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ONE APPLICATION OF DIFFERENTIAL EQUATIONS IN DYNAMIC MODELING OF ROBOTIC MANIPULATORS

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Dynamics has found extensive application across a wide range of disciplines, including spacecraft dynamics, mechanical system design, and the rapidly evolving field of robotics. In robotics, dynamic modeling plays a crucial role in the design, analysis, simulation, and control of robotic systems.

Robotic manipulators, in particular, benefit greatly from dynamic modeling. They are widely used to perform tasks that are hazardous, repetitive, or physically demanding. Accurate dynamic models help ensure that these robots operate safely and efficiently in various environments.

The study of robot dynamics primarily concerns itself with the derivation and mathematical formulation of the equations that govern the motion of robotic arms and manipulators. These equations are crucial as they characterize the dynamic behavior of the robot system, offering understanding forces and torques over time [1].

Such mathematical models are essential for practical applications such as computer-based simulations, trajectory planning, and real-time motion control of robotic arms [2]. Modern robotics relies on mathematical modeling to describe the motion, forces, and interactions within mechanical systems. One of the most important tools for this modeling is ordinary differential equations (ODE). They allow you to describe the behavior of a robot over time, taking into account such parameters as mass, acceleration, moment of inertia, and external forces. ODEs are especially relevant when analyzing the dynamics of manipulators - multi-link robotic arms used in industry, medicine, logistics, and other areas. Dynamic models based on ODEs allow you to synthesize control systems, predict the behavior of a robot, and optimize its movements [1].

Over the years, a variety of methodological frameworks have been developed to formulate these dynamic equations of motion. It is necessary to note the Lagrange-Euler formulation [3], the Newton-Euler method [4], the recursive Lagrangian technique, the Generalized D'Alembert Principle [5], and Kane's method, which is also referred to as the Lagrangian form of D'Alembert's principle. Each of these approaches comes with its own set of strengths and limitations, depending on the specific application, complexity of the robotic system, and computational resources available. Nevertheless, the two most widely adopted conventional methods - Lagrange-Euler and Newton-Euler - continue to be the predominant choices in both academic research and industrial development for generating the dynamic models necessary to control and simulate robotic manipulator motion effectively.

The modeling is based on Newton's second law. For rotational motion it has the form:

$$\tau = I \cdot \alpha, \tag{1}$$

where τ is a moment of force; I is a moment of inertia and α is an angular acceleration.

For complex systems it is more convenient to use the Lagrange formalism [1]:

$$L = T - V, \tag{2}$$

where L is a smooth function called a Lagrangian; T is the kinetic energy and V is the potential energy of the system, respectively.

Equation of motion can be written as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i, \tag{3}$$

where q_i are generalized coordinates, rotation angles of links and Q_i are generalized forces (moments).

Let us consider a two-link manipulator (Fig.1) with two rotating links (arm and forearm), with lengths l_1, l_2, masses m_1, m_2, and rotation angles θ_1, θ_2.

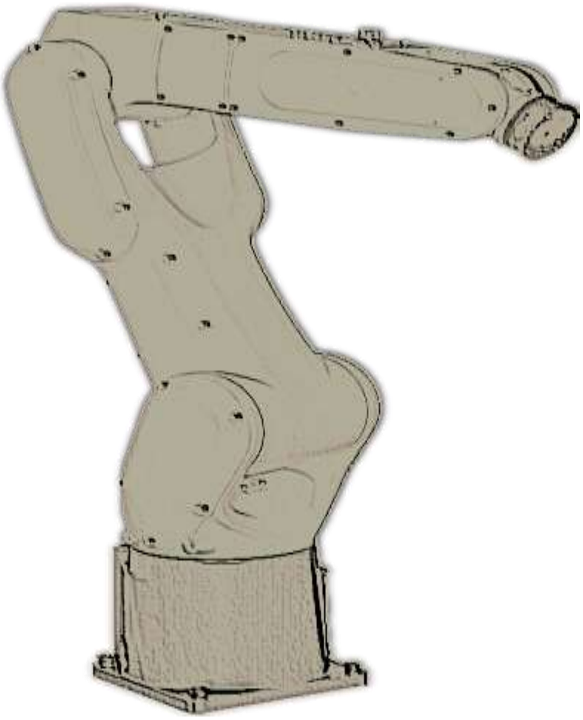

Figure 1. Scheme of a Robot Manipulator.

The model in the form of Euler-Lagrange equations [3] can be written as:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \tau, \tag{4}$$

where $\theta = [\theta_1, \theta_2]^T$; $M(\theta)$ is the mass matrix; $C(\theta,\dot{\theta})$ is the matrix Corriolis function; $G(\theta)$ is the vector of gravitational forces; $\tau = [\tau_1, \tau_2]^T$ is the vector of control moments. The forms of these matrices (for example, in a simplified flat model) has the next view:

$$M(\theta) = \begin{bmatrix} a + 2b\cos\theta_2 & d + b\cos\theta_2 \\ d + b\cos\theta_2 & d \end{bmatrix},$$

$$C(\theta,\dot{\theta}) = \begin{bmatrix} -b\sin\theta_2 \cdot \dot{\theta}_2 & -b\sin\theta_2 \cdot (\dot{\theta}_1 + \dot{\theta}_2) \\ b\sin\theta_2 \cdot \dot{\theta}_1 & 0 \end{bmatrix}, \tag{5}$$

$$G(\theta) = \begin{bmatrix} g_1\cos\theta_1 + g_2\cos(\theta_1 + \theta_2) \\ g_2\cos(\theta_1 + \theta_2) \end{bmatrix},$$

where a, b, d, g_1, g_2 depend on the masses and lengths of the links.

Unlike kinematic analysis, dynamic modeling takes into account real forces and moments [6], which is critical for designing energy-efficient systems, creating realistic simulators (e.g. in Gazebo, PyBullet, MuJoCo), and real-time control (e.g. on ROS or ROS2 platforms).

As a conclusion, it could be underliened that a method of differential equations to simulate the motion of a multi-link robotic arm with rotational joints was presented in this work. It treats each link as being in equilibrium under inertial, gravitational, and driving forces during movement. The method is simple and relies on repeating mathematical steps. The resulting equations are easy to program and integrate with standard tools. The algorithm is validated through comparison with popular simulation software.

References

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