\int\limits_0^{\sqrt[3]{12}} \frac{x^3 dx}{\sqrt{x^4+4}}$.

9. $\int\limits_0^{\frac{\pi}{2}} 5\sin x\cos^4 x\, dx$.

10. $\int\limits_e^{e^e} \frac{dx}{x\ln x}$.

11. $\int\limits_1^{\sqrt{3}} \frac{24\arctan^3 x dx}{1+x^2}$.

12. $\int\limits_0^{\frac{\sqrt{3}}{2}} \frac{e^{\arcsin x}dx}{\sqrt{1-x^2}}$.

13. $\int\limits_0^{\sqrt{\pi}} x\cos\left(\frac{x^2}{2}\right)dx$.

14. $\int\limits_0^{\ln 6} e^x\sqrt{10-e^x}\,dx$.

15. $\int\limits_0^{\frac{\sqrt{\pi}}{2}} x\sin\left(2x^2\right)$.

Task 1.3. Evaluate the improper integrals or determine its divergence.

1. a) $\int\limits_{-1}^{\infty} \frac{dx}{9x^2+6x+5}$; b) $\int\limits_1^2 \frac{dx}{\sqrt{(2-x)^3}}$.

2. a) $\int\limits_{-2}^{\infty} xe^{4-x^2}dx$; b) $\int\limits_{-5}^{0} \frac{dx}{\sqrt{4+x}}$.

3. a) $\int\limits_1^{\infty} \frac{\arctan x dx}{1+x^2}$; b) $\int\limits_{-1}^{0} \frac{dx}{x+1}$.

4. a) $\int_{2}^{\infty} \frac{x^3 dx}{7+x^4}$; b) $\int_{3}^{11} \frac{dx}{\sqrt[3]{x-3}}$.

5. a) $\int_{\sqrt{\pi}}^{\infty} \frac{2x dx}{16+x^4}$; b) $\int_{-3}^{1} \frac{dx}{\sqrt[4]{(x+3)^3}}$.

6. a) $\int_{e}^{\infty} \frac{dx}{x \ln^3 x}$; b) $\int_{0}^{1} \frac{dx}{\sqrt[3]{(x-1)^4}}$.

7. a) $\int_{0}^{\infty} \frac{(3x+7)dx}{3x^2+14x+e}$; b) $\int_{\frac{3}{2}}^{1} \frac{dx}{3-2x}$.

8. a) $\int_{3}^{\infty} \frac{dx}{x^2-3x+2}$; b) $\int_{5}^{6} \frac{dx}{\sqrt{(x-5)^3}}$.

9. a) $\int_{0}^{\infty} \frac{e^x dx}{2+e^x}$; b) $\int_{1}^{3} \frac{dx}{\sqrt[3]{(x-1)^2}}$.

10. a) $\int_{-3}^{\infty} \frac{dx}{\sqrt{16+x^2}}$; b) $\int_{-2}^{30} \frac{dx}{\sqrt[5]{x+2}}$.

11. a) $\int_{0}^{\infty} \frac{x dx}{4-x^4}$; b) $\int_{\frac{3}{4}}^{1} \frac{dx}{4x-3}$.

12. a) $\int_{2}^{\infty} \frac{\ln(x-1)dx}{x-1}$; b) $\int_{-1}^{0} \frac{dx}{\sqrt{(1+x)^5}}$.

13. a) $\int_{3}^{\infty} \frac{dx}{x^2+4x+4}$; b) $\int_{1}^{2} \frac{dx}{\sqrt[5]{(x-1)^2}}$.

14. a) $\int_{-1}^{\infty} \frac{(3x^2+1)dx}{e^{x^3+x+1}}$; b) $\int_{7}^{15} \frac{dx}{\sqrt[3]{x-7}}$.

15. a) $\int_{e}^{\infty} \frac{dx}{x(1+\ln^2 x)}$; b) $\int_{-1}^{0} \frac{dx}{\sqrt[4]{(1+x)^3}}$.

Task 1.4. Find the area of the figure bounded by the curves.

1. $y = x^2 - 4,\ y = 2x + 4.$
2. $y = x^2 + 1,\ y = 3 - x,\ y = 1.$
3. $y = x^2,\ y = \frac{1}{x},\ y = 0,\ x = e.$
4. $y = \sin x,\ y = 1,\ x = 0 (x \geq 0).$
5. $y = x^2 - 1,\ y = 2 - 2x,\ x = 0 (x \geq 0).$
6. $y = x - 1,\ y = 1 - x,\ x = 0.$
7. $y = \sqrt{x+1},\ y = \frac{1}{2}\sqrt{4-x}, y = 0.$
8. $y = x^2,\ y = \sqrt{x}.$
9. $y = e^x,\ y = \frac{x}{2},\ x = 0,\ x = 1.$
10. $y = \frac{1}{x},\ y = x,\ x = 2.$
11. $y = (x-1)^2,\ y = 1 - x^2.$
12. $y = \cos x,\ y = -\frac{2}{\pi}x + 1.$
13. $y = \frac{1}{x^2},\ y = \sqrt{x},\ x = 4.$
14. $y = x^2 + 2,\ y = 2x,\ x = 0,\ x = 1.$
15. $y = e^x - 1,\ y = e,\ x = 0.$

Task 1.5. Find the arc length of the curve between the given points or the given values of *t*.

1. $x = 2\cos^3 t,\ y = 2\sin^3 t,\ \frac{\pi}{4} \leq t \leq \pi.$
2. $y = x^2 + 1,\ 0 \leq x \leq 2.$

3. $x = 3(1-\cos t),\ y = 3(1-\sin t),\ 0 \le t \le \frac{\pi}{4}$.

4. $y = \ln(\cos x) - 1,\ 0 \le x \le \frac{\pi}{3}$.

5. $x = \frac{1}{2}(t^2 - 1),\ y = t - 3,\ 0 \le t \le 1$.

6. $y = 2(1 + \ln(x^2 - 4)),\ 3 \le x \le 4$.

7. $x = 3\cos^2 t,\ y = 3\sin^2 t,\ 0 \le t \le \frac{\pi}{4}$.

8. $y = \ln(\sin x) + 3,\ \frac{\pi}{3} \le x \le \frac{\pi}{2}$.

9. $x = 2(1-\cos t),\ y = 2(1+\sin t),\ \frac{\pi}{3} \le t \le \frac{\pi}{2}$.

10. $y = e^x + 11,\ \ln\sqrt{3} \le x \le \ln\sqrt{8}$.

11. $x = 5\cos t,\ y = 5\sin t,\ \frac{\pi}{4} \le t \le \frac{3\pi}{4}$.

12. $y = 3(1 + \ln(x^2 - 9)),\ 4 \le x \le 5$.

13. $x = 4t + 5,\ x = 2(t^2 + 3),\ 0 \le t \le 2$.

14. $y = e^x - 2,\ \ln\sqrt{8} \le x \le \ln\sqrt{15}$.

15. $x = 4\cos^3 t,\ y = 4\sin^3 t,\ \frac{\pi}{3} \le t \le \frac{\pi}{2}$.

Task 1.6. The plane area is bounded by the given curves. Find the volume generated when this area is revolved about the indicated axis.

1. $y = 2\cos x,\ x = \frac{\pi}{6},\ x = \frac{\pi}{4},\ Ox$.
2. $y = \sin 2x,\ x = 0,\ y = 1,\ Oy$.
3. $y = 4 - (x-3)^2,\ y = 0,\ Ox$.
4. $x^2 + y^2 = 9,\ x = 0,\ y = 0,\ Oy$.
5. $y = \frac{2}{x},\ x = 1,\ x = e,\ Ox$.

6. $y=\sqrt{2+x}$, $x=0$, $y=0$, Oy.
7. $y=x^3-1$, $x=2$, $y=0$, Ox.
8. $\frac{x^2}{1}+\frac{y^2}{4}=1$, $x=0$, $y=0$, Oy.
9. $y=x$, $y=2-x$, $y=0$, Ox.
10. $\sqrt{y}=x$, $x=0$, $y=9$, Oy.
11. $y=e^x$, $x=\ln 2$, $x=\ln 5$, Ox.
12. $y=\sqrt{x}$, $x=0$, $y=2$, Oy.
13. $y=\frac{1}{\sqrt{x}}$, $x=1$, $x=\ln 3$, Ox.
14. $y=\cos 2x$, $y=0$, Oy.
15. $y=x^3+1$, $x=1$, $y=0$, Ox.

2 MULTIPLE INTEGRALS, LINE INTEGRALS AND SURFACE INTEGRALS

Task 2.1. Evaluate the double integral.

1. $\iint\limits_D \left(x^2+4y\right)dxdy, \qquad D:\begin{cases}0\le x\le 3,\\ 2\le y\le 4.\end{cases}$
2. $\iint\limits_D \left(2x-3y^2\right)dxdy, \qquad D:\begin{cases}2\le x\le 3,\\ 0\le y\le 2.\end{cases}$
3. $\iint\limits_D \left(8x^3-y\right)dxdy, \qquad D:\begin{cases}0\le x\le 2,\\ 4\le y\le 6.\end{cases}$
4. $\iint\limits_D \left(x+6y^2\right)dxdy, \qquad D:\begin{cases}2\le x\le 4,\\ 0\le y\le 3.\end{cases}$
5. $\iint\limits_D \left(4x^3+2y\right)dxdy, \qquad D:\begin{cases}0\le x\le 2,\\ 2\le y\le 3.\end{cases}$
6. $\iint\limits_D \left(5x-4y^3\right)dxdy, \qquad D:\begin{cases}2\le x\le 3,\\ 0\le y\le 2.\end{cases}$
7. $\iint\limits_D \left(x^2-4y\right)dxdy, \qquad D:\begin{cases}0\le x\le 3,\\ 2\le y\le 3.\end{cases}$
8. $\iint\limits_D \left(2x+5y^4\right)dxdy, \qquad D:\begin{cases}2\le x\le 4,\\ 0\le y\le 2.\end{cases}$
9. $\iint\limits_D \left(3x^2+2y\right)dxdy, \qquad D:\begin{cases}0\le x\le 2,\\ 1\le y\le 4.\end{cases}$
10. $\iint\limits_D \left(6x-5y^3\right)dxdy, \qquad D:\begin{cases}1\le x\le 3,\\ 0\le y\le 2.\end{cases}$
11. $\iint\limits_D \left(8x^3-y\right)dxdy, \qquad D:\begin{cases}0\le x\le 1,\\ 2\le y\le 4.\end{cases}$

12. $\iint\limits_D \left(x+7y^6\right)dxdy, \qquad D:\begin{cases}2\le x\le 6,\\ 0\le y\le 1.\end{cases}$

13. $\iint\limits_D \left(2x^2+3y\right)dxdy, \qquad D:\begin{cases}0\le x\le 3,\\ 2\le y\le 4.\end{cases}$

14. $\iint\limits_D \left(3x-10y^4\right)dxdy, \qquad D:\begin{cases}2\le x\le 4,\\ 0\le y\le 1.\end{cases}$

15. $\iint\limits_D \left(5x+12y^5\right)dxdy, \qquad D:\begin{cases}0\le x\le 2,\\ 1\le y\le 2.\end{cases}$

Task 2.2. Using the polar coordinates, compute the double integral. Find the area of the domain D.

1. $\iint\limits_D 2\cos\left(x^2+y^2\right)dxdy, \qquad D:\begin{cases}x^2+y^2=9,\\ y\ge 0,\ y\le \sqrt{3}x.\end{cases}$

2. $\iint\limits_D \frac{dxdy}{\sqrt{5+x^2+y^2}}, \qquad D:\begin{cases}x^2+y^2=4,\\ y\ge 0,\ x\le 0.\end{cases}$

3. $\iint\limits_D e^{-\left(x^2+y^2\right)}dxdy, \qquad D:\begin{cases}x^2+y^2=1,\\ y\ge 0,\ y\ge \frac{\sqrt{3}}{3}x.\end{cases}$

4. $\iint\limits_D \sqrt{x^2+y^2}\,dxdy, \qquad D:\begin{cases}x^2+y^2\ge 2y,\\ x^2+y^2\le 4y.\end{cases}$

5. $\iint\limits_D \frac{dxdy}{\sqrt{x^2+y^2}\sin^2\sqrt{x^2+y^2}}, \qquad D:\begin{cases}x^2+y^2=9,\\ x\ge 0,\ y\ge \sqrt{3}x.\end{cases}$

6. $\iint\limits_D \sqrt{16+x^2+y^2}\,dxdy, \qquad D:\begin{cases}x^2+y^2=9,\\ y\ge x,\ y\ge -x.\end{cases}$

7. $\iint\limits_D \frac{dxdy}{e^{x^2+y^2}}, \qquad D:\begin{cases}x^2+y^2=4,\\ y\ge 0,\ y\le x.\end{cases}$

8. $\iint\limits_D \frac{dxdy}{4+x^2+y^2}$, $D: \begin{cases} x^2+y^2=1, \\ y \le \frac{\sqrt{3}}{3}x,\ y \ge -\sqrt{3}\,x. \end{cases}$

9. $\iint\limits_D \frac{6xy}{\sqrt{x^2+y^2}}dxdy$, $D: \begin{cases} x^2+y^2=4, \\ x \ge 0. \end{cases}$

10. $\iint\limits_D \frac{\ln\sqrt{x^2+y^2}}{x^2+y^2}dxdy$, $D: \begin{cases} x^2+y^2 \ge 1, \\ x^2+y^2 \le 4, \\ y \ge 0. \end{cases}$

11. $\iint\limits_D \frac{dxdy}{\sqrt{9+x^2+y^2}}$, $D: \begin{cases} x^2+y^2=4, \\ y \ge \sqrt{3}x,\ y \ge -x. \end{cases}$

12. $\iint\limits_D \sqrt{10-x^2-y^2}\,dxdy$, $D: \begin{cases} x^2+y^2=9, \\ y \ge 0,\ y \ge x. \end{cases}$

13. $\iint\limits_D \frac{4xy}{x^2+y^2}dxdy$, $D: \begin{cases} x^2+y^2=2y, \\ x \ge 0. \end{cases}$

14. $\iint\limits_D \frac{dxdy}{\sqrt{x^2+y^2}}$, $D: \begin{cases} x^2+y^2=2x, \\ x^2+y^2=6x. \end{cases}$

15. $\iint\limits_D \sqrt{3+x^2+y^2}\,dxdy$, $D: \begin{cases} x^2+y^2=1, \\ x \ge 0, y \le x. \end{cases}$

Task 2.3. Find the mass of the plate D bounded by the given curves. Its mass density is given by the function $\gamma = \gamma(x, y)$.

1. D: $y=\sqrt{x}$, $y=\sqrt{2-x}$, $y=0$, $\gamma=2xy$.
2. D: $y=2-x^2$, $y=-x$, $\gamma=4-2x$.
3. D: $y=\sqrt{x}$, $y=0$, $y=2-x$, $\gamma=2xy$.
4. D: $y=x^3$, $y=4x$, $y \ge 0$, $\gamma=2y-x$.
5. D: $y=x-1$, $y=3x-3$, $x=1$, $x=3$, $\gamma=4xy$.

6. $D:\ y=\sqrt{x},\ yx=1,\ x=4,\ \gamma=5x-4y$.
7. $D:\ y=\sqrt{x-1},\ y=\sqrt{-x-1},\ y=0,\ y=2,\ \gamma=4xy$.
8. $D:\ y=x^3-1,\ y=-x^2+1,\ x\ge 0,\ \gamma=x-2y$.
9. $D:\ y=4-x^2,\ y=x^2-2x,\ \gamma=x-2y$.
10. $D:\ yx=4,\ y=\sqrt{x-3},\ x=3,\ \gamma=4yx^2$.
11. $D:\ y=\sqrt{4-x},\ x=y^2+2,\ y=0,\ \gamma=6x^2y$.
12. $D:\ x+y=4,\ y=3x,\ x=0,\ \gamma=2x-4y$.
13. $D:\ y=x^2+4x+5,\ y=-x^2+5,\ \gamma=4x^3$.
14. $D:\ y=\dfrac{x}{4},\ y=\sqrt[3]{x},\ \gamma=3y^2-4x$.
15. $D:\ y=x^2-2x+4,\ y=x,\ x=1,\ x=3,\ \gamma=2x$.

Task 2.4. Find the volume of the body bounded by the given surfaces.

1. $G:\begin{cases} x^2+y^2=4, \\ z=5-x^2-y^2, \\ z=0. \end{cases}$
2. $G:\begin{cases} z=\sqrt{x^2+y^2}, \\ x^2+y^2=9, \\ z=0. \end{cases}$
3. $G:\begin{cases} z\le 3+x^2+y^2, \\ x^2+y^2=1, \\ z\ge 0. \end{cases}$
4. $G:\begin{cases} z\le x^2+y^2, \\ x^2+y^2=2y, \\ z\ge 0. \end{cases}$
5. $G:\begin{cases} z=\sqrt{x^2+y^2}, \\ z=2-x^2-y^2. \end{cases}$
6. $G:\begin{cases} z\le x^2+y^2, \\ x^2+y^2=2x, \\ z\ge 0. \end{cases}$
7. $G:\begin{cases} z\le x^2+y^2, \\ x^2+y^2=4y, \\ z\ge 0. \end{cases}$
8. $G:\begin{cases} z=1-x^2, \\ y\ge 0,\ y\le 1, \\ z\ge 0. \end{cases}$

9. $G:\begin{cases} z \le 10 - x^2 - y^2, \\ x^2 + y^2 = 1, \\ z \ge 0. \end{cases}$

10. $G:\begin{cases} z = \sqrt{x^2 + y^2}, \\ x^2 + y^2 = 4, \\ y \ge 0. \end{cases}$

11. $G:\begin{cases} z \le x^2 + y^2, \\ x^2 + y^2 = 6y, \\ z \ge 0. \end{cases}$

12. $G:\begin{cases} z = \sqrt{x^2 + y^2}, \\ z = 6 - x^2 - y^2. \end{cases}$

13. $G:\begin{cases} z = 4 - x^2 - y^2, \\ x + y = 1, \\ y \ge 0. \end{cases}$

14. $G:\begin{cases} z \le 25 - x^2 - y^2, \\ x^2 + y^2 = 16, \\ z \ge 0. \end{cases}$

15. $G:\begin{cases} z = 4 - x^2, \\ y \ge 0,\ y \le 2, \\ z \ge 0. \end{cases}$

Task 2.5. Compute the triple integral.

1. $\iiint\limits_G (2x - y + 2z)\,dxdydz, \quad G:\begin{cases} 0 \le x \le 2, \\ -1 \le y \le 4, \\ 0 \le z \le 2. \end{cases}$

2. $\iiint\limits_G 2z\,(x - 2y)\,dxdydz, \quad G:\begin{cases} 0 \le x \le 2, \\ 1 \le y \le 2, \\ -1 \le z \le 3. \end{cases}$

3. $\iiint\limits_G (2xz - y)dxdydz, \quad G:\begin{cases} 2 \le x \le 4, \\ 0 \le y \le 2, \\ -1 \le z \le 1. \end{cases}$

4. $\iiint\limits_G (x - 2yz)\,dxdydz, \quad G:\begin{cases} -2 \le x \le 4, \\ 1 \le y \le 2, \\ 0 \le z \le 5. \end{cases}$

5. $\iiint\limits_G (2z - xy)\,dxdydz, \qquad G: \begin{cases} 0 \le x \le 2, \\ 1 \le y \le 3, \\ -1 \le z \le 1. \end{cases}$

6. $\iiint\limits_G (6yz^2 + x)\,dxdydz, \qquad G: \begin{cases} 2 \le x \le 4, \\ -1 \le y \le 1, \\ 0 \le z \le 1. \end{cases}$

7. $\iiint\limits_G (3xy + 2z)\,dxdydz, \qquad G: \begin{cases} -1 \le x \le 2, \\ 0 \le y \le 4, \\ 1 \le z \le 3. \end{cases}$

8. $\iiint\limits_G (2y - 3xz^2)\,dxdydz, \qquad G: \begin{cases} 2 \le x \le 3, \\ -1 \le y \le 1, \\ 0 \le z \le 2. \end{cases}$

9. $\iiint\limits_G (3x + 2y^2 z)\,dxdydz, \qquad G: \begin{cases} 1 \le x \le 2, \\ -1 \le y \le 2, \\ 0 \le z \le 4. \end{cases}$

10. $\iiint\limits_G (2y - 4z + x)\,dxdydz, \qquad G: \begin{cases} 1 \le x \le 2, \\ 0 \le y \le 1, \\ -1 \le z \le 2. \end{cases}$

11. $\iiint\limits_G (6z + 3xy)\,dxdydz, \qquad G: \begin{cases} -2 \le x \le 0, \\ 0 \le y \le 2, \\ 1 \le z \le 3. \end{cases}$

12. $\iiint\limits_G (8x - 3z^2 y)\,dxdydz, \qquad G: \begin{cases} 1 \le x \le 2, \\ -1 \le y \le 2, \\ 0 \le z \le 1. \end{cases}$

13. $\iiint\limits_G (4y + x - 6z)\,dxdydz, \qquad G: \begin{cases} 0 \le x \le 2, \\ -1 \le y \le 1, \\ 1 \le z \le 3. \end{cases}$

14. $\iiint\limits_G \left(12x^2y^3 - z\right)dxdydz, \qquad G: \begin{cases} -1 \le x \le 0, \\ 0 \le y \le 2, \\ 2 \le z \le 4. \end{cases}$

15. $\iiint\limits_G \left(2y - 3x^2z\right)dxdydz, \qquad G: \begin{cases} 1 \le x \le 2, \\ -1 \le y \le 2, \\ 0 \le z \le 2. \end{cases}$

Task 2.6. Using the cylindrical coordinates, compute the volume of the body bounded by the given surfaces.

1. $G: \begin{cases} x^2 + y^2 - 4y = 0, \\ z = x^2 + y^2,\ z = 0. \end{cases}$

2. $G: \begin{cases} x^2 + y^2 - 4x = 0, \\ z = \sqrt{x^2 + y^2}, z = 0. \end{cases}$

3. $G: \begin{cases} z = 9 - x^2 - y^2, \\ z = x^2 + y^2 + 1. \end{cases}$

4. $G: \begin{cases} z = 2 + \sqrt{x^2 + y^2}, \\ z = 4 - x^2 - y^2. \end{cases}$

5. $G: \begin{cases} z = 8 - x^2 - y^2, \\ z = x^2 + y^2. \end{cases}$

6. $G: \begin{cases} z = \sqrt{x^2 + y^2}, \\ x^2 + y^2 = 4, z = 0. \end{cases}$

7. $G: \begin{cases} z = \sqrt{x^2 + y^2}, \\ z = \sqrt{8 - x^2 - y^2}. \end{cases}$

8. $G: \begin{cases} z = 10 - x^2 - y^2, \\ z = 2 + x^2 + y^2. \end{cases}$

9. $G: \begin{cases} z = \sqrt{18 - x^2 - y^2}, \\ z = \sqrt{x^2 + y^2}. \end{cases}$

10. $G: \begin{cases} z = \sqrt{4 - x^2 - y^2}, \\ x^2 + y^2 = 1,\ z = 0. \end{cases}$

11. $G: \begin{cases} z = 10 - x^2 - y^2, \\ z = x^2 + y^2 + 2. \end{cases}$

12. $G: \begin{cases} z = x^2 + y^2 + 1, \\ x^2 + y^2 = 4. \end{cases}$

13. $G: \begin{cases} z = 4 - x^2 - y^2, \\ x^2 + y^2 - 2y = 0. \end{cases}$

14. $G: \begin{cases} z = \sqrt{9 - x^2 - y^2}, \\ x^2 + y^2 = 4. \end{cases}$

15. $G:\begin{cases} z = 9 - x^2 - y^2, \\ x^2 + y^2 - 2x = 1. \end{cases}$

Task 2.7. Evaluate the line integral of the first kind.

1. $\int\limits_L \frac{xy}{x^2 + y^2}\,dl$, $\quad L:\begin{cases} x = 4\cos t, \\ y = 4\sin t, \end{cases} \; t \in \left[\frac{\pi}{12}, \frac{\pi}{8}\right].$
2. $\int\limits_L (2x + 3y)\,dl$, $\quad L: y = 2x - 3$ from $A(0,-3)$ to $B(2,1)$.
3. $\int\limits_L (2x - y)\,dl$, $\quad L:\begin{cases} x = 2\cos^3 t, \\ y = 2\sin^3 t, \end{cases} \; t \in \left[\frac{\pi}{2}, \pi\right].$
4. $\int\limits_L (3y - 5x)\,dl$, $\quad L: y = 3x - 1$ from $A(1,2)$ to $B(3,8)$.
5. $\int\limits_L \left(2\sqrt{y} + x\right)dl$, $\quad L:\begin{cases} x = 2t, \\ y = t^2, \end{cases} \; t \in \left[0, \sqrt{3}\right].$
6. $\int\limits_L (4x + 3y - 1)\,dl$, $\quad L: y = 2x - 1$ from $A(-1,-3)$ to $B(4,7)$.
7. $\int\limits_L \frac{x}{y^2}\,dl$, $\quad L:\begin{cases} x = 2\cos t, \\ y = 2\sin t, \end{cases} \; t \in \left[\frac{\pi}{6}, \frac{\pi}{4}\right].$
8. $\int\limits_L (2x + 3y + 5)\,dl$, $\quad L: y = 8x - 3$ from $A(1,5)$ to $B(2,13)$.
9. $\int\limits_L (x + 2y)\,dl$, $\quad L:\begin{cases} x = 4\cos^3 t, \\ y = 4\sin^3 t, \end{cases} \; t \in \left[0, \frac{\pi}{2}\right].$
10. $\int\limits_L (3y - x - 1)\,dl$, $\quad L: y = 3x + 1$ from $A(1,4)$ to $B(3,10)$.
11. $\int\limits_L \left(3x - \sqrt{y + 1}\right)dl$, $\quad L:\begin{cases} x = 2t, \\ y = t^2 - 1, \end{cases} \; t \in \left[0, 2\sqrt{2}\right].$
12. $\int\limits_L (3x - y + 4)\,dl$, $\quad L: y = 5x + 1$ from $A(-1,-4)$ to $B(2,11)$.

13. $\int\limits_L \frac{3x^3}{\sqrt{x^2+y^2}}\,dl$, $L:\begin{cases} x=2\cos t, \\ y=2\sin t, \end{cases} t\in[0,\pi].$

14. $\int\limits_L (4y-10x-3)dl$, $L: y=4x-1$ from $A(1,3)$ to $B(3,11)$.

15. $\int\limits_L (4x+3y)dl$, $L:\begin{cases} x=\cos^3 t, \\ y=\sin^3 t, \end{cases} t\in[0,\pi].$

Task 2.8. Evaluate the line integral of the second kind.

1. $\int\limits_L (6x^3-2y)dx-7xy\,dy$, $L: y=x^3-1$ from $A(0,-1)$ to $B(2,7)$.

2. $\int\limits_L 6xy\,dx-x^2\,dy$, $L: y=x^2+1$ from $A(-1,2)$ to $B(2,5)$.

3. $\int\limits_L (x+y)dx+3xy\,dy$, $L: y=2x+3$ from $A(0,3)$ to $B(2,7)$.

4. $\int\limits_L (2x-y)dx+3x\,dy$, $L: y=x^2+2x$ from $A(0,0)$ to $B(2,8)$.

5. $\int\limits_L (y^2-x)dx+4xy\,dy$, $L: y=\sqrt{x-1}$ from $A(1,0)$ to $B(2,1)$.

6. $\int\limits_L 4y\,dx+(y-x^3)dy$, $L: y=x^3+2$ from $A(-1,1)$ to $B(1,3)$.

7. $\int\limits_L (x^2+2y)dx+y\,dy$, $L: y=4x^2$ from $A(1,4)$ to $B(2,16)$.

8. $\int\limits_L (y^2+3x)dx+4y\,dy$, $L: y=\sqrt{x+2}$ from $A(2,2)$ to $B(7,3)$.

9. $\int\limits_L (y-x^2)dx+6x\,dy$, $L: y=x^2-4x$ from $A(1,-3)$ to $B(2,-4)$.

10. $\int\limits_L 2x\,dx+(y-2x^3)dy$, $L: y=2x^3-8$ from $A(1,-6)$ to $B(2,8)$.

11. $\int\limits_L y^2\,dx+(y+3x)dy$, $L: y=3x-2$ from $A(0,-2)$ to $B(2,4)$.

12. $\int\limits_L \left(2y-5x^2\right)dx+4x^2\,dy$, $L: y=x^2+6x$ from $A(0,0)$ to $B(1,7)$.

13. $\int\limits_L \left(2y^2-1\right)dx-4xy\,dy$, $L: y=\sqrt{x-3}$ from $A(3,0)$ to $B(4,1)$.

14. $\int\limits_L 4y\,dx+\left(x^3-y\right)dy$, $L: y=x^3-2$ from $A(0,-2)$ to $B(2,6)$.

15. $\int\limits_L (y-2)dx+3xy\,dy$, $L: y=4x+6$ from $A(-1,2)$ to $B(0,6)$.

Task 2.9. Find the work done by the force $\overline{F}$ on an object moving along the given curve.

1. $\overline{F}=(1-y,\,x)$, $L: \begin{cases} x=t-\sin t, \\ y=1-\cos t, \end{cases} t\in\left[0,\dfrac{\pi}{2}\right]$.

2. $\overline{F}=\left(y^2-x^2,\,1-y\right)$, $L: \begin{cases} x=2\cos t, \\ y=2\sin t, \end{cases} t\in\left[\dfrac{\pi}{2},\pi\right]$.

3. $\overline{F}=\left(x^3,\,6y-1\right)$, $L: \begin{cases} x=\cos^3 t, \\ y=\sin^3 t, \end{cases} t\in\left[0,\dfrac{\pi}{2}\right]$.

4. $\overline{F}=\left(2y+x,\,y^2\right)$, $L: \begin{cases} x=3\cos t, \\ y=4\sin t, \end{cases} t\in[0,\pi]$.

5. $\overline{F}=\left(x^2-1,\,2y-x\right)$, $L: \begin{cases} x=4\cos t, \\ y=4\sin t, \end{cases} t\in\left[\dfrac{\pi}{2},\pi\right]$.

6. $\overline{F}=\left(4xy,\,x^4+4y\right)$, $L: \begin{cases} x=\sqrt{t}, \\ y=t^2-2, \end{cases} t\in[0,1]$.

7. $\overline{F}=(y,\,2x-1)$, $L: \begin{cases} x=2\cos^3 t, \\ y=2\sin^3 t, \end{cases} t\in[0,\pi]$.

8. $\overline{F}=\left(y^2,\,2x-y\right)$, $L: \begin{cases} x=4\cos t, \\ y=3\sin t, \end{cases} t\in\left[0,\dfrac{\pi}{2}\right]$.

9. $\overline{F}=\left(4x-y,\, y^3\right)$, $L:\begin{cases} x=\cos t, \\ y=\sin t, \end{cases} t\in\left[\frac{\pi}{2},\pi\right]$.

10. $\overline{F}=\left(2y,\, x-1\right)$, $L:\begin{cases} x=t-\sin t, \\ y=1-\cos t, \end{cases} t\in[0,\pi]$.

11. $\overline{F}=\left(y-x^2-1,\, 2y\right)$, $L:\begin{cases} x=2t, \\ y=6t^2-5, \end{cases} t\in[0,1]$.

12. $\overline{F}=\left(x-3y,\, y^2\right)$, $L:\begin{cases} x=3\cos t, \\ y=3\sin t, \end{cases} t\in\left[0,\frac{\pi}{2}\right]$.

13. $\overline{F}=\left(x-2y,\, x\right)$, $L:\begin{cases} x=\cos^3 t, \\ y=\sin^3 t, \end{cases} t\in[0,\pi]$.

14. $\overline{F}=\left(2y,\, 5x-1\right)$, $L:\begin{cases} x=5\cos t, \\ y=4\sin t, \end{cases} t\in\left[\frac{\pi}{2},\pi\right]$.

15. $\overline{F}=\left(4y-x,\, 6x\right)$, $L:\begin{cases} x=4\cos t, \\ y=5\sin t, \end{cases} t\in\left[0,\frac{\pi}{2}\right]$.

Task 2.10. Evaluate the following surface integral $\iint\limits_S f(x,y,z)dS$, where S is the part of the plane P bounded by the coordinate planes.

1. $\iint\limits_S (x+2y-z)dS$, $P: x+2y+z=2$.
2. $\iint\limits_S (x-2z)dS$, $P: x-2y+2z=2$.
3. $\iint\limits_S (4x-3y+z)dS$, $P: 2x-y+z=4$.
4. $\iint\limits_S (3x-y-2z)dS$, $P: x-y+2z=2$.
5. $\iint\limits_S (x+2y+3z)dS$, $P: 2x+y-3z=6$.

6. $\iint\limits_S (y-2x+z)dS$, $P: 4x-y+z=4$.
7. $\iint\limits_S (2y-3x+2z)dS$, $P: x+2y-2z=2$.
8. $\iint\limits_S (x-2y+z)dS$, $P: -x-4y+z=4$.
9. $\iint\limits_S (2x-y+3z)dS$, $P: 2x+y+z=2$.
10. $\iint\limits_S (3x+2y+z)dS$, $P: 3x-2y+z=6$.
11. $\iint\limits_S (4x-y+2z)dS$, $P: 2x+3y+2z=6$.
12. $\iint\limits_S (y-4z)dS$, $P: 2x-y+4z=4$.
13. $\iint\limits_S (3x+3y+5z)dS$, $P: x+y-5z=10$.
14. $\iint\limits_S (3x+2y-z)dS$, $P: 3x-2y-z=6$.
15. $\iint\limits_S (x-4y+3z)dS$, $P: x-2y+z=2$.

Task 2.11. Evaluate the given surface integral over the surface σ. σ is the part of the plane P bounded by the coordinate planes and oriented in the direction of the positive z-axis.

1. $\iint\limits_\sigma (x-2z)dxdy$, $P: x-2y+z=2$.
2. $\iint\limits_\sigma (x+2y)dydz$, $P: x-2y-2z=2$.
3. $\iint\limits_\sigma (y+2z)dxdz$, $P: 2x-y+2z=2$.
4. $\iint\limits_\sigma (x+2z)dxdy$, $P: x-2y+2z=2$.

5. $\iint\limits_{\sigma}(x-2y)dydz$, $P: x+2y-2z=2$.

6. $\iint\limits_{\sigma}(y-2z)dxdz$, $P: 2x-y-2z=2$.

7. $\iint\limits_{\sigma}(2x-z)dxdy$, $P: 4x-2y+z=4$.

8. $\iint\limits_{\sigma}(x-4y)dydz$, $P: x-2y+2z=2$.

9. $\iint\limits_{\sigma}(2y+z)dxdz$, $P: 2x+2y-z=2$.

10. $\iint\limits_{\sigma}(2x+z)dxdy$, $P: 2x-2y-z=2$.

11. $\iint\limits_{\sigma}(x+4z)dydz$, $P: x-2y+2z=2$.

12. $\iint\limits_{\sigma}(2y-z)dxdz$, $P: 2x+2y+z=2$.

13. $\iint\limits_{\sigma}(4x+z)dxdy$, $P: 2x-2y+z=2$.

14. $\iint\limits_{\sigma}(x-4z)dydz$, $P: x+4y+4z=4$.

15. $\iint\limits_{\sigma}(3y+z)dxdz$, $P: 2x+y-z=2$.

3 ELEMENTS OF FIELD THEORY

Task 3.1. The vector field $\overline{F}(M)$ is given. Find its divergence and its curl.

1. $\overline{F} = 2xy\,\overline{i} + y^2 z\,\overline{j} + x(y+z)\overline{k}$.
2. $\overline{F} = zx^3\,\overline{i} + 2z^2\,\overline{j} + xyz\overline{k}$.
3. $\overline{F} = 3x(y+x)\overline{i} + (z-y)\,\overline{j} + 5y^2\,\overline{k}$.
4. $\overline{F} = (z-5x)\overline{i} - 2yz\,\overline{j} - y(2-x)\overline{k}$.
5. $\overline{F} = -yz^2\,\overline{i} + 5z^2\,\overline{j} + x(z+5y)\overline{k}$.
6. $\overline{F} = 4(x^2+y)\overline{i} + x^3\,\overline{j} + 12xz\overline{k}$.
7. $\overline{F} = yz\,\overline{i} + (y+2z)\,\overline{j} + x(y^2+3)\overline{k}$.
8. $\overline{F} = x^3\,\overline{i} - 3y^2 x\,\overline{j} + z(2y+z)\overline{k}$.
9. $\overline{F} = -x^2 y\,\overline{i} + (y-z)\,\overline{j} - 5xz\overline{k}$.
10. $\overline{F} = y^3\,\overline{i} + x(z-y)\,\overline{j} + (2x-y)\overline{k}$.
11. $\overline{F} = x(z+y)\overline{i} - 5xz\,\overline{j} - 2z^2\,\overline{k}$.
12. $\overline{F} = zy^2\,\overline{i} - 3x^2\,\overline{j} + 3xz\overline{k}$.
13. $\overline{F} = 3(y^2 - z)\overline{i} + y^3 x\,\overline{j} + 2yz\overline{k}$.
14. $\overline{F} = -y^2 x\,\overline{i} - (x+z)\,\overline{j} + 4xy\overline{k}$.
15. $\overline{F} = z^3\,\overline{i} + 7x^2 z\,\overline{j} + 3x(y-z)\overline{k}$.

Task 3.2. Calculate the flux of the vector field $\overline{F}(M)$ through the surface of the pyramid bounded by the plane P and the coordinate planes with outward orientation:

a) using the definition;

b) using the Gauss-Ostrogradsky formula.

1. $\overline{F} = 2x\,\overline{i} + (y+2z)\,\overline{j} - (x+z)\overline{k}$, $\quad P: 2x+4y+2z=2$.
2. $\overline{F} = (x+2y)\overline{i} + (2-z)\,\overline{j} + (x-y)\overline{k}$, $\quad P: x+6y-3z=3$.

3. $\overline{F} = -3y\,\overline{i} - (x+2y)\,\overline{j} + (y-z)\,\overline{k}$, $P: -x+3y+z=3$.
4. $\overline{F} = (z-x)\,\overline{i} + 5y\,\overline{j} + (2z-3x)\,\overline{k}$, $P: x-2y+3z=2$.
5. $\overline{F} = 4z\,\overline{i} + 11y\,\overline{j} + 2(y+3z)\,\overline{k}$, $P: 2x+y-4z=2$.
6. $\overline{F} = (x-y)\,\overline{i} + 3x\,\overline{j} + 2(x+z)\,\overline{k}$, $P: x-y+z=3$.
7. $\overline{F} = (y-x)\,\overline{i} - (x+z)\,\overline{j} - (y-z)\,\overline{k}$, $P: -4x+2y+z=4$.
8. $\overline{F} = 4y\,\overline{i} + 2z\,\overline{j} - 11z\,\overline{k}$, $P: 2x+3y-3z=6$.
9. $\overline{F} = (x+y)\,\overline{i} - (y+z)\,\overline{j} + 5y\,\overline{k}$, $P: -x+4y-2z=8$.
10. $\overline{F} = (z-y)\,\overline{i} + (x-z)\,\overline{j} + (y-x)\,\overline{k}$, $P: 5x-y+5z=5$.
11. $\overline{F} = -3x\,\overline{i} - (x+y)\,\overline{j} - 3z\,\overline{k}$, $P: -2x+4y+4z=8$.
12. $\overline{F} = (x-z)\,\overline{i} - 3y\,\overline{j} + 5(x-z)\,\overline{k}$, $P: 2x+4y+z=4$.
13. $\overline{F} = 3y\,\overline{i} + 2x\,\overline{j} + 2(z-2x)\,\overline{k}$, $P: -3x+6y+z=6$.
14. $\overline{F} = 2x\,\overline{i} + (x-z)\,\overline{j} + 4z\,\overline{k}$, $P: -x+3y-z=3$.
15. $\overline{F} = (x-4z)\,\overline{i} + (1-x)\,\overline{j} - (y-z)\,\overline{k}$, $P: x-y+2z=4$.

Task 3.3. Find the circulation of the vector field $\overline{F}(M)$ over the triangle formed by the intersection of coordinate planes and the plane P (the curve is oriented counterclockwise as viewed from above):

a) using definition;

b) using Stokes' formula.

1. $\overline{F} = -3y\,\overline{i} - (y+2z)\,\overline{j} + (2x+z)\,\overline{k}$, $P: x-2y+2z=4$.
2. $\overline{F} = 2(z-y)\,\overline{i} + (y+3x)\,\overline{j} + 5z\,\overline{k}$, $P: 4x-2y+8z=8$.
3. $\overline{F} = (5y+z)\,\overline{i} + 3x\,\overline{j} + (x-2z)\,\overline{k}$, $P: x-3y+6z=6$.
4. $\overline{F} = 4(y+x)\,\overline{i} + (x+3y)\,\overline{j} - y\,\overline{k}$, $P: -3x-9y+3z=9$.
5. $\overline{F} = 2x\,\overline{i} - 3z\,\overline{j} + (5y-3z)\,\overline{k}$, $P: x+y-z=5$.
6. $\overline{F} = 3y\,\overline{i} + (z+x)\,\overline{j} - 2(y+z)\,\overline{k}$, $P: x+3y+3z=3$.
7. $\overline{F} = (y+2x)\,\overline{i} - 2(x+y)\,\overline{j} - (y+z)\,\overline{k}$, $P: 4x-2y+4z=4$.
8. $\overline{F} = (4z-3y)\,\overline{i} + 2y\,\overline{j} + 11z\,\overline{k}$, $P: 7x+y-7z=7$.

9. $\overline{F} = (x-2z)\bar{i} - (z-x)\bar{j} + 5x\bar{k}$, $P: 4x - 4y - 2z = 8$.
10. $\overline{F} = (z-2y)\bar{i} + 2(x+z)\bar{j} + (y+x)\bar{k}$, $P: 5x - 2y + 5z = 10$.
11. $\overline{F} = 2x\bar{i} - (x+z)\bar{j} + 5z\bar{k}$, $P: x + 4y + 2z = 4$.
12. $\overline{F} = (x+2y)\bar{i} - 3z\bar{j} + 5(2x+z)\bar{k}$, $P: x - 4y + 2z = 4$.
13. $\overline{F} = 3(y-z)\bar{i} - x\bar{j} + 2(z+x)\bar{k}$, $P: -x + 2y + z = 2$.
14. $\overline{F} = (2y-x)\bar{i} + (x+z)\bar{j} + (z+2y)\bar{k}$, $P: -2x + y - 6z = 6$.
15. $\overline{F} = (x+3z)\bar{i} + (1+y)\bar{j} + 3(z-y)\bar{k}$, $P: 5x - y + 5z = 5$.

4 SERIES

Task 4.1. Examine the series for convergence.

1. a) $\sum_{n=1}^{\infty} \frac{5^n}{7n+4}$;

   b) $\sum_{n=1}^{\infty} \left(\frac{2n}{2n-1}\right)^{n^2}$;

   c) $\sum_{n=1}^{\infty} \frac{1}{(n+1)\ln^2(n+1)}$;

   d) $\sum_{n=1}^{\infty} \frac{2n-3}{n^3+n}$.

2. a) $\sum_{n=1}^{\infty} \frac{2n+7}{3^n}$;

   b) $\sum_{n=1}^{\infty} \left(\sin\frac{\pi n}{3n+5}\right)^n$;

   c) $\sum_{n=1}^{\infty} \frac{\arctan(2n)}{1+4n^2}$;

   d) $\sum_{n=1}^{\infty} \frac{n^2+5n-3}{n^3-11n}$.

3. a) $\sum_{n=1}^{\infty} \frac{n^2+2}{2^n}$;

   b) $\sum_{n=1}^{\infty} \left(\frac{7n+4}{7n-2}\right)^{n^2}$;

   c) $\sum_{n=1}^{\infty} \frac{\ln^3(2n+3)}{(2n+3)}$;

   d) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^4+5}$.

4. a) $\sum_{n=1}^{\infty} \frac{4^n}{11n+5}$;

   b) $\sum_{n=1}^{\infty} \left(\arctan\frac{\sqrt{3}n-1}{3n+2}\right)^n$;

   c) $\sum_{n=1}^{\infty} \frac{2n\ln(n^2+3)}{n^2+3}$;

   d) $\sum_{n=1}^{\infty} \frac{n^2+7n}{3n^5-11n^3}$.

5. a) $\sum_{n=1}^{\infty} \frac{e^n}{n-13}$;

   b) $\sum_{n=1}^{\infty} \left(\frac{5n^2+7}{5n^2-3}\right)^{n^3}$;

   c) $\sum_{n=1}^{\infty} \frac{1}{(4n+2)\ln(2n+1)}$;

   d) $\sum_{n=1}^{\infty} \frac{n+4}{n^4+7n^2-11}$.

6. a) $\sum_{n=1}^{\infty} \frac{1}{2^n(n-1)}$;

   b) $\sum_{n=1}^{\infty} \left(\frac{n+6}{n-2}\right)^{n^2}$;

   c) $\sum_{n=1}^{\infty} \frac{\arctan^2(n)}{1+n^2}$;

   d) $\sum_{n=1}^{\infty} \frac{n^3+5n^2+3n-3}{n^5-11n+4}$.

7. a) $\sum_{n=1}^{\infty}\frac{2^n}{2n^2-1}$;

b) $\sum_{n=1}^{\infty}\left(\frac{3n+2}{3n}\right)^{n^2}$;

c) $\sum_{n=1}^{\infty}\frac{2^{\frac{1}{n}}}{n^2}$;

d) $\sum_{n=1}^{\infty}\frac{5n}{n^2+11n-1}$.

8. a) $\sum_{n=1}^{\infty}\frac{7n+1}{6^n}$;

b) $\sum_{n=1}^{\infty}\left(\ln\frac{4n+3}{2n-1}\right)^n$;

c) $\sum_{n=1}^{\infty}\frac{1}{2\sqrt{n}\ln\sqrt{n}}$;

d) $\sum_{n=1}^{\infty}\frac{3n^2+1}{n^5+3n^2+4}$.

9. a) $\sum_{n=1}^{\infty}\frac{2^n}{3n-2}$;

b) $\sum_{n=1}^{\infty}\left(\arctan\frac{4n+1}{4n+7}\right)^n$;

c) $\sum_{n=1}^{\infty}\frac{2^{\sqrt{n}}}{6\sqrt{n}}$;

d) $\sum_{n=1}^{\infty}\frac{5n-7}{n^4+n^3-1}$.

10. a) $\sum_{n=1}^{\infty}\frac{1}{e^n(n+5)}$;

b) $\sum_{n=1}^{\infty}\left(\frac{6n}{6n-1}\right)^{n^2}$;

c) $\sum_{n=1}^{\infty}ne^{-n^2}$;

d) $\sum_{n=1}^{\infty}\frac{7n-1}{n^2+2n-3}$.

11. a) $\sum_{n=1}^{\infty}\frac{4^n}{n-7}$;

b) $\sum_{n=1}^{\infty}\left(\frac{3n-1}{3n+1}\right)^{n^2}$;

c) $\sum_{n=1}^{\infty}\frac{\arctan\left(\frac{1}{n}\right)}{n^2}$;

d) $\sum_{n=1}^{\infty}\frac{n^3-5n^2+3}{n^4+11n^3+2n-1}$.

12. a) $\sum_{n=1}^{\infty}\frac{e^{-n}}{2n-1}$;

b) $\sum_{n=1}^{\infty}\left(\frac{7n}{7n+1}\right)^n 5^n$;

c) $\sum_{n=1}^{\infty}\frac{1}{\left(\sqrt{n}+7\right)\ln^3\left(\sqrt{n}+7\right)}$;

d) $\sum_{n=1}^{\infty}\frac{n^3+3}{n^5-n^3+n}$.

13. a) $\sum_{n=1}^{\infty}\frac{e^n}{3^n(n+4)}$;

14. a) $\sum_{n=1}^{\infty}\frac{2^n}{4n-3}$;

b) $\sum_{n=1}^{\infty} \ln^n \frac{en}{2n+7}$;

c) $\sum_{n=1}^{\infty} \frac{\ln\sqrt{n+1}}{\sqrt{n+1}}$;

d) $\sum_{n=1}^{\infty} \frac{4n-n^2}{n^4+7n}$.

b) $\sum_{n=1}^{\infty} \left(\frac{3n-1}{3n+4}\right)^{n^2}$;

c) $\sum_{n=1}^{\infty} \frac{\arctan\left(e^n\right)}{1+e^{2n}}$;

d) $\sum_{n=1}^{\infty} \frac{n^2+2n}{n^5+4}$.

15. a) $\sum_{n=1}^{\infty} \frac{2^n(n+4)}{3^n}$;

b) $\sum_{n=1}^{\infty} \sin^n \frac{1}{n}$;

c) $\sum_{n=1}^{\infty} \frac{n}{2^{n^2}}$;

d) $\sum_{n=1}^{\infty} \frac{n^3+1}{7n^5+1}$.

Task 4.2. Determine if the series is absolutely convergent, conditionally convergent or divergent.

1. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{3n+8}}$.

2. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\ln(5n+1)}$.

3. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n^2+2}}$.

4. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2n+1}{5n+1}\right)^n$.

5. $\sum_{n=1}^{\infty} (-1)^n \frac{4n+3}{5n+3}$.

6. $\sum_{n=1}^{\infty} (-1)^n \frac{n+5}{n^3+11n-3}$.

7. $\sum_{n=1}^{\infty} (-1)^n \frac{4}{3n+3}$.

8. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{2n+7}{n+1}\right)^n$.

9. $\sum_{n=1}^{\infty} (-1)^n \frac{12n+1}{n!}$.

10. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+5}$.

11. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt[4]{2n+3}}$.
12. $\sum_{n=1}^{\infty}(-1)^n \left(\frac{3n-2}{2n+5}\right)^n$.
13. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}\ln\sqrt{n}}$.
14. $\sum_{n=1}^{\infty}(-1)^n \frac{\ln\left(\sqrt{n}+1\right)}{\sqrt{n}+1}$.
15. $\sum_{n=1}^{\infty}(-1)^n \frac{n+5}{n^2+n+5}$.

Task 4.3. Expand the function into the Maclaurin series and find its domain of convergence.

1. $f(x)=x\sin(x)$.
2. $f(x)=\frac{\cos(x)-1}{x}$.
3. $f(x)=\frac{\sin\left(x^2\right)}{x}$.
4. $f(x)=\frac{1}{x}\ln\left(1+x^2\right)$.
5. $f(x)=xe^x$.
6. $f(x)=\ln\left(1-x^2\right)$.
7. $f(x)=e^{x^2}$.
8. $f(x)=\frac{e^x-1}{x}$.
9. $f(x)=x\ln\left(1+x^2\right)$.
10. $f(x)=\frac{\sin x}{x}$.
11. $f(x)=x\cos(x)$.
12. $f(x)=1-x\cos x$.
13. $f(x)=x(\cos x-1)$.
14. $f(x)=x\ln(1-x)$.
15. $f(x)=\frac{\cos(x)-1}{x^2}$.

Task 4.4. Find the expansion of the 2π –periodic function $f(x)$ into the Fourier series.

1. $f(x)=2x+5$.
2. $f(x)=3-x$.
3. $f(x)=4x-3$.
4. $f(x)=x+7$.
5. $f(x)=2x-1$.
6. $f(x)=3x+1$.
7. $f(x)=2-3x$.
8. $f(x)=1+4x$.
9. $f(x)=-x+5$.
10. $f(x)=2x+1$.
11. $f(x)=x-4$.
12. $f(x)=5x+1$.
13. $f(x)=3x+2$.
14. $f(x)=x-1$.
15. $f(x)=3x+2$.

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Appendix A

Table A.1 – Table of Derivatives
($u = u(x)$ is a differentiable function)

1. $(C)' = 0,\ C - const$;	11. $(\tan u)' = \frac{1}{\cos^2 u} \cdot u'$;
2. $(x)' = 1$;	12. $(\cot u)' = -\frac{1}{\sin^2 u} \cdot u'$;
3. $(u^n)' = n \cdot u^{n-1} \cdot u'$;	13. $(\arcsin u)' = \frac{1}{\sqrt{1-u^2}} \cdot u'$;
4. $(\sqrt{u})' = \frac{1}{2\sqrt{u}} \cdot u'$;	14. $(\arccos u)' = -\frac{1}{\sqrt{1-u^2}} \cdot u'$;
5. $(a^u)' = a^u \cdot \ln a \cdot u',\ a - const$;	15. $(\arctan u)' = \frac{1}{1+u^2} \cdot u'$;
6. $(e^u)' = e^u \cdot u'$;	16. $(\cot^{-1} u)' = -\frac{1}{1+u^2} \cdot u'$;
7. $(\log_a u)' = \frac{1}{u \cdot \ln a} \cdot u'$;	17. $(\sinh u)' = \cosh u \cdot u'$;
8. $(\ln u)' = \frac{1}{u} \cdot u'$;	18. $(\cosh u)' = \sinh u \cdot u'$;
9. $(\sin u)' = \cos u \cdot u'$;	19. $(\tanh u)' = \frac{1}{\cosh^2 u} \cdot u'$;
10. $(\cos u)' = -\sin u \cdot u'$;	20. $(\coth u)' = -\frac{1}{\sinh^2 u} \cdot u'$.

Appendix B

Table B.1 – Table of Basic Integrals
($u = u(x)$ is a differentiable function)

1. $\int du = u + C$;	12. $\int \frac{du}{\sin^2 u} = -\operatorname{ctg} u + C$;
2. $\int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1} + C$, $\alpha \neq -1$;	13. $\int \frac{du}{\sin u} = \ln \left\lvert \operatorname{tg} \frac{u}{2} \right\rvert + C$;
3. $\int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$;	14. $\int \frac{du}{\cos u} = \ln \left\lvert \operatorname{tg}\left(\frac{u}{2} + \frac{\pi}{4} \right) \right\rvert + C$;
4. $\int \frac{du}{u} = \ln \lvert u \rvert + C$;	15. $\int \operatorname{sh} u \, du = \operatorname{ch} u + C$;
5. $\int a^u du = \frac{a^u}{\ln a} + C$;	16. $\int \operatorname{ch} u \, du = \operatorname{sh} u + C$;
6. $\int e^u du = e^u + C$;	17. $\int \frac{du}{\operatorname{ch}^2 u} = \operatorname{th} u + C$;
7. $\int \sin u \, du = -\cos u + C$;	18. $\int \frac{du}{\operatorname{sh}^2 u} = -\operatorname{cth} u + C$;
8. $\int \cos u \, du = \sin u + C$;	19. $\int \frac{du}{u^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{u}{a} + C$;
9. $\int \operatorname{tg} u \, du = -\ln \lvert \cos u \rvert + C$;	20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left\lvert \frac{u-a}{u+a} \right\rvert + C$;
10. $\int \operatorname{ctg} u \, du = \ln \lvert \sin u \rvert + C$;	21. $\int \frac{du}{\sqrt{u^2 \pm a^2}} = \ln \left\lvert u + \sqrt{u^2 \pm a^2} \right\rvert + C$;
11. $\int \frac{du}{\cos^2 u} = \operatorname{tg} u + C$;	22. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$;

Table B.1 (continued)

23. $\int \sqrt{a^2-u^2}\,du = \frac{1}{2}u\sqrt{a^2-u^2} + \frac{1}{2}a^2 \arcsin\frac{u}{a} + C$;	25. $\int \frac{f'(x)}{f(x)}dx = \ln\lvert f(x)\rvert + C$;
24. $\int \sqrt{u^2 \pm a^2}\,du = \frac{1}{2}u\sqrt{u^2 \pm a^2} \pm \pm\frac{1}{2}a^2 \ln\left\lvert u + \sqrt{u^2 \pm a^2}\right\rvert + C$;	26. $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + C$.