MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
National University Zaporizhzhia Polytechnic

CALCULATION TASKS ON HIGHER MATHEMATICS (1st module)

for students majoring in
G3 Electrical Engineering

2026

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INTRODUCTION

One of the types of unaided work of students is the execution of calculation tasks during the semester. Their purpose is to develop practical skills of students. They are designed to help students capture theoretical material more deeply and learn how to apply the acquired knowledge to solve practical problems. The offered typical calculation tasks correspond to the course "Higher Mathematics" taught to students majoring in G3 Electrical Engineering full-time in the first module of the two-semester course.

This task book covers virtually all major sections of the first module of the course on higher mathematics; it contains assignments for all the topics of the course. A list of recommended literature is also provided.

Calculation tasks are executed during the semester, which provides students with a systematic study of the course. While carrying out these tasks, students work with recommended textbooks and manuals, independently search for necessary literary sources and materials, analyze them and summarize, independently research and make written presentation of practical assignments.

The student chooses the option (the number of the variant) according to his number in the register list. The work is done in English in writing, preferably in a notebook in a cell. Note fields must be left blank. The name of the subject, major, group and course, surname, first name and patronymic of the student, name of the teacher who accepts the work should be indicated on the title page of the work.

While performing the work, the student should solve the offered tasks by the methods specified in the tasks, as well as make all the necessary drawings (graphical solutions). The student must show the acquired theoretical knowledge of the course.

When doing the work, the student can use both the lecture and practical material, as well as the supporting literature listed at the end of this manual. Before starting the work, it is recommended to study the relevant theoretical material, then to understand the solutions of the tasks that were performed in the practical classes, and only after that to start the actual calculation work.

When evaluating a work, the indicator of its quality is, first of all, how the student independently and correctly solved the tasks and understood the content of the obtained solutions. That is why additional

questions may be asked to protect the student's work, including the theoretical material presented for the exam.

Completed work is submitted to the teacher for verification and subsequent protection in the form of an interview (usually during modular control). The student must be able to:

– present the content of the tasks and to prove their solutions;
– answer questions about the content of the solutions obtained;
– answer additional questions.

If the work is successfully protected, the student receives a certain number of rating points. If the specified requirements are not fulfilled, then the work is returned to the student for completion, indicating the term of re-protection.

1 LINEAR ALGEBRA AND VECTOR ALGEBRA

Task 1.1. Matrices $A=\begin{pmatrix}2 & -3 & 4\\ 1 & 0 & -2\end{pmatrix}$, $B=\begin{pmatrix}1 & 4 & 5\\ 3 & 0 & -2\end{pmatrix}$, $C=\begin{pmatrix}2 & 1\\ 0 & -1\\ 3 & 4\end{pmatrix}$, $D=\begin{pmatrix}-2 & 0\\ 3 & 1\\ 0 & 5\end{pmatrix}$, $F=\begin{pmatrix}2 & 0\\ 4 & -3\end{pmatrix}$, $K=\begin{pmatrix}-3 & 1\\ 5 & 0\end{pmatrix}$, $L=\begin{pmatrix}2 & 0 & -4\\ 1 & 3 & 2\\ -3 & 5 & 0\end{pmatrix}$, $M=\begin{pmatrix}3 & 1 & 2\\ 0 & -4 & 1\\ 2 & 0 & 6\end{pmatrix}$ are given. Find:

1. $\left(A+2D^T\right)\cdot L$.
2. $\left(B-4C^T\right)\cdot M$.
3. $A\cdot L-5D^T$.
4. $B^T\cdot A-2M$.
5. $A^T\cdot B+3L$.
6. $3C-D\cdot K^T$.
7. $D\cdot\left(F^T-4I\right)$.
8. $C\cdot\left(K^T+2I\right)$.
9. $\left(4L^T-M\right)\cdot D$.
10. $\left(2L-M^T\right)\cdot C$.
11. $\left(M+2L^T\right)\cdot D$.
12. $L\cdot\left(B^T+2D\right)$.
13. $D\cdot\left(2I-K^T\right)$.
14. $C\cdot\left(2K^T-F\right)$.
15. $M^T\cdot D-3C$.

Task 1.2. Calculate the determinant.

1. $\begin{vmatrix}1 & 0 & 2 & 0\\ 3 & 2 & 1 & -2\\ -1 & 0 & 4 & 0\\ 2 & -1 & 0 & 3\end{vmatrix}$.
2. $\begin{vmatrix}2 & 1 & 0 & 3\\ 1 & 0 & -2 & 0\\ -4 & 3 & 1 & -1\\ 0 & 2 & 0 & 5\end{vmatrix}$.
3. $\begin{vmatrix}2 & 0 & 4 & 3\\ 0 & 1 & -2 & 0\\ 3 & 6 & -1 & 2\\ 0 & 2 & 0 & -5\end{vmatrix}$.
4. $\begin{vmatrix}3 & 5 & 2 & 0\\ 2 & 6 & -1 & 2\\ -1 & 0 & 4 & 0\\ 0 & 5 & 0 & -3\end{vmatrix}$.

5. $\begin{vmatrix} -5 & 0 & 1 & 3 \\ 0 & 7 & -1 & 0 \\ 4 & 9 & -3 & -2 \\ 0 & 5 & 0 & 2 \end{vmatrix}$.

6. $\begin{vmatrix} 6 & 0 & -3 & 0 \\ 2 & 5 & 4 & -1 \\ 8 & 0 & -5 & 0 \\ 3 & -1 & 0 & 2 \end{vmatrix}$.

7. $\begin{vmatrix} -2 & 4 & 0 & 5 \\ 6 & 0 & -1 & 0 \\ 9 & 3 & 6 & 1 \\ 0 & 8 & 0 & -5 \end{vmatrix}$.

8. $\begin{vmatrix} 4 & 3 & -1 & 0 \\ -1 & 2 & -6 & 5 \\ 5 & 0 & 7 & 0 \\ 0 & 3 & 0 & -4 \end{vmatrix}$.

9. $\begin{vmatrix} 7 & 0 & 8 & -1 \\ 0 & 2 & -3 & 0 \\ -3 & 5 & -1 & 9 \\ 0 & 4 & 0 & -7 \end{vmatrix}$.

10. $\begin{vmatrix} 3 & 0 & 2 & -1 \\ 0 & 2 & -4 & 0 \\ 5 & -7 & -2 & 6 \\ 0 & 3 & 0 & -8 \end{vmatrix}$.

11. $\begin{vmatrix} 9 & 0 & -6 & 0 \\ 2 & 3 & 7 & -1 \\ -3 & 0 & -8 & 0 \\ 5 & 2 & 0 & 3 \end{vmatrix}$.

12. $\begin{vmatrix} 2 & -5 & 8 & 0 \\ 7 & -2 & 9 & -4 \\ 2 & 0 & -3 & 0 \\ 0 & -1 & 0 & 5 \end{vmatrix}$.

13. $\begin{vmatrix} 1 & 3 & 2 & 0 \\ 4 & 9 & -5 & -6 \\ -3 & 0 & 7 & 0 \\ 0 & 1 & 0 & 4 \end{vmatrix}$.

14. $\begin{vmatrix} -6 & 5 & 0 & 7 \\ 5 & 0 & 2 & 0 \\ 8 & -3 & 1 & -2 \\ 0 & 9 & 0 & 4 \end{vmatrix}$.

15. $\begin{vmatrix} 8 & 0 & 7 & 0 \\ 1 & -2 & -6 & 4 \\ 2 & 0 & 5 & 0 \\ -1 & 3 & 0 & 2 \end{vmatrix}$.

Task 1.3. Solve the following system of linear equations using:
a) the matrix method;
b) Cramer's rule;
c) Gaussian elimination.

1. $\begin{cases} 2x-y-2z=3, \\ x-2y-3z=1, \\ 3x+2y+z=7. \end{cases}$

2. $\begin{cases} 2x+3y+2z=5, \\ x-2y-4z=4, \\ 3x-y+3z=2. \end{cases}$

3. $\begin{cases} 2x+2y-3z=-1, \\ x+3y-z=4, \\ 4x+5y-3z=3. \end{cases}$

4. $\begin{cases} 3x-3y-5z=4, \\ x+y+z=2, \\ 5x+y-z=10. \end{cases}$

5. $\begin{cases} x+5y+6z=1, \\ 4x-3y-z=6, \\ 2x+y+7z=-2. \end{cases}$

6. $\begin{cases} x-y-2z=-5, \\ 3x+2y-2z=-1, \\ 2x+4y-4z=2. \end{cases}$

7. $\begin{cases} 3x-y-3z=6, \\ x-3y-5z=2, \\ 4x-2y-4z=6. \end{cases}$

8. $\begin{cases} 5x+2y+5z=7, \\ 3x+y-2z=9, \\ x-2y-4z=4. \end{cases}$

9. $\begin{cases} 2x+y+4z=4, \\ x+5y-6z=3, \\ -2x-4y+5z=-1. \end{cases}$

10. $\begin{cases} x-2y-3z=1, \\ -x+4y+5z=1, \\ 3x-3y-5z=4. \end{cases}$

11. $\begin{cases} x+2y-z=5, \\ 7x-4y+2z=8, \\ 2x-8y+2z=-6. \end{cases}$

12. $\begin{cases} 3x+4y-3z=2, \\ -x+2y-2z=3, \\ 2x+5y-6z=2. \end{cases}$

13. $\begin{cases} x-7y-9z=-2, \\ 3x+2y+z=7, \\ 3x-5y-7z=2. \end{cases}$

14. $\begin{cases} 3x-2y+z=3, \\ x+4y+6z=0, \\ 4x-3y+4z=1. \end{cases}$

15. $\begin{cases} -2x+3y-5z=3, \\ x-2y+3z=-2, \\ 4x-y+2z=-4. \end{cases}$

Task 1.4. Solve the system provided it is consistent.

1. $\begin{cases} 5x_1 + 4x_2 - 17x_3 + 6x_4 = 13, \\ 2x_1 + x_2 - 5x_3 + 3x_4 = 4, \\ 2x_1 + 3x_2 - 11x_3 + x_4 = 8, \\ x_1 - x_3 + 2x_4 = 1. \end{cases}$

2. $\begin{cases} x_1 + x_2 - 3x_3 + 3x_4 = 1, \\ 3x_1 + x_2 - 7x_3 + 5x_4 = 5, \\ 5x_1 + 2x_2 - 12x_3 + 9x_4 = 8, \\ x_1 - x_2 - x_3 - x_4 = 3. \end{cases}$

3. $\begin{cases} 3x_1 + x_2 + 10x_3 - 2x_4 = 5, \\ 2x_1 + 3x_2 + 9x_3 + x_4 = 1 \\ x_1 + x_2 + 4x_3 = 1, \\ x_1 + 2x_2 + 5x_3 + x_4 = 0. \end{cases}$

4. $\begin{cases} x_1 + 3x_2 - 2x_3 + 4x_4 = 6, \\ 2x_1 + 3x_2 + 2x_3 + 5x_4 = 9, \\ 5x_1 + 20x_2 + 5x_4 = 15, \\ x_1 - x_2 + 6x_3 = 2. \end{cases}$

5. $\begin{cases} 7x_1 + 3x_2 - 2x_3 + 11x_4 = -9, \\ 3x_1 + x_2 + 5x_4 = -5, \\ x_1 + x_2 - 2x_3 + x_4 = 1, \\ 2x_1 + x_2 - x_3 + 3x_4 = -2. \end{cases}$

6. $\begin{cases} 2x_1 + 3x_2 - 5x_3 - 2x_4 = 14, \\ x_1 + x_2 - x_3 - x_4 = 5, \\ 3x_1 + 4x_2 - 6x_3 - 3x_4 = 19, \\ x_1 + 2x_2 - 4x_3 - x_4 = 9. \end{cases}$

7. $\begin{cases} 4x_1+3x_2+26x_3-x_4=10, \\ 2x_1+x_2+12x_3-x_4=4, \\ 3x_1+2x_2+19x_3-x_4=7, \\ x_1+x_2+7x_3=3. \end{cases}$

8. $\begin{cases} 2x_1+x_2+8x_4=4, \\ 4x_1+x_2-4x_3+14x_4=6, \\ x_1+x_2+2x_3+5x_4=3, \\ 5x_1+2x_2-2x_3+19x_4=9. \end{cases}$

9. $\begin{cases} 3x_1+2x_2+14x_3-7x_4=7, \\ 2x_1+x_2+8x_3-5x_4=4, \\ x_1+x_2+6x_3-2x_4=3, \\ 4x_1+3x_2+20x_3-9x_4=10. \end{cases}$

10. $\begin{cases} 2x_1+x_2+5x_3+2x_4=5, \\ x_1+x_2+4x_3=3, \\ 3x_1+x_2+6x_3+4x_4=7, \\ x_1+2x_2+7x_3-2x_4=4. \end{cases}$

11. $\begin{cases} 2x_1+x_2-x_3+6x_4=0, \\ x_1+x_2+x_3+5x_4=1, \\ 3x_1+2x_2+11x_4=1, \\ 3x_1+x_2-3x_3+7x_4=-1. \end{cases}$

12. $\begin{cases} 3x_1+x_2+10x_3=2, \\ 5x_1+2x_2+18x_3-x_4=5, \\ x_1+x_2+6x_3-2x_4=4, \\ 2x_1+x_2+8x_3-x_4=3. \end{cases}$

13. $\begin{cases} 3x_1+x_2-3x_3-x_4=-5, \\ 5x_1+2x_2-4x_3-3x_4=-8, \\ x_1+x_2+x_3-3x_4=-1, \\ 2x_1+x_2-x_3-2x_4=-3. \end{cases}$

14. $\begin{cases} 4x_1+3x_2+x_3-11x_4=9, \\ 7x_1+4x_2-2x_3-13x_4=12, \\ x_1+x_2+x_3-4x_4=3, \\ 5x_1+3x_2-x_3-10x_4=9. \end{cases}$

15. $\begin{cases} 2x_1+x_2+8x_3+x_4=5, \\ x_1+2x_2+10x_3+5x_4=4, \\ 2x_1+x_2+8x_3+x_4=5, \\ 3x_1+x_2+10x_3=7. \end{cases}$

Task 1.5. Three points A, B, C are given. Find $\cos\left(\bar{a},\hat{}\,\bar{b}\right)$.

1. $A(1,2,0)$, $B(3,-2,3)$, $C(2,1,-1)$, $\bar{a}=2\overline{AB}-\overline{AC}$, $\bar{b}=\overline{AB}$.
2. $A(2,-1,1)$, $B(1,2,2)$, $C(3,2,-2)$, $\bar{a}=\overline{BA}-3\overline{AC}$, $\bar{b}=\overline{AC}$.
3. $A(1,0,-1)$, $B(2,-1,3)$, $C(4,1,0)$, $\bar{a}=2\overline{AB}+\overline{BC}$, $\bar{b}=\overline{BC}$.
4. $A(0,-1,2)$, $B(3,2,4)$, $C(2,-2,1)$, $\bar{a}=\overline{BA}-4\overline{AC}$, $\bar{b}=\overline{AB}$.
5. $A(3,1,-2)$, $B(4,3,1)$, $C(-1,0,1)$, $\bar{a}=\overline{AB}-2\overline{CB}$, $\bar{b}=\overline{CB}$.
6. $A(4,0,-1)$, $B(5,1,1)$, $C(1,-1,0)$, $\bar{a}=2\overline{BA}+\overline{BC}$, $\bar{b}=\overline{BA}$.
7. $A(2,1,0)$, $B(3,-1,2)$, $C(1,0,-2)$, $\bar{a}=\overline{AB}-3\overline{BC}$, $\bar{b}=\overline{BC}$.
8. $A(0,3,-1)$, $B(2,2,0)$, $C(-1,3,1)$, $\bar{a}=3\overline{AB}+\overline{CB}$, $\bar{b}=\overline{AB}$.
9. $A(1,-2,0)$, $B(3,1,-1)$, $C(1,0,2)$, $\bar{a}=\overline{AB}-4\overline{CA}$, $\bar{b}=\overline{CA}$.
10. $A(4,-1,2)$, $B(2,0,3)$, $C(1,-1,4)$, $\bar{a}=2\overline{CB}-\overline{AB}$, $\bar{b}=\overline{AB}$.
11. $A(3,-2,1)$, $B(1,3,5)$, $C(0,1,-2)$, $\bar{a}=4\overline{BA}+\overline{CA}$, $\bar{b}=\overline{BA}$.
12. $A(-1,0,3)$, $B(3,-2,1)$, $C(2,3,-1)$, $\bar{a}=\overline{BC}-5\overline{AC}$, $\bar{b}=\overline{BC}$.
13. $A(1,2,-1)$, $B(4,1,1)$, $C(2,-1,0)$, $\bar{a}=\overline{AB}-4\overline{BC}$, $\bar{b}=\overline{AB}$.
14. $A(0,-1,3)$, $B(2,1,0)$, $C(1,2,-1)$, $\bar{a}=3\overline{BA}-\overline{CB}$, $\bar{b}=\overline{BA}$.
15. $A(2,1,-1)$, $B(3,2,1)$, $C(-2,3,1)$, $\bar{a}=\overline{AC}-3\overline{AB}$, $\bar{b}=\overline{AC}$.

Task 1.6. Find the value of a parameter p to satisfy the required condition.

1. $\bar{a}=(2,-4,p)$, $\bar{b}=(10,-20,15)$, and $\bar{a}\parallel\bar{b}$.
2. $\bar{a}=(-2,5,-2)$, $\bar{b}=(3,-4,p)$, and $\bar{a}\perp\bar{b}$.
3. $\bar{a}=(12,18,-36)$, $\bar{b}=(8,p,-24)$, and $\bar{a}\parallel\bar{b}$.
4. $\bar{a}=(4,3,-7)$, $\bar{b}=(-3,p,6)$, and $\bar{a}\perp\bar{b}$.
5. $\bar{a}=(-3,6,-15)$, $\bar{b}=(p,-4,10)$, and $\bar{a}\parallel\bar{b}$.
6. $\bar{a}=(5,-2,7)$, $\bar{b}=(p,3,-2)$, and $\bar{a}\perp\bar{b}$.
7. $\bar{a}=(20,p,-32)$, $\bar{b}=(-15,6,24)$, and $\bar{a}\parallel\bar{b}$.
8. $\bar{a}=(p,5,-3)$, $\bar{b}=(-2,3,7)$, and $\bar{a}\perp\bar{b}$.
9. $\bar{a}=(-40,10,p)$, $\bar{b}=(48,-12,8)$, and $\bar{a}\parallel\bar{b}$.
10. $\bar{a}=(-2,6,p)$, $\bar{b}=(4,-2,-5)$, and $\bar{a}\perp\bar{b}$.
11. $\bar{a}=(-5,6,-3)$, $\bar{b}=(20,p,12)$, and $\bar{a}\parallel\bar{b}$.
12. $\bar{a}=(p,-6,16)$, $\bar{b}=(8,4,-2)$, and $\bar{a}\perp\bar{b}$.
13. $\bar{a}=(-15,6,9)$, $\bar{b}=(20,p,-12)$, and $\bar{a}\parallel\bar{b}$.
14. $\bar{a}=(-6,-4,3)$, $\bar{b}=(p,9,4)$, and $\bar{a}\perp\bar{b}$.
15. $\bar{a}=(20,p,10)$, $\bar{b}=(-8,10,-4)$, and $\bar{a}\parallel\bar{b}$.

Task 1.7. Two vectors $\bar{m}$ and $\bar{n}$ are given. Find:
a) the scalar product $\bar{m}\cdot\bar{n}$;
b) the area S of the parallelogram formed by the vectors $\bar{m}$ and $\bar{n}$.

1. $\bar{m}=2\bar{a}-\bar{b}$, $\bar{n}=3\bar{a}+\bar{b}$, $|\bar{a}|=2$, $|\bar{b}|=3$, $\angle(\bar{a},\bar{b})=\pi/3$.
2. $\bar{m}=\bar{a}-5\bar{b}$, $\bar{n}=4\bar{a}+3\bar{b}$, $|\bar{a}|=1$, $|\bar{b}|=2$, $\angle(\bar{a},\bar{b})=\pi/6$.
3. $\bar{m}=4\bar{a}+\bar{b}$, $\bar{n}=\bar{a}-2\bar{b}$, $|\bar{a}|=3$, $|\bar{b}|=\sqrt{2}$, $\angle(\bar{a},\bar{b})=3\pi/4$.
4. $\bar{m}=\bar{a}+3\bar{b}$, $\bar{n}=5\bar{a}-2\bar{b}$, $|\bar{a}|=2$, $|\bar{b}|=1$, $\angle(\bar{a},\bar{b})=5\pi/6$.
5. $\bar{m}=3\bar{a}-\bar{b}$, $\bar{n}=6\bar{a}+\bar{b}$, $|\bar{a}|=\sqrt{2}$, $|\bar{b}|=1$, $\angle(\bar{a},\bar{b})=\pi/4$.

6. $\overline{m}=\overline{a}-2\overline{b}$, $\overline{n}=3\overline{a}+4\overline{b}$, $|\overline{a}|=1$, $|\overline{b}|=3$, $\angle(\overline{a},\overline{b})=\pi/2$.
7. $\overline{m}=4\overline{a}-3\overline{b}$, $\overline{n}=\overline{a}+5\overline{b}$, $|\overline{a}|=3$, $|\overline{b}|=2$, $\angle(\overline{a},\overline{b})=2\pi/3$.
8. $\overline{m}=3\overline{a}+2\overline{b}$, $\overline{n}=6\overline{a}-\overline{b}$, $|\overline{a}|=1$, $|\overline{b}|=4$, $\angle(\overline{a},\overline{b})=\pi/6$.
9. $\overline{m}=2\overline{a}+\overline{b}$, $\overline{n}=3\overline{a}-4\overline{b}$, $|\overline{a}|=2$, $|\overline{b}|=\sqrt{2}$, $\angle(\overline{a},\overline{b})=3\pi/4$.
10. $\overline{m}=\overline{a}-3\overline{b}$, $\overline{n}=2\overline{a}+5\overline{b}$, $|\overline{a}|=4$, $|\overline{b}|=3$, $\angle(\overline{a},\overline{b})=5\pi/6$.
11. $\overline{m}=4\overline{a}+3\overline{b}$, $\overline{n}=\overline{a}-5\overline{b}$, $|\overline{a}|=2\sqrt{2}$, $|\overline{b}|=1$, $\angle(\overline{a},\overline{b})=\pi/4$.
12. $\overline{m}=3\overline{a}-2\overline{b}$, $\overline{n}=\overline{a}+7\overline{b}$, $|\overline{a}|=1$, $|\overline{b}|=3$, $\angle(\overline{a},\overline{b})=\pi/3$.
13. $\overline{m}=\overline{a}+3\overline{b}$, $\overline{n}=2\overline{a}-\overline{b}$, $|\overline{a}|=5$, $|\overline{b}|=2$, $\angle(\overline{a},\overline{b})=\pi/2$.
14. $\overline{m}=\overline{a}+5\overline{b}$, $\overline{n}=\overline{a}-2\overline{b}$, $|\overline{a}|=4$, $|\overline{b}|=1$, $\angle(\overline{a},\overline{b})=2\pi/3$.
15. $\overline{m}=\overline{a}-5\overline{b}$, $\overline{n}=4\overline{a}+\overline{b}$, $|\overline{a}|=2$, $|\overline{b}|=5$, $\angle(\overline{a},\overline{b})=\pi/6$.

Task 1.8. The points A, B, and the force $\overline{F}$ are given. Find:

a) the work done by the force $\overline{F}$ that moves an object from the point A to the point B along a straight line;

b) the magnitude of the moment $\overline{M}$ of the force $\overline{F}$, applied at the point A, with respect to the point B.

1. $\overline{F}=(2,1,-4)$, $A(0,-1,3)$, $B(2,1,0)$.
2. $\overline{F}=(1,-2,3)$, $A(1,2,-5)$, $B(3,-3,-2)$.
3. $\overline{F}=(4,3,-2)$, $A(5,-3,1)$, $B(2,4,-3)$.
4. $\overline{F}=(-2,6,-3)$, $A(4,7,-5)$, $B(6,5,-1)$.
5. $\overline{F}=(8,-5,2)$, $A(-2,1,3)$, $B(5,4,1)$.
6. $\overline{F}=(3,-2,1)$, $A(1,-5,2)$, $B(4,-1,3)$.
7. $\overline{F}=(-7,4,2)$, $A(3,1,-5)$, $B(6,3,-2)$.
8. $\overline{F}=(5,-3,8)$, $A(2,-4,6)$, $B(5,-3,7)$.
9. $\overline{F}=(-3,2,-5)$, $A(4,5,-1)$, $B(5,2,2)$.
10. $\overline{F}=(4,-1,2)$, $A(-2,1,3)$, $B(1,-4,2)$.

11. $\overline{F} = (2,3,-1)$, $A(3,-1,2)$, $B(4,0,5)$.
12. $\overline{F} = (6,-3,1)$, $A(6,5,-1)$, $B(7,8,2)$.
13. $\overline{F} = (5,1,-1)$, $A(-4,2,-2)$, $B(0,3,1)$.
14. $\overline{F} = (-2,3,4)$, $A(3,-1,2)$, $B(4,1,3)$.
15. $\overline{F} = (5,-1,2)$, $A(-1,0,3)$, $B(2,4,5)$.

Task 1.9. The tetrahedron is given by its vertices A, B, C, and D. Find the altitude from the point D to the base ABC.

1. $A(2,-4,-3)$, $B(-2,-3,7)$, $C(-1,-2,7)$, $D(-3,-1,-8)$.
2. $A(1,3,5)$, $B(3,1,2)$, $C(5,3,11)$, $D(6,5,1)$.
3. $A(1,4,-7)$, $B(-3,5,3)$, $C(-2,6,3)$, $D(2,7,-12)$.
4. $A(-1,-2,-1)$, $B(-1,-6,-9)$, $C(3,-4,-6)$, $D(7,-7,-4)$.
5. $A(1,-1,6)$, $B(5,-2,-4)$, $C(4,-3,-4)$, $D(6,-4,11)$.
6. $A(-3,8,-6)$, $B(-5,10,-3)$, $C(-7,8,-12)$, $D(-8,6,-2)$.
7. $A(1,4,-4)$, $B(5,3,-14)$, $C(4,2,-14)$, $D(0,1,1)$.
8. $A(1,-1,5)$, $B(6,6,9)$, $C(10,9,7)$, $D(3,10,2)$.
9. $A(3,-1,0)$, $B(-2,-8,-4)$, $C(-6,-11,-2)$, $D(1,-12,3)$.
10. $A(4,-3,-6)$, $B(2,-1,-3)$, $C(0,-3,-12)$, $D(-1,-5,-2)$.
11. $A(6,-1,-4)$, $B(4,1,-1)$, $C(2,-1,-10)$, $D(1,-3,0)$.
12. $A(0,4,7)$, $B(4,3,-3)$, $C(3,2,-3)$, $D(-1,1,12)$.
13. $A(3,0,-3)$, $B(3,4,5)$, $C(-1,2,2)$, $D(-5,5,0)$.
14. $A(3,2,2)$, $B(8,9,6)$, $C(12,12,4)$, $D(5,13,-1)$.
15. $A(-1,5,2)$, $B(-1,1,-6)$, $C(3,3,-3)$, $D(7,0,-1)$.

2 ANALYTICAL GEOMETRY

Task 2.1. Let ABC be a triangle with the vertices at the points A, B and C in the xy–plane. Sketch the drawing. Find:

a) the length of the side BC and its equations;
b) the equation of the median AM from the vertex A;
c) the length of the altitude AH from the vertex A and its equation;
d) the equation of the line l passing through the point A parallel to BC;
e) the tangent of the angle B.

1. $A(7,-1)$, $B(-1,1)$, $C(2,5)$.
2. $A(-2,-3)$, $B(5,-2)$, $C(2,2)$.
3. $A(3,4)$, $B(-5,-2)$, $C(-1,1)$.
4. $A(3,2)$, $B(-3,-1)$, $C(1,-4)$.
5. $A(-4,1)$, $B(0,-1)$, $C(-3,-5)$.
6. $A(-2,4)$, $B(-1,-1)$, $C(3,2)$.
7. $A(6,3)$, $B(1,2)$, $C(4,-2)$.
8. $A(-2,-1)$, $B(4,1)$, $C(1,5)$.
9. $A(0,7)$, $B(-1,1)$, $C(-5,4)$.
10. $A(2,-4)$, $B(3,1)$, $C(-1,-2)$.
11. $A(-1,2)$, $B(2,-1)$, $C(5,3)$.
12. $A(-1,-5)$, $B(6,-3)$, $C(3,1)$.
13. $A(-1,3)$, $B(1,-4)$, $C(5,-1)$.
14. $A(3,1)$, $B(-2,-2)$, $C(2,-5)$.
15. $A(-2,3)$, $B(3,6)$, $C(6,2)$.

Task 2.2. The points A, B, C and D are given. Find:

a) the equation of the plane (ABC) in the intercept form and sketch the drawing;
b) the equation of the line l_1 passing through the point C and being parallel to the line AB;

c) the equation of the plane passing through the point C and being perpendicular to the line AB;

d) the equation of the line l_2 passing through the point D and being perpendicular to the plane (ABC);

e) the equation of the plane passing through the point D and being parallel to the plane (ABC);

f) the distance from the point D to the plane (ABC);

g) the point P of intersection of the line l_2 and the plane (ABC);

h) the angle between the lines AB and CD;

i) the angle between the line CD and the plane (ABC).

1. $A(-6,4,3)$, $B(6,3,-2)$, $C(0,1,3)$, $D(8,1,8)$.
2. $A(1,3,-1)$, $B(2,-5,2)$, $C(3,-1,-1)$, $D(3,4,1)$.
3. $A(-7,3,2)$, $B(5,2,-3)$, $C(-1,0,2)$, $D(7,0,7)$.
4. $A(2,0,-2)$, $B(0,-2,1)$, $C(4,-4,1)$, $D(8,6,-2)$.
5. $A(2,3,0)$, $B(-1,5,4)$, $C(4,5,-1)$, $D(-1,6,0)$.
6. $A(4,3,-1)$, $B(1,4,3)$, $C(4,4,-3)$, $D(5,7,-2)$.
7. $A(-8,2,1)$, $B(4,1,-4)$, $C(-2,-1,1)$, $D(6,-2,6)$.
8. $A(2,1,-3)$, $B(-1,2,1)$, $C(2,2,-5)$, $D(3,5,-4)$.
9. $A(0,1,-2)$, $B(-3,3,2)$, $C(2,3,-3)$, $D(-3,4,-2)$.
10. $A(2,6,2)$, $B(-1,4,4)$, $C(4,4,-1)$, $D(3,5,5)$.
11. $A(3,3,4)$, $B(4,4,8)$, $C(4,3,6)$, $D(4,6,3)$.
12. $A(5,4,0)$, $B(2,5,4)$, $C(5,5,-2)$, $D(6,8,-1)$.
13. $A(3,3,3)$, $B(5,4,5)$, $C(4,4,3)$, $D(2,5,6)$.
14. $A(2,3,2)$, $B(3,5,4)$, $C(1,5,2)$, $D(1,2,5)$.
15. $A(2,3,0)$, $B(-1,5,4)$, $C(4,5,-1)$, $D(-1,6,0)$.

Task 2.3. Reduce the equations of the curves to their canonical forms. If the curve is an ellipse or a hyperbola find its center and its semi-axes. If the curve is a parabola find its vertex, the intersection points with the coordinate axes, the equation of its axis of symmetry. Sketch the graphs of the curves.

1. a) $9x^2+4y^2-54x+16y+61=0$;
   b) $4x^2-25y^2+16x+50y-109=0$;
   c) $x^2+2x-8y+9=0$.
2. a) $25x^2+9y^2-100x+54y-44=0$;
   b) $4x^2-25y^2+24x-50y-89=0$;
   c) $y^2-8y-2x+12=0$.
3. a) $36x^2+9y^2-72x+72y-144=0$;
   b) $25x^2-16y^2-250x-128y-31=0$;
   c) $x^2-6x-6y-15=0$.
4. a) $x^2+16y^2+4x-192y+564=0$;
   b) $4x^2-49y^2-64x-98y+11=0$;
   c) $y^2+10y-8x+73=0$.
5. a) $25x^2+16y^2+250x-64y+286=0$;
   b) $9x^2-64y^2-18x-512y-1591=0$;
   c) $x^2+4x+10y+34=0$.
6. a) $36x^2+25y^2+216x-250y+49=0$;
   b) $9x^2-4y^2-36x-48y-144=0$;
   c) $y^2-12y-2x+44=0$.
7. a) $25x^2+36y^2+250x-288y+301=0$;
   b) $9x^2-4y^2-72x-40y+8=0$;
   c) $x^2-2x-12y-59=0$.
8. a) $4x^2+36y^2-64x+144y+256=0$;
   b) $49x^2-9y^2+98x+126y-833=0$;
   c) $y^2+6y+4x-11=0$.
9. a) $64x^2+16y^2+640x-32y+592=0$;
   b) $9x^2-81y^2-54x-648y-1944=0$;

c) $x^2+8x-6y+22=0$.

10. a) $9x^2+4y^2-108x+56y+484=0$;
    b) $100x^2-25y^2+1200x-100y+1000=0$;
    c) $y^2-4y-14x-10=0$.
11. a) $100x^2+64y^2+800x-640y-3200=0$;
    b) $49x^2-16y^2+294x+64y-407=0$;
    c) $x^2-12x+8y+76=0$.
12. a) $25x^2+4y^2-350x+16y+1141=0$;
    b) $16x^2-4y^2+96x+40y-20=0$;
    c) $y^2+18y-4x+97=0$.
13. a) $49x^2+25y^2+784x-150y+2136=0$;
    b) $64x^2-4y^2-768x-40y+1948=0$;
    c) $x^2+4x-6y+32=0$.
14. a) $16x^2+9y^2+128x-18y+121=0$;
    b) $49x^2-16y^2-588x-32y+964=0$;
    c) $y^2-6y+4x+17=0$.
15. a) $81x^2+49y^2+1134x-392y+784=0$;
    b) $4x^2-64y^2+16x+384y-816=0$;
    c) $x^2-10x+2y+31=0$.

Task 2.4. Reduce the equation of the surface to the canonical form. Determine the type of the surface and sketch the drawing.

1. a) $9x^2+4y^2+36z^2-18x+16y-216z+313=0$;
   b) $y^2-2y-10x+11=0$.
2. a) $4x^2+9y^2+36z^2+8x-54y+288z+625=0$;
   b) $9x^2+36y^2-4z^2-36x+72y-24z=0$.

3. a) $36x^2+9y^2+4z^2+72x-54y-16z+97=0$;
   b) $9x^2+4y^2-18x+24y+9=0$.
4. a) $4x^2+36y^2+9z^2+24x-72y+36z+72=0$;
   b) $9x^2-4y^2+36x+8y-4=0$.
5. a) $36x^2+9y^2+4z^2-144x+54y+32z+253=0$;
   b) $9x^2+4y^2-36z^2-36x+24y+36=0$.
6. a) $9x^2+4y^2+36z^2+36x+32y-216z+388=0$;
   b) $y^2-6y-10x+39=0$.
7. a) $36x^2+4y^2+9z^2+144x-32y-54z+253=0$;
   b) $x^2+y^2+8x-2y+8=0$.
8. a) $9x^2+4y^2+36z^2+72x-16y+216z+448=0$;
   b) $4x^2+y^2-4z^2-8x+4y-8z=0$.
9. a) $9x^2+36y^2+4z^2-36x-216y-32z+388=0$;
   b) $36x^2+4y^2-9z^2-72x+8y-36z-32=0$.
10. a) $4x^2+9y^2+36z^2-24x+36y-288z+612=0$;
    b) $y^2-4y-8x-6=0$.
11. a) $9x^2+36y^2+4z^2-36x+288y+24z+612=0$;
    b) $9x^2+4y^2+36x-32y+64=0$.
12. a) $36x^2+9y^2+4z^2+216x-72y+24z+468=0$;
    b) $x^2+y^2-6x+2y+6=0$.
13. a) $4x^2+36y^2+9z^2-32x+216y-36z+388=0$;
    b) $x^2+9y^2-9z^2-4x+36y-18z+31=0$.
14. a) $9x^2+36y^2+4z^2-54x+288y-24z+657=0$;
    b) $x^2-4x-6y-8=0$.
15. a) $4x^2+9y^2+36z^2-24x-36y+288z+612=0$;
    b) $9x^2+4y^2+18x+40y+73=0$.

3 LIMITS OF FUNCTIONS OF A SINGLE VARIABLE AND THEIR DIFFERENTIAL CALCULUS

Task 3.1. Calculate the limits.

1. a) $\lim\limits_{x\to -3}\dfrac{2x^2-x-21}{x^2+x-6}$;

   b) $\lim\limits_{x\to \infty}\dfrac{4x^3-x^2+8x-2}{3x^2+7x+1}$;

   c) $\lim\limits_{x\to \infty}\dfrac{5x^2-x+8}{4x^2+6x+3}$;

   d) $\lim\limits_{x\to \infty}\dfrac{8-x+2x^2}{x^5-4x+2}$;

   e) $\lim\limits_{x\to 1}\dfrac{2-\sqrt{3x+1}}{x^2+x-2}$.

2. a) $\lim\limits_{x\to 1}\dfrac{2x^2-8x+6}{3x^2+9x-12}$;

   b) $\lim\limits_{x\to \infty}\dfrac{5x^4-7x+17}{11x^3+x-1}$;

   c) $\lim\limits_{x\to \infty}\dfrac{9x^2-4x+11}{6x^2-x-1}$;

   d) $\lim\limits_{x\to \infty}\dfrac{4+x+3x^2}{x^4-x^3+7}$;

   e) $\lim\limits_{x\to -1}\dfrac{2-\sqrt{2-2x}}{x^2-x-2}$.

3. a) $\lim\limits_{x\to -3}\dfrac{4x^2+16x+12}{2x^2+2x-12}$;

   b) $\lim\limits_{x\to \infty}\dfrac{x^3+4x^2+9x-11}{x^2-13x+4}$;

   c) $\lim\limits_{x\to \infty}\dfrac{x^2-12x+1}{5x^2-x-1}$;

   d) $\lim\limits_{x\to \infty}\dfrac{5+3x-3x^2}{-x^3+x^2-3}$;

   e) $\lim\limits_{x\to -1}\dfrac{\sqrt{1-8x}-3}{x^2-4x-5}$.

4. a) $\lim\limits_{x\to -1}\dfrac{x^2-6x-7}{3x^2-12x-15}$;

   b) $\lim\limits_{x\to \infty}\dfrac{-x^5+2x^3-x^2-12}{4x^4+17x^3+13}$;

   c) $\lim\limits_{x\to \infty}\dfrac{5x^2-7x+11}{x^2+4x+13}$;

   d) $\lim\limits_{x\to \infty}\dfrac{11+x+x^2-2x^3}{x^4-4x^3+x+3}$;

   e) $\lim\limits_{x\to -3}\dfrac{\sqrt{1-5x}-4}{x^2+x-6}$.

5. a) $\lim\limits_{x\to 2}\dfrac{3x^2+3x-18}{2x^2-10x+12}$;

   b) $\lim\limits_{x\to \infty}\dfrac{-2x^3+3x^2+11}{2x^2+14x+2}$;

   c) $\lim\limits_{x\to \infty}\dfrac{3x^2+x+1}{3x^2-5x+4}$;

6. a) $\lim\limits_{x\to 3}\dfrac{5x^2-20x+15}{4x^2-20x+24}$;

   b) $\lim\limits_{x\to \infty}\dfrac{x^4+x^3+3x^2-1}{7x^2+x-11}$;

   c) $\lim\limits_{x\to \infty}\dfrac{8x^2+2x+7}{3x^2+7x+1}$;

d) $\lim\limits_{x\to\infty}\dfrac{8-2x+x^3+x^4}{2x^5+x^4-3x^2+2}$;

e) $\lim\limits_{x\to 3}\dfrac{5-\sqrt{8x+1}}{x^2-5x+6}$.

7. a) $\lim\limits_{x\to -2}\dfrac{2x^2+2x-4}{4x^2-8x-32}$;

b) $\lim\limits_{x\to\infty}\dfrac{2x^3-6x+2}{x^2-4x+2}$;

c) $\lim\limits_{x\to\infty}\dfrac{2x^2+2x+7}{4x^2-5x+6}$;

d) $\lim\limits_{x\to\infty}\dfrac{-1-x-x^2}{x^3+x^2+x+1}$;

e) $\lim\limits_{x\to 2}\dfrac{3-\sqrt{2x+5}}{x^2-6x+4}$.

9. a) $\lim\limits_{x\to 4}\dfrac{x^2-7x+12}{2x^2-10x+8}$;

b) $\lim\limits_{x\to\infty}\dfrac{x^4-x^3+2x^2-x+1}{x^2+15x+7}$;

c) $\lim\limits_{x\to\infty}\dfrac{2x^2-x+5}{4x^2+3x+1}$;

d) $\lim\limits_{x\to\infty}\dfrac{7+3x-x^2}{2x^3-x^2-4x+2}$;

e) $\lim\limits_{x\to 3}\dfrac{2-\sqrt{2x-2}}{x^2-4x+3}$.

11. a) $\lim\limits_{x\to -2}\dfrac{x^2+x-2}{3x^2+15x+18}$;

b) $\lim\limits_{x\to\infty}\dfrac{x^5-x^3-7x-2}{-x^4+2x^2+11}$;

c) $\lim\limits_{x\to\infty}\dfrac{7x^2+9x+3}{4x^2+5x+2}$;

d) $\lim\limits_{x\to\infty}\dfrac{8-x+2x^2}{x^5-4x+2}$;

e) $\lim\limits_{x\to 2}\dfrac{3-\sqrt{4x+1}}{x^2-5x+6}$.

8. a) $\lim\limits_{x\to 2}\dfrac{3x^2-15x+18}{4x^2-12x+8}$;

b) $\lim\limits_{x\to\infty}\dfrac{3x^3-7x^2+3}{2x^2-3x+3}$;

c) $\lim\limits_{x\to\infty}\dfrac{3x^2+3x+8}{5x^2-4x+7}$;

d) $\lim\limits_{x\to\infty}\dfrac{1-2x-3x^2}{x^3+3x^2+4x+1}$;

e) $\lim\limits_{x\to -2}\dfrac{2-\sqrt{6+x}}{x^2-2x-16}$.

10. a) $\lim\limits_{x\to 3}\dfrac{3x^2-6x-9}{2x^2-8x+6}$;

b) $\lim\limits_{x\to\infty}\dfrac{x^4+3x^2+2}{2x^3+x^2+16x+8}$;

c) $\lim\limits_{x\to\infty}\dfrac{x^2+6}{5x^2+4x+2}$;

d) $\lim\limits_{x\to\infty}\dfrac{5+x-3x^3}{4x^4+x^2-2x+4}$;

e) $\lim\limits_{x\to 4}\dfrac{\sqrt{1+2x}-3}{x^2-5x+4}$.

12. a) $\lim\limits_{x\to 1}\dfrac{2x^2-6x+4}{3x^2-12x+9}$;

b) $\lim\limits_{x\to\infty}\dfrac{x^5+2x^2+x+1}{x^3-3x-5}$;

c) $\lim\limits_{x\to\infty}\dfrac{5x^2+11x+1}{6x^2+3x+4}$;

d) $\lim\limits_{x\to\infty}\dfrac{2+x-x^4}{3x^5-7x^4+1}$;

e) $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{x^2-4x+3}$.

13. a) $\lim\limits_{x\to -1}\dfrac{3x^2-3x-6}{4x^2+12x+8}$;

b) $\lim\limits_{x\to\infty}\dfrac{x^5+x^2-11x+4}{-5x^2-7x+11}$;

c) $\lim\limits_{x\to\infty}\dfrac{9x^2-2x+11}{-3x^2-8x+4}$;

d) $\lim\limits_{x\to\infty}\dfrac{4+4x-x^2}{2x^3+4x-11}$;

e) $\lim\limits_{x\to 2}\dfrac{\sqrt{4x-4}-2}{x^2-3x+2}$.

15. a) $\lim\limits_{x\to 4}\dfrac{2x^2-12x+16}{x^2-3x-4}$;

b) $\lim\limits_{x\to\infty}\dfrac{-x^4+x^3-3x^2+4x+7}{-2x^2-4x+2}$;

c) $\lim\limits_{x\to\infty}\dfrac{5x^2+3x+5}{x^2-3x+1}$;

d) $\lim\limits_{x\to\infty}\dfrac{3+4x-2x^2+x^3}{2x^5+3x^3+2x-1}$;

e) $\lim\limits_{x\to -4}\dfrac{\sqrt{1-2x}-3}{x^2+5x+4}$.

d) $\lim\limits_{x\to\infty}\dfrac{4-x^2+x^3}{x^4-5x^2+3}$;

e) $\lim\limits_{x\to -2}\dfrac{4-\sqrt{10-3x}}{x^2+5x+6}$.

14. a) $\lim\limits_{x\to 2}\dfrac{3x^2-9x+6}{x^2-x-2}$;

b) $\lim\limits_{x\to\infty}\dfrac{3x^5-x^4-9x+2}{-7x^2-9x+13}$;

c) $\lim\limits_{x\to\infty}\dfrac{8x^2-3x+10}{-2x^2-9x+5}$;

d) $\lim\limits_{x\to\infty}\dfrac{-3+2x-x^2}{-x^4+5x-1}$;

e) $\lim\limits_{x\to -1}\dfrac{2-\sqrt{1-3x}}{x^2+3x+2}$.

Task 3.2. Use the most important limits to calculate the limits.

1. a) $\lim\limits_{x\to 0}\dfrac{1-\cos 8x}{3x\tan 5x}$;

b) $\lim\limits_{x\to\infty}\left(\dfrac{2x-4}{2x+5}\right)^{2x-1}$.

2. a) $\lim\limits_{x\to 0}\dfrac{\cos 3x-\cos 5x}{x\sin 2x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{3x-1}{3x-3}\right)^{4x+5}$.

3. a) $\lim\limits_{x\to 0}\dfrac{\arcsin 4x^2}{\sin x\tan 2x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{5x-3}{5x+1}\right)^{x+4}$.

4. a) $\lim\limits_{x\to 0}\dfrac{1-\cos 4x}{\sin 2x\arctan 4x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{x-1}{x+4}\right)^{2x+1}$.

5. a) $\lim\limits_{x\to 0}\dfrac{3x\left(e^{2x}-1\right)}{\cos 7x-\cos x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{4x+3}{4x+1}\right)^{4x+7}$.

6. a) $\lim\limits_{x\to 0}\dfrac{\sin x\ln(1+5x)}{1-\cos 5x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{2x-3}{2x+8}\right)^{x+1}$.

7. a) $\lim\limits_{x\to 0}\dfrac{2x\ln(1+5x)}{\arcsin 3x\tan 4x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{3x+1}{3x-1}\right)^{4x-1}$.

8. a) $\lim\limits_{x\to 0}\dfrac{\left(e^{7x}-1\right)\sin 3x}{\tan x\arctan 2x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{3x+4}{3x+2}\right)^{2x+5}$.

9. a) $\lim\limits_{x\to 0}\dfrac{1-\cos 3x}{2x\arcsin 4x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{5x-1}{5x+1}\right)^{3x+4}$.

10. a) $\lim\limits_{x\to 0}\dfrac{\cos 5x-\cos 3x}{\left(e^{3x}-1\right)\sin 5x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{2x-7}{2x+4}\right)^{x-2}$.

11. a) $\lim\limits_{x\to 0}\dfrac{\sin 2x\tan 4x}{1-\cos 2x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{5x-3}{5x+1}\right)^{4x-5}$.

12. a) $\lim\limits_{x\to 0}\dfrac{\sin 2x\ln(1+x)}{\sin 3x\tan 4x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{3x-4}{3x+3}\right)^{2x+1}$.

13. a) $\lim\limits_{x\to 0}\dfrac{\left(e^{2x}-1\right)\tan 5x}{\arcsin 3x\tan 2x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+3}\right)^{3x-1}$.

14. a) $\lim\limits_{x\to 0}\dfrac{1-\cos 5x}{x\arctan 2x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{4x+3}{4x-3}\right)^{x+11}$.

15. a) $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos 3x}{\left(e^{3x}-1\right)5x}$; b) $\lim\limits_{x\to\infty}\left(\dfrac{x+4}{x-6}\right)^{5x+1}$.

Task 3.3. Differentiate the following functions.

1. a) $y = 3\sqrt[3]{x^2} - 5x^8 + \frac{3}{x^4} + 8e$;

   b) $y = \arcsin 4x \cdot \cos^3 4x - \frac{\sqrt{x^2 - 2x + 3}}{5^{\tan x}}$;

   c) $y = (\sin 3x)^{\ln x}$; d) $x^2 - y^2 + 3x - y = 0$.

2. a) $y = 5\sqrt[5]{x^3} + x^6 + \frac{2}{x^5} + \sin 5$;

   b) $y = \sqrt{x^3 + x - 11} \cdot e^{\cos x} - \frac{\tan 11x}{\ln(x^2 + 3)}$;

   c) $y = (\cos 2x)^{e^x}$; d) $x^2 + y^2 - 2x + 6y = 0$.

3. a) $y = 3\sqrt[3]{x^5} - \frac{3}{4}x^4 + \frac{5}{x^5} + 5\ln 17$;

   b) $y = \arccos 2x \cdot \tan^2 x + \frac{\sqrt{2x^2 - 3}}{7^{\sin x}}$;

   c) $y = (\ln 2x)^{\sin 2x}$; d) $x^2 + y^2 - x + 4y = 7$.

4. a) $y = 5\sqrt[5]{x^7} + \frac{5}{2}x^6 - \frac{3}{2x^4} + e^3$;

   b) $y = \sqrt{x^4 + 2^2 + 3} \cdot 5^{\ln x} - \frac{\arcsin 2x}{\cos^2 4x}$;

   c) $y = (\ln 3x)^{e^x}$; d) $x^2 + \frac{1}{2}y^2 + 6x + y = 0$.

5. a) $y = 5\sqrt[10]{x} + 4x^4 - \frac{2}{x^{11}} + \arctan 17$;

   b) $y = \arctan 3x \cdot \sin^2 2x - \frac{\sqrt{x^3 + x + 1}}{3^{\ln x}}$;

   c) $y = (x^3 + 15)^{\tan x}$; d) $\frac{1}{2}x^2 + 3y^2 + 2x + 6y = 0$.

6. a) $y = 3\sqrt[4]{x^5} + 3x^5 + \dfrac{5}{x^5} + 17\arcsin e$;

b) $y = \ln(2x+5)\cdot \tan^3 4x - \dfrac{\arctan 3x}{\sin^4 x}$;

c) $y = (\arctan x)^{x^2}$; d) $x^2 + y^3 + 4x - 3y = 0$.

7. a) $y = 5\sqrt[7]{x^{10}} - x^6 + \dfrac{1}{x^5} + \ln 15$;

b) $y = \tan 4x \cdot e^{3x+5} - \dfrac{\sqrt{x^4 + 5x^2 + 1}}{5^{2^x}}$;

c) $y = \left(x^3 + 5x - 17\right)^{\tan x}$; d) $x^3 + 2y^2 - 3x + 6y = 0$.

8. a) $y = 4\sqrt[6]{x^5} + 2x^{12} - \dfrac{2}{x^5} + 12\tan\dfrac{7\pi}{13}$;

b) $y = \sin\left(x^2 - 3\right)\cdot \tan^2 x - \dfrac{\ln\left(5 - x^3\right)}{\arcsin 2x}$;

c) $y = (\tan 3x)^{e^x}$; d) $x^3 + y^2 - 3x^2 + 8y = 0$.

9. a) $y = 3\sqrt[3]{x^7} + 3x^5 + \dfrac{5}{4x^4} + \sqrt{e+6}$;

b) $y = \tan^3 4x \cdot \arccos 5x - \dfrac{\sqrt{x^3 - x + 10}}{e^{x^2+5}}$;

c) $y = (\arctan 3x)^{\ln 2x}$; d) $\dfrac{3}{4}x^4 - 2y^2 - 3x + 4y = 0$.

10. a) $y = 5\sqrt[5]{x^3} - \dfrac{5}{4}x^{12} - \dfrac{2}{3x^3} + 8^e$;

b) $y = \sqrt{3x^2 - 4x + 1}\cdot \ln^3 3x - \dfrac{e^{x^2-2x+3}}{\tan 7x}$;

c) $y = \left(\ln(5-3x)\right)^{x^3-1}$; d) $x^2 + \dfrac{1}{3}y^3 + 2x - 4y = 0$.

11. a) $y = 6\sqrt[3]{x^7} + 3x^7 - \dfrac{5}{7x^7} + \tan^3 e$;

b) $y=\sin^3 4x\cdot\ln^4 5x-\dfrac{\sqrt{-2x^2+3}}{e^{\tan x}}$;

c) $y=(\cos 2x)^{\sin 7x}$; d) $\dfrac{7}{2}x^2-2y^2+7x+6y=0$.

12. a) $y=8\sqrt[4]{x^7}+3x^5-\dfrac{11}{3x^6}-\dfrac{e}{\tan 15}$;

b) $y=\sqrt{x^3-5x^2+2x-3}\cdot e^{3x^2-1}-\dfrac{\arccos 5x}{\ln(x+7)}$;

c) $y=(\tan 5x)^{\cos 7x}$; d) $x^2+\dfrac{7}{2}y^2-2x-14y=15$.

13. a) $y=5\sqrt[5]{x^2}+3x^4-\dfrac{8}{7x^7}+\ln 17$;

b) $y=\arccos 2x\cdot\tan^5 5x-\dfrac{\ln(x^2-5x-2)}{e^{\sin x}}$;

c) $y=(\ln 3x)^{7x^2-2}$; d) $x^3+y^3-6x+9y=0$.

14. a) $y=\dfrac{4}{5}\sqrt[4]{x^5}+x^7+\dfrac{7}{5x^5}+\sqrt{8e+5}$;

b) $y=\ln(4x^2+3x-2)\cdot\sin^2 2x-\dfrac{\arctan 5x}{\cos(5x+1)}$;

c) $y=(x^3+x^2+5)^{x^2+7}$; d) $x^3+y^3-3x^2-6y+7=0$.

15. a) $y=5\sqrt[5]{x^8}+5x^4-\dfrac{1}{5x^{10}}+\arctan 25$;

b) $y=-\arccos 3x\cdot\tan^4 2x+\dfrac{\sqrt{2x^2+x-7}}{\ln(3x-7)}$;

c) $y=(\arctan 3x)^{\ln 2x}$; d) $x^2+y^4-6x+4y=4$.

Task 3.4. Find the first and second derivatives of the parametric function.

1. $\begin{cases} x = 4(t - \sin t), \\ y = 4(1 - \cos t). \end{cases}$
2. $\begin{cases} x = 6t^3 - t - 7, \\ y = t^5 + 11t. \end{cases}$
3. $\begin{cases} x = 2\sin^3 t, \\ y = 2\cos^3 t. \end{cases}$
4. $\begin{cases} x = \ln t, \\ y = t^3 + \dfrac{1}{t}. \end{cases}$
5. $\begin{cases} x = t^4 + 2t^2, \\ y = \operatorname{arctan} t. \end{cases}$
6. $\begin{cases} x = 3\sin 2t, \\ y = 4\cos^2 t. \end{cases}$
7. $\begin{cases} x = \sin 2t, \\ y = \cos 2t. \end{cases}$
8. $\begin{cases} x = 3(t - e^t), \\ y = 2(1 + e^t). \end{cases}$
9. $\begin{cases} x = t\sin 2t, \\ y = 2t\cos 2t. \end{cases}$
10. $\begin{cases} x = e^{t^2}, \\ y = 5 + \ln 2t. \end{cases}$
11. $\begin{cases} x = 4\cos\dfrac{t}{2}, \\ y = e^{2t}. \end{cases}$
12. $\begin{cases} x = \cos 2t, \\ y = 1 - \sin t. \end{cases}$
13. $\begin{cases} x = \dfrac{t^2}{2} + t, \\ y = \ln(t + 1). \end{cases}$
14. $\begin{cases} x = \dfrac{t - 1}{t^2 + 1}, \\ y = 2arcctgt. \end{cases}$
15. $\begin{cases} x = e^{t^2 + t}, \\ y = \ln(2t + 1). \end{cases}$

Task 3.5. Applying L'Hospital's rule find the following limits.

1. $\lim\limits_{x \to \infty} \dfrac{3x^2 - 2e^{5x}}{7e^{2x}}$.
2. $\lim\limits_{x \to 0} \dfrac{\tan x}{\sin x - x}$.
3. $\lim\limits_{x \to \infty} \left(\dfrac{\pi}{2} - \operatorname{arctan} x \right) \ln x$.
4. $\lim\limits_{x \to \infty} \left(e^{\frac{1}{x}} - 1 \right) x$.

5. $\lim\limits_{x\to 1}\dfrac{x^3-2x^2-x+2}{x^3-7x+6}$.

6. $\lim\limits_{x\to 0}\dfrac{x\cos x-\sin x}{x^3}$.

7. $\lim\limits_{x\to 0}\dfrac{\cosh x-1}{1-\cos x}$.

8. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{\tan x}{\tan 3x}$.

9. $\lim\limits_{x\to 0}(1-\cos x)\cot x$.

10. $\lim\limits_{x\to \infty} x\sin\left(\dfrac{3}{x}\right)$.

11. $\lim\limits_{x\to 0}\dfrac{\tan x-\sin x}{4x-\sin x}$.

12. $\lim\limits_{x\to 1}\dfrac{1-x}{1-\sin\left(\dfrac{\pi x}{2}\right)}$.

13. $\lim\limits_{x\to 1}\dfrac{1-4\sin^2\left(\dfrac{\pi x}{6}\right)}{1-x^2}$.

14. $\lim\limits_{x\to \infty}\dfrac{\ln(x+3)}{\sqrt{x+5}}$.

15. $\lim\limits_{x\to -1}\dfrac{\sqrt[3]{1+2x}+1}{\sqrt{2+x}+x}$.

Task 3.6. Sketch the graphs of the functions.

1. $y=\dfrac{2x^2}{x-3}$.

2. $y=\dfrac{x^3}{2x+1}$.

3. $y=\dfrac{5x}{(x-1)^2}$.

4. $y=2x+\dfrac{3}{x^3}$.

5. $y=\dfrac{x^3+1}{(x+1)^2}$.

6. $y=x-\dfrac{1}{x^4}$.

7. $y=\dfrac{1-x^3}{x^2-4}$.

8. $y=\dfrac{2x+5}{x^2+1}$.

9. $y=\dfrac{1}{x^2+2}-x$.

10. $y=\dfrac{4-x^2}{x+5}$.

11. $y=\dfrac{x+3}{x^3+8}$.

12. $y=x^2+\dfrac{8}{x^3}$.

13. $y=\dfrac{-x^2}{x+4}$.

14. $y=\dfrac{x^3}{x-3}$.

15. $y=\dfrac{2-x^2}{(x-1)^3}$.

4 DIFFERENTIAL CALCULUS OF FUNCTIONS OF SEVERAL INDEPENDENT VARIABLES

Task 4.1. Find and sketch the domain of definition of the function of two variables.

1. $z = \dfrac{\sqrt{9 - x^2 - y^2}}{\ln(x-3)}$.
2. $z = \ln(4 - x^2 - y^2) + \sqrt{x+1}$.
3. $z = \dfrac{\sqrt{9 - y^2}}{x^2 - 6x + 5}$.
4. $z = \arcsin(x+1) + \sqrt[3]{1-y}$.
5. $z = \arccos(3 + x^2) + \ln(1+y)$.
6. $z = \cos(xy+1) + \sqrt{x - y^2}$.
7. $z = \ln(36 - x^2 - y^2) + \dfrac{1}{x-y}$.
8. $z = \dfrac{\cos(x+y)}{\sqrt{16 - x^2 - y^2}}$.
9. $z = \dfrac{\sqrt{4 - x^2}}{\ln(x^2 - 1)}$.
10. $z = \sqrt{2 + 6x - 2y} - \log_2(x-y)$.
11. $z = \arccos(1 + x - y)$.
12. $z = \sqrt[4]{1 - x^2 + y^2} + e^{x+y^3}$.
13. $z = \sqrt{5-y} - 2^{\arcsin(x)}$.
14. $z = \dfrac{\ln(5x - y)}{2x + 3y - 6}$.
15. $z = \dfrac{\sqrt{4 - y^2}}{4 - x^2 - y^2}$.

Task 4.2. The function $z = f(x, y)$ is given. Find dz.

1. $z = \ln(5 + xy) + 3x^2 + 7y$.
2. $z = 5e^{x^2+5y} - 2xy + x + 7y$.
3. $z = \ln(5 - x + 3y) + \arctan(xy)$.
4. $z = \sin(x^2 - 4xy) + \cos(x - y)$.
5. $z = e^{x^2y+2x} + 7y^2x$.
6. $z = \tan(x + y) + 2\sqrt{x^2 + y^2}$.
7. $z = e^{\frac{x}{2}}(2x^2 + y^4)$.
8. $z = \sqrt{5} + x^3y + 7y^2 - 3xy$.
9. $z = \arcsin(xy) + \ln(x + 3y)$.
10. $z = e^{2x-3y}\ln(xy + 1)$.
11. $z = \sin(x^2 + 5xy - y^2)$.
12. $z = \tan(x)\cos(y^2 + 1)$.
13. $z = \frac{x^3y}{6} + 15\ln(5x + 3y)$.
14. $z = \arcsin(x + y) + e^{15-xy+y^2}$.
15. $z = x\sqrt{y} + 5(x + y) + 11x^2y^2$.

Task 4.3. Find the derivatives of composite functions:

a) $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$;

b) $\frac{dz}{dt}$; **c)** $\frac{dz}{dx}$.

1. a) $z = \sin(yx^2)$, $x = \sqrt{2u - v}$, $y = \sqrt{uv}$;

   b) $z = e^{2x-y-3}$, $x = \sin 2t$, $y = \cos 3t$;

   c) $z = xe^{2y}$, $y = \ln(2x - 3)$.

2. a) $z=\dfrac{1}{y^2+xy},\ x=\sin(2uv),\ y=u\sqrt{v}$;

   b) $z=\ln(x+y-1),\ x=\dfrac{1}{t},\ y=\tan t$;

   c) $z=\sqrt[3]{x+xy^2},\ y=2^{x+3}$.

3. a) $z=\sqrt{y-2x},\ x=\arctan(uv),\ y=u^2e^v$;

   b) $z=\dfrac{xy}{x^2+y^2},\ x=e^{2-t},\ y=\cos^3 t$;

   c) $z=y\ln(x+y),\ y=\sqrt[5]{x^3-x+7}$.

4. a) $z=\sinh\left(y\sqrt{x}\right),\ x=\dfrac{u}{v+5},\ y=e^{u^2+v^2}$;

   b) $z=\arcsin(3-x+4y),\ x=\cos t^2,\ y=\tan t$;

   c) $z=\sin x\cosh y,\ y=\cot(2x+1)$.

5. a) $z=\dfrac{1}{yx^2},\ x=\ln\left(3u+v^2\right),\ y=\cosh uv$;

   b) $z=e^{2xy+7},\ x=\arctan t,\ y=\cos\sqrt{t}$;

   c) $z=\sin(2y)e^x,\ y=\tan\left(x^2\right)$.

6. a) $z=\sqrt{yx+y^3+x},\ x=\sqrt[4]{5u^2v},\ y=\cos uv$;

   b) $z=\arcsin(x-y),\ x=\dfrac{1}{t^2},\ y=\ln t^2$;

   c) $z=\sin\left(y\sqrt{x}\right),\ y=\cot(3x+4y)$.

7. a) $z=\dfrac{2-x}{xy},\ x=e^{u(1+v)},\ y=\ln(3-u+2v)$;

   b) $z=\sinh\left(xy^2\right),\ x=\ln\sqrt{t},\ y=\sqrt[3]{t^2-1}$;

   c) $z=y^3e^{2-x},\ y=\tan x^2$.

8. a) $z=\arccos(xy),\ x=5^u+v^2,\ y=\dfrac{1}{u+v}$;

b) $z = \ln\left(x^2 + \sqrt{y}\right),\ x = \arcsin t,\ y = \dfrac{1}{\sqrt{t}}$;

c) $z = \sin\dfrac{x}{y},\ y = \sqrt[7]{5x+2}$.

9. a) $z = \ln\dfrac{x}{\sqrt{y}},\ x = \sin\left(\sqrt{uv}\right),\ y = e^{2uv^2}$;

b) $z = \sin(1-2x+3y),\ x = \sqrt{1-t^2},\ y = t^3 + 12t$;

c) $z = \tan(xy),\ y = \dfrac{1}{\sqrt{1-x}}$.

10. a) $z = \cot(x+y),\ x = \ln(u+v),\ y = u\sqrt[3]{v}$;

b) $z = x\sin y^2,\ x = \arcsin t,\ y = \ln t^2$;

c) $z = \sinh(5xy),\ y = e^{4-x}$.

11. a) $z = \sin\left(x + y^2\right),\ x = e^{uv},\ y = \ln\left(u + v^2\right)$;

b) $z = \arccos\left(y^2 - x\right),\ x = \sqrt{t^2 + 3t},\ y = \dfrac{1}{t+1}$;

c) $z = y(x+y)\ \ y = \sinh 2x$.

12. a) $z = \sqrt{y}\tan 3x,\ x = \dfrac{\ln v}{u},\ y = u\sqrt{v}$;

b) $z = e^{x+11y},\ x = \dfrac{t^2}{t+1},\ y = \cos\sqrt{t}$;

c) $z = x^3 \ln y,\ \ y = \arcsin(x+1)$.

13. a) $z = \tan(x-y),\ x = u\cos v,\ y = \sqrt{u - v^2}$;

b) $z = 2^{y-2x+1},\ x = \ln t,\ y = \sqrt{\cos t}$;

c) $z = \sinh\left(x^2 + y^2\right),\ \ y = \dfrac{2}{\sqrt{x}+4}$.

14. a) $z = \sin\left(1 - x^2 - y^2\right),\ x = v\sin u,\ y = u\cos v$;

b) $z = \arctan(x+2y),\ x = 2\sin 3t,\ y = 3\cos 2t$;

c) $z = \ln\left(x + e^y\right),\ \ y = \tan\left(2 - 3x^3\right)$.

15. a) $z = e^{\sqrt{x+y}}$, $x = u^2\sqrt{v+u}$, $y = 2^{u-2v}$;

b) $z = \arcsin(1-xy)$, $x = e^{\sqrt{t}}$, $y = t - 2\tan t$;

c) $z = x(xy - \sin y)$, $y = 2(5 + x^2)$.

Task 4.4. Examine $z = f(x, y)$ for maximum and minimum values.

1. $z = x^3 + 8y^3 - 6xy + 5$.
2. $z = 1 + 15x - 2x^2 - xy - 2y^2$.
3. $z = 1 + 6x - x^2 - xy - y^2$.
4. $z = x^3 + y^2 - 6xy - 39x + 18y + 20$.
5. $z = 2x^3 + 2y^3 - 6xy + 5$.
6. $z = 3x^3 + 3y^3 - 9xy + 10$.
7. $z = x^2 + xy + y^2 + x - y + 1$.
8. $z = 4(x - y) - x^2 - y^2$.
9. $z = x^2 + xy + y^2 - 6x - 9y$.
10. $z = 6(x - y) - 3x^2 - 3y^2$.
11. $z = x^3 - 3xy + y^3$.
12. $z = (x-2)^2 + 2y^2 - 10$.
13. $z = 2xy - 2x^2 - 4y^2$.
14. $z = (x-5)^2 + y^2 + 1$.
15. $z = xy(12 - x - y)$.

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Appendix A

Table A.1 – Equations and Graphs of Quadratic Curves

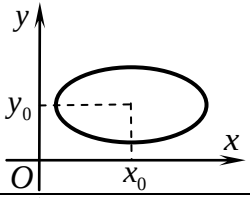
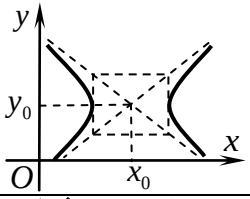
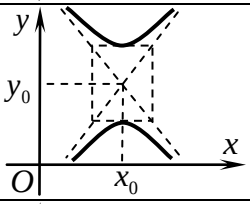
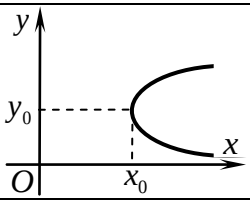
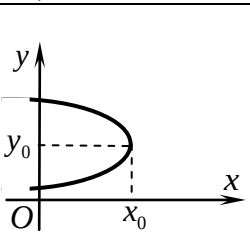
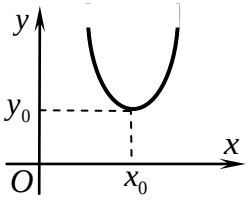
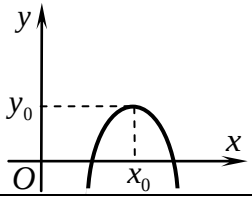
	The type of the curve	Equation	Graph
1	Ellipse	$\frac{(x-x_0)^2}{a^2}+\frac{(y-y_0)^2}{b^2}=1$	
2	Hyperbola	$\frac{(x-x_0)^2}{a^2}-\frac{(y-y_0)^2}{b^2}=1$	
		$\frac{(y-y_0)^2}{b^2}-\frac{(x-x_0)^2}{a^2}=1$	
3	Parabola	$(y-y_0)^2=2p(x-x_0)$, the equation of the axis of symmetry is $y=y_0$	
		$(y-y_0)^2=-2p(x-x_0)$, the equation of the axis of symmetry is $y=y_0$	

Table A.1 (continued)

	The type of the curve	Equation	Graph
3	Parabola	$(x-x_0)^2 = 2p(y-y_0)$, the equation of the axis of symmetry is $x = x_0$	
		$(x-x_0)^2 = -2p(y-y_0)$, the equation of the axis of symmetry is $x = x_0$	

Appendix B

Table B.1 – Canonical Equations of the Second-order Surfaces and their Graphs

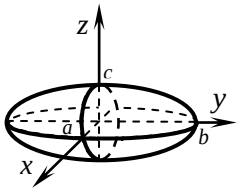
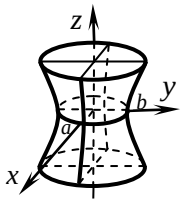
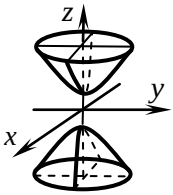
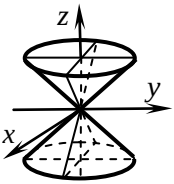
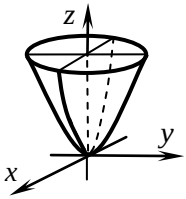
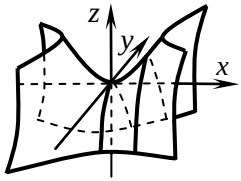
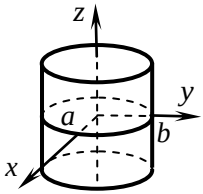
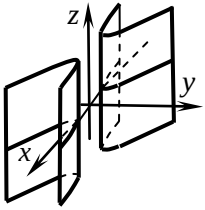
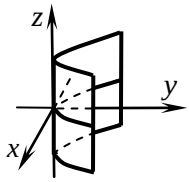
	The type of the surface	Equation	Graph
1	Ellipsoid	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	
2	Hyperboloid of one sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	
3	Hyperboloid of two sheets	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1$	
4	Second-order cone	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=0$	
5	Elliptic paraboloid	$\frac{x^2}{p}+\frac{y^2}{q}=2z$ $(p>0,\ q>0)$	

Table B.1 (continued)

	The type of the surface	Equation	Graph
6	Hyperbolic paraboloid	$\frac{x^2}{p} - \frac{y^2}{q} = 2z$ $(p > 0,\ q > 0)$	
7	Elliptic cylinder	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	
8	Hyperbolic cylinder	$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	
9	Parabolic cylinder	$x^2 = 2py$ $(p > 0)$	

Appendix C

Table C.1 – Table of Derivatives
($u = u(x)$ is a differentiable function)

1. $(C)' = 0,\ C - const$;	11. $(\tan u)' = \frac{1}{\cos^2 u} \cdot u'$;
2. $(x)' = 1$;	12. $(\cot u)' = -\frac{1}{\sin^2 u} \cdot u'$;
3. $(u^n)' = n \cdot u^{n-1} \cdot u'$;	13. $(\arcsin u)' = \frac{1}{\sqrt{1-u^2}} \cdot u'$;
4. $(\sqrt{u})' = \frac{1}{2\sqrt{u}} \cdot u'$;	14. $(\arccos u)' = -\frac{1}{\sqrt{1-u^2}} \cdot u'$;
5. $(a^u)' = a^u \cdot \ln a \cdot u',\ a - const$;	15. $(\arctan u)' = \frac{1}{1+u^2} \cdot u'$;
6. $(e^u)' = e^u \cdot u'$;	16. $(\cot^{-1} u)' = -\frac{1}{1+u^2} \cdot u'$;
7. $(\log_a u)' = \frac{1}{u \cdot \ln a} \cdot u'$;	17. $(\sinh u)' = \cosh u \cdot u'$;
8. $(\ln u)' = \frac{1}{u} \cdot u'$;	18. $(\cosh u)' = \sinh u \cdot u'$;
9. $(\sin u)' = \cos u \cdot u'$;	19. $(\tanh u)' = \frac{1}{\cosh^2 u} \cdot u'$;
10. $(\cos u)' = -\sin u \cdot u'$;	20. $(\coth u)' = -\frac{1}{\sinh^2 u} \cdot u'$.