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METHODICAL INSTRUCTIONS
for laboratory works in Theoretical Electrical Engineering
for the students of specialty 141 "Electricity, electrical engineering and electromechanics" for full-time and part-time forms of education
Part 2

2025

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CONTENT

Laboratory work №7
THE RESEARCH OF THE THREE-PHASE "WYE" CONNECTION

The object of the work is to investigate a three-phase circuit at the "wye" connection with a neutral wire and without it in different operating modes and get skills in the analysis of three-phase circuits by means of vector diagrams.

7.1 Short theoretical information
7.1.1 The main definitions

For "wye" (or "star") connection the ends of three phases are connected together to form a node called a neutral point ***N*** (for the generator phases) and ***n*** (for the load phases) (see fig. 7.1).

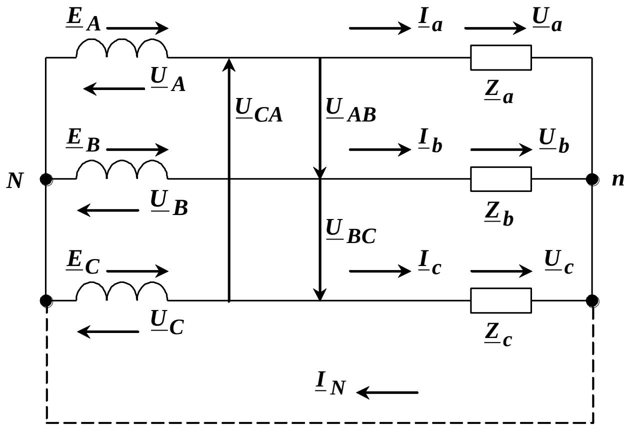


Figure 7.1 - The "wye" connection of the generator and receivers

The wire connecting two neutral points is referred to as the *neutral wire*. The neutral wire carries a *neutral current*, $\underline{I}_N$, the positive direction for which is from *n* to *N***.** The wires connecting the generator terminals *A*, *B* and *C* and the load terminals *a*, *b* and *c* are termed the *line wires*.

There are two kinds of "wye" connection: a "star" connection without a neutral wire or three-wire "star" connection, and a "star" connection with a neutral wire or four-wire "star" connection.

The voltage between two wires is termed the line voltage. As with any voltage, it symbol has a two-letter subscript, for example, $\underline{U}_{AB}$, which is the line voltage between the terminals ***A*** and ***B***. The symbol for the magnitude of the line voltage is U_l. The voltage between the beginning and the end of a phase, or the voltage from line to earth is called a phase voltage, U_p. The currents in the line wires are referred to as the line current. They are marked with one letter as $\underline{I}_A$, $\underline{I}_B$ and $\underline{I}_C$. The currents in the phases of either the supply or the load are called the phase currents, I_p. The directions of line and phase currents and voltages are shown in Fig. 7.1.

7.1.2 The main equations

A three-phase generator is called symmetrical (or *balanced)* if it has three identical (phase) coils rigidly attached to one another and rotating in a uniform magnetic field at a constant angular speed ω. The coils are displaced at 120° from one another, and the sine wave e.m.f.s induced in them, are also displaced at 120° in time phase.

We may express this system of e.m.f.s in a complex state as

$$\underline{E}_A = E_P e^{j0^o};\ \underline{E}_B = E_P e^{-j120^o};\ \underline{E}_C = E_P e^{+j120^o}, \qquad (7.1)$$

where $E_p = \dfrac{E_m}{\sqrt{2}}$ is the effective value of a generator phase e.m.f.

The phase voltages across generator terminals are similar to electromotive forces and also make a symmetrical system

$$\underline{U}_A = U_p e^{j0^o};\ \underline{U}_B = U_p e^{-j120^o};\ \underline{U}_C = U_p e^{+j120^o}, \qquad (7.2)$$

For a wye connection according to KVL line voltages are equal to geometrical difference of the corresponding phase voltages

$$\underline{U}_{AB} = \underline{U}_A - \underline{U}_B = U_p e^{j0^o} - U_p e^{-j120^o} = U_l e^{j30^o};$$
$$\underline{U}_{BC} = \underline{U}_B - \underline{U}_C = U_p e^{-j120^o} - U_p e^{j120^o} = U_l e^{-j90^o}; \qquad (7.3)$$
$$\underline{U}_{CA} = \underline{U}_C - \underline{U}_A = U_p e^{j120^o} - U_p e^{j0^o} = U_l e^{j150^o},$$

where $U_l = \sqrt{3}U_p$ is a line voltage of the generator.

The voltage appearing between the neutral points of the load and the supply is called the *neutral voltage shift* or *the bias neutral voltage* $\underline{U}_N$.

The load may be symmetrical (balanced) and non-symmetrical. The load is balanced if there are equal complex impedances in different phases

$$\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = Z_P e^{j\varphi_ô} \quad , \tag{7.4}$$

Asymmetry occurs when there is an open circuit or a short circuit in a phase of symmetric receivers or when there are different loads in phases

The effective value of a neutral voltage shift

$$\underline{U}_N = \frac{\underline{U}_A \underline{Y}_a + \underline{U}_B \underline{Y}_b + \underline{U}_C \underline{Y}_c}{\underline{Y}_a + \underline{Y}_b + \underline{Y}_c + \underline{Y}_N}, \tag{7.5}$$

where $\underline{Y}_a = \frac{1}{\underline{Z}_a}$, $\underline{Y}_b = \frac{1}{\underline{Z}_b}$, $\underline{Y}_c = \frac{1}{\underline{Z}_c}$, are complex conductances of the phases, $\underline{Y}_N = \frac{1}{\underline{Z}_N}$ is a complex conductance of a neutral wire ($\underline{Y}_N = \infty$, if there is a neutral wire in the circuit, and $\underline{Y}_N = 0$ if it is not).

If $\underline{U}_N \neq 0$ phase voltages differ from each other and from generator phase voltages. By the KVL the complex phase voltages across load:

$$\underline{U}_a = \underline{U}_A - \underline{U}_N ; \quad \underline{U}_b = \underline{U}_B - \underline{U}_N ; \quad \underline{U}_c = \underline{U}_C - \underline{U}_N , \tag{7.6}$$

Phase (and line) currents by Ohm's law

$$\underline{I}_a = \underline{U}_a \underline{Y}_a ; \quad \underline{I}_b = \underline{U}_b \underline{Y}_b ; \quad \underline{I}_c = \underline{U}_c \underline{Y}_c , \tag{7.7}$$

When there is a neutral wire in the circuit by KCL law:

$$\underline{I}_a + \underline{I}_b + \underline{I}_c = \underline{I}_N ; \tag{7.8}$$

7.1.3 A symmetrical rate

If any electric circuit consists of the symmetrical generator and symmetrical load it works in a symmetric mode. In this case

$$\underline{U}_N = \frac{(\underline{U}_A + \underline{U}_B + \underline{U}_C)\underline{Y}_a}{3\underline{Y}_a + \underline{Y}_N} = 0 , \tag{7.9}$$

Therefore the voltages across the phases of receiver are equal to corresponding generator voltages (see eq. 7.6). Phase currents according to formulas (7.8) form a symmetrical system (that is they are equal in values and are displaced at $\pm {}^{2\pi}\!/_{3}{}^{o}$ from one another, and are shifted at the same phase angles concerning corresponding phase voltages

$\varphi_a = \varphi_b = \varphi_c = \varphi_P$. The vector diagram for the case of a symmetrical rate is presented in figure 7.2.

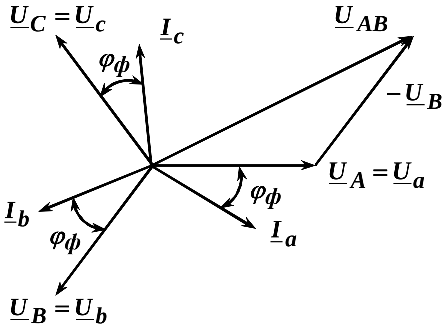


Figure 7.2 – The vector diagram for a case of a symmetrical rate

So, the algebraic sum of phase currents are

$$\underline{I}_a + \underline{I}_b + \underline{I}_c = \underline{I}_N = 0; \quad (7.10)$$

Therefore, a neutral wire is not used for symmetric receivers

Active power in the case of a symmetrical rate

$$P = 3U_ф I_ф cos\varphi_ф = \sqrt{3} U_л I_л cos\varphi_ф; \quad (7.11)$$

7.1.4 Asymmetrical rate with a Neutral Wire

When asymmetrical load $\underline{Z}_a \neq \underline{Z}_b \neq \underline{Z}_c$ and there is a neutral wire with $\underline{Z}_N \approx 0$, the potentials of two neutral points are equal, and a neutral voltage shift $\underline{U}_N = 0$.

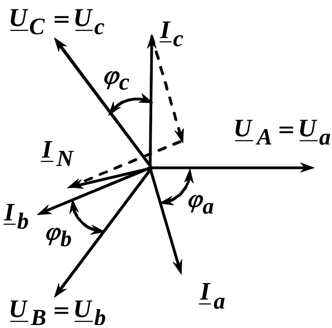


Figure 7.3 – The vector diagram for asymmetrical rate with a neutral wire

Thus the phase voltages across a receiver are equal to corresponding generator phase voltages (eq. 7.6). The asymmetry of load in this case causes asymmetrical system of phase (line) currents. It appears a current in a neutral wire $\underline{I}_N = \underline{I}_a + \underline{I}_b + \underline{I}_c \neq 0$. The vector diagram for the general case of asymmetrical load $\underline{Z}_a \neq \underline{Z}_b \neq \underline{Z}_c$ is resulted in Figure 7.3.

The active power here is determined as the sum of phase powers

$$P = U_a I_a cos\varphi_a + U_b I_b cos\varphi_b + U_c I_c cos\varphi_c ; \qquad (7.12)$$

7.1.5 Asymmetrical rate without a neutral wire

For asymmetrical rate without a neutral wire $\underline{U}_N \neq 0$. We will analyse this fact by means of a vector diagram. The drawing is begun from the vectors of generator phase voltages $\underline{U}_A$, $\underline{U}_B$, $\underline{U}_C$ (Fig. 7.4). From the ends of these vectors we draw circles with radiuses which in scale are equal to the phase voltages of the load: $\underline{U}_a$, $\underline{U}_b$, $\underline{U}_c$. The point of cross-section of these three circles is the end of vector $\underline{U}_N$. Phase voltages across the load are equal to geometrical differences of vectors $\underline{U}_a$, $\underline{U}_b$, $\underline{U}_c$ and $\underline{U}_N$. This method is called a method of cuts. The vectors of phase currents $\underline{I}_a$, $\underline{I}_b$, $\underline{I}_c$ are drawn from the end of the vector head $\underline{U}_N$, as from the origin of coordinates.

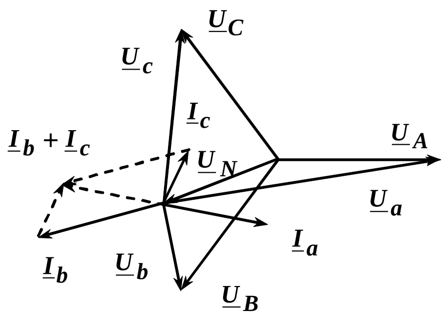


Figure 7.4 – A vector diagram asymmetric rate without a neutral wire

The point of cross-section of these three circles is the end of vector $\underline{U}_N$. Phase voltages across the load are equal to geometrical differences of vectors $\underline{U}_a$, $\underline{U}_b$, $\underline{U}_c$ and $\underline{U}_N$. This method is called a method of cuts. The vectors of phase currents are drawn from the end of $\underline{U}_N$, as from the origin of coordinates.

Phase currents do not organise the symmetrical system, but by KCL their geometrical sum is equal to zero. The active power in this case is determined as the sum of phase powers (7.14). It can be measured by means of three single-phase wattmeters which are connected through corresponding phase currents and voltages, or by a method of two wattmeters.

7.2 The order of the working out of the spade-work

7.2.1 Learn theoretical material and prepare answers to control questions.

7.2.2 Prepare the report blank form to which it should enter the operation purpose, the electric circuit design (fig. 7.5) and tables 7.2, 7.3.

7.2.3 Build a vector diagram corresponding to the load (Table 7.1).

Your variant is determined by numeral in the register of your group. The values of active resistances and reactances are considered to be equal.

Table 7.1 - The character of load in phases

	Variant									
Phase	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
A	X_C	X_L	R	X_C	R	X_L	X_C	X_C	R	X_L
B	X_L	X_C	X_L	R	R	R	R	R	X_C	R
C	R	R	R	X_L	X_C	X_L	R	X_C	R	R

7.3 The description of the laboratory plant

For fulfilling of the laboratory work one must use:

- a three-phase sinusoidal voltage source (terminals ***A***, ***B***, ***C, N***);
- active resistances $R14$, $R15$, $R16$;
- the voltmeters of electro-magnetic system $V3$, $V4$;
- the ampere-meters of electro-magnetic system $A5$, $A6$, $A7$, $A8$;
- the wattmeters of electro-dynamic system $W1$, $W2$;
- capacitors $C3-C7$; - keys S_1, S_2.

7.4 The order of carrying out of the work

7.4.1 Make up an electric circuit (fig. 7.4). Invite a lecturer for checking up the correctness of the circuit.

7.4.2 Measure line and phase voltages U_{AB}, U_{BC}, U_{CA}, U_A, U_B, U_C. The line source voltage must be ***80…100*** *V*. Write down the voltages into table 7.2.

7.4.3 Symmetrical load with a neutral wire. Turn on keys S_1, S_2. Measure phase voltages U_a, U_b, U_c, phase currents I_a, I_b, I_c, the current in a neutral wire I_N, active powers P_1, P_2.

Table 7.2 – Line and phase generator voltages

U_{AB}, V	U_{BC}, V	U_{CA}, V	U_A, V	U_B, V	U_C, V

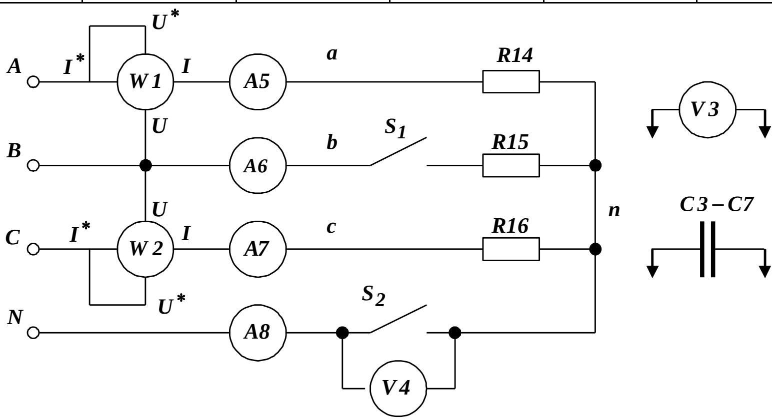


Figure 7.4 – A wye connection for investigation

7.4.4 Symmetrical load without a neutral wire. Turn off the key S_2. Measure phase voltages U_a, U_b, U_c, phase currents I_a, I_b, I_c, the neutral voltage shift U_N, active powers P_1, P_2.

7.4.5 An open circuit in the phase "***b***" (with a neutral wire). Turn on key S_2, Turn off key S_1. Measure the same values as in case 7.4.3.

7.4.6 An open circuit in "***b***" (without a neutral wire). Turn off keys S_1, S_2. Measure all the values as in case 7.4.4.

7.4.7 A short circuit in phase "***b***". ***ATTENTION! The switch*** S_2 ***must be disconnected obviously***. Switch off the stand. Do a short circuit across the resistor ***R15***. Turn on the stand. Measure phase voltages U_a, U_b, U_c, phase currents I_a, I_b, I_c, a voltage U_N, powers P_1, P_2. All the results must be written down into table 7.3.

Table 7.3 –The result of examination of the three-phase circuit

Working rates	№ test	The kind of a connection	Measured										Calculated		
			***U**$_a$*, *V*	***U**$_b$*, *V*	***U**$_c$*, *V*	***I**$_a$*, *A*	***I**$_b$*, *A*	***I**$_c$*, *A*	***U**$_N$*, *V*	***I**$_N$*, *A*	***P**$_1$*, *Wt*	***P**$_2$*, *Wt*	***P**$_a$*, *Wt*	***P**$_b$*, *Wt*	***P**$_c$*, *Wt*
Symmetrical rate	7.4.3	***YN***													
	7.4.4	***Y***													
Open circuit in phase "***b***"	7.4.5	***YN***													
	7.4.6	***Y***													
Short circuit in phase "***b***"	7.4.7	***Y***													
Asymmetrical rate	7.4.8	***YN***													
	7.4.9	***Y***													

7.4.8 Asymmetrical load with a neutral wire. Switch off the stand. Connect a capacitor $C6$ instead of resistor $R14$. Turn on the switches S_1 and S_2. Measure phase voltages U_a, U_b, U_c, phase currents I_a, I_b, I_c, a current in neutral wire I_N, active powers P_1, P_2. Write down the results into the table 7.3.

7.4.9 Asymmetrical load without a neutral wire. Turn off the switch S_2. Measure phase voltages U_a, U_b, U_c, phase currents I_a, I_b, I_c, a neutral voltage shift U_N, and powers P_1, P_2. Write down the results into the table 7.3.

7.5 The processing of experimental results

7.5.1 Calculate active powers in each load phase for each rate.

7.5.2 Using the results of experiences (7.4.3, 7.4.4, 7.4.5, 7.4.6, 7.4.7) construct vector diagrams for different rates.

7.5.3 Using the results of experiences (7.4.8, 7.4.9) construct vector diagrams for a rate of asymmetrical load at the presence of a neutral wire and its absence. Do some conclusions on work.

7.6 Control questions

7.6.1 Which rate is called asymmetrical?

7.6.2 Prove that the algebraic sum of line voltages is equal to zero and the geometrical sum of line currents is equal to zero in a symmetrical rate.

7.6.3 Write down line and phase voltages in a complex form.

7.6.4 What is a neutral current equal to? In which cases does it appear?

7.6.5 Show how we must measure line and phase voltages in the circuit.

7.6.6. Name advantages of four-wire wye connections.

7.6.7 What is called a neutral voltage shift? How is it measured?

7.6.8 When is wye connection applied in practice?

7.6.9 How can a vector diagram be constructed for three-wire connection by the method of notches?

Laboratory work №8
THE RESEARCH OF THE THREE-PHASE "DELTA" CONNECTION

The object of the work is to investigate a "delta" connection and get skills in the analysis of three-phase circuits by means of vector diagrams.

8.1 Short theoretical information
8.1.1 The main definitions

The windings of generators, transformers and consumers in three-phase circuits may be joined in "delta" connection (see fig. 8.1).

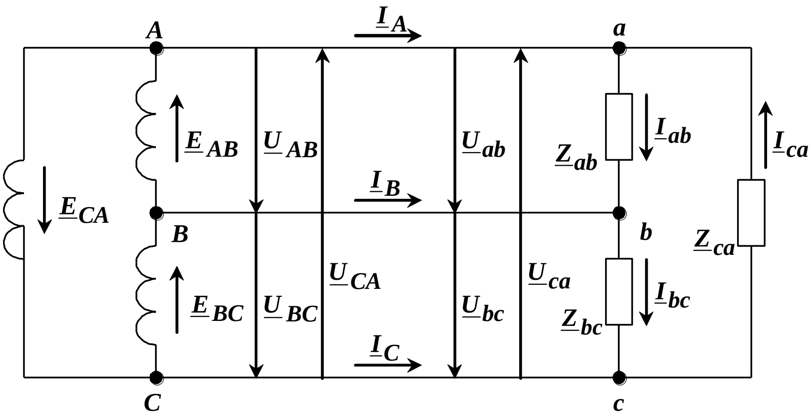


Figure 8.1 – The "delta" connection of the generator and receivers

Generator terminals are marked with letters ***A, B, C***, and load terminals - with letters ***a, b, c.*** When there are no impedances in line wires the generator voltages are equal to the load voltages. When a generator is symmetrical its phase e.m.fs are equal to each other and are shifted by 120^o.

$$\underline{U}_{AB} = U_l e^{j30^o}; \quad \underline{U}_{BC} = U_l e^{-j90^o}; \quad \underline{U}_{CA} = U_l e^{j150^o}, \qquad (8.1)$$

where $U_l = U_p$ line (phase) generator voltage.

The complex phase currents are determined by Ohm's law:

$$\underline{I}_{ab} = \frac{\underline{U}_{AB}}{\underline{Z}_{ab}}; \quad \underline{I}_{bc} = \frac{\underline{U}_{BC}}{\underline{Z}_{bc}}; \quad \underline{I}_{ca} = \frac{\underline{U}_{CA}}{\underline{Z}_{ca}}, \qquad (8.2)$$

where $\underline{Z}_{ab}$, $\underline{Z}_{bc}$, $\underline{Z}_{ca}$ complex phase impedances.

The complex line currents are determined by the first Kirchhoff's law

$$\underline{I}_A = \underline{I}_{ab} - \underline{I}_{ca}; \quad \underline{I}_B = \underline{I}_{bc} - \underline{I}_{ab}; \quad \underline{I}_C = \underline{I}_{ca} - \underline{I}_{bc}. \qquad (8.3)$$

8.1.2 A symmetrical load

For this case complex phase impedances are equal to each other.

$$\underline{Z}_{ab} = \underline{Z}_{bc} = \underline{Z}_{ca} = Z_P e^{j\varphi_{\hat{o}}}. \qquad (8.4)$$

Then the effective values of phase currents are equal too

$$I_{ab} = I_{bc} = I_{ca} = I_P. \qquad (8.5)$$

A vector diagram for this case is shown in figure 8.2.

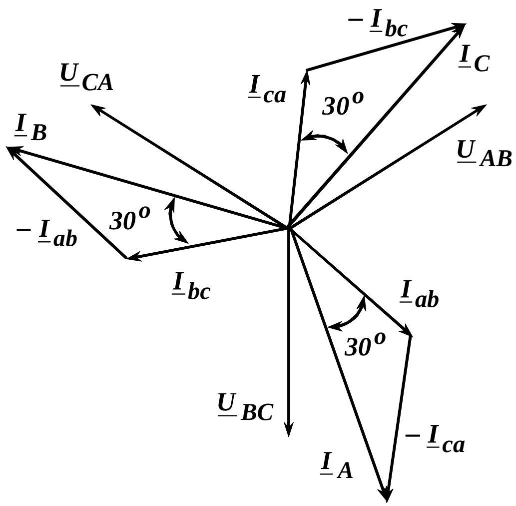


Figure 8.2 – A vector diagram for a symmetrical load

As we can see from a vector diagram the currents form a symmetric system. From similar triangles one can determine a line current

$$I_l = 2I_p \cos 30^o = 2I_p \frac{\sqrt{3}}{2} = \sqrt{3} I_p. \qquad (8.6)$$

The active power for a symmetrical load

$$P = 3U_p I_p \cos\varphi_{\hat{o}} = \sqrt{3} U_l I_l \cos\varphi_p. \qquad (8.7)$$

8.1.3 Asymmetrical load

For $\underline{Z}_{ab} \neq \underline{Z}_{bc} \neq \underline{Z}_{ca}$ phase voltages create symmetrical system but line currents are asymmetrical. Phase currents are found by eq. (8.2), and line ones - by eq. (8.3). If add the right and left parts of the equations (8.3), we'll get

$$\underline{I}_A + \underline{I}_B + \underline{I}_C = 0. \qquad (8.8)$$

A vector diagram for this case is shown in figure 8.3. The active power in this case is defined as the algebraic sum of separate phase powers

$$P = P_{ab} + P_{bc} + P_{ca} = \\ = U_{ab} I_{ab} \cos\varphi_{ab} + U_{bc} I_{bc} \cos\varphi_{bc} + U_{ca} I_{ca} \cos\varphi_{ca}. \qquad (8.9)$$

It may be measured by means of three single-phase wattmeters which are connected on corresponding phase currents and voltages, or by a method of two wattmeters (the scheme is shown in figure 8.4).

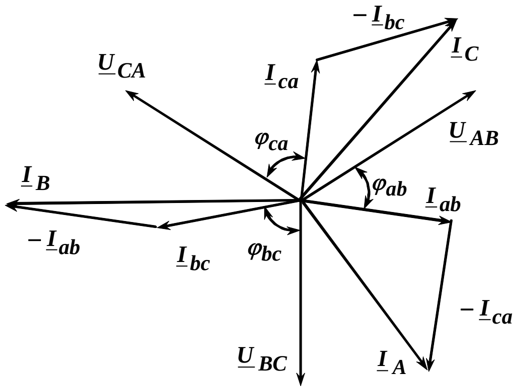


Figure 8.3 – A vector diagram for the asymmetrical load

8.1.4 The measurement of active and reactive power by a method of two wattmeters

The instantaneous value of active power:

$$p = p_1 + p_2 = u_{AB} i_A + u_{CB} i_C. \qquad (8.10)$$

As $i_A = i_{ab} - i_{ca}$; and $i_C = i_{ca} - i_{bc}$,

$$u_{CB} = -u_{BC} = -(u_{AB} + u_{CA}), \qquad (8.11)$$

we can substitute (8.11) into (8.10):

$$p = u_{AB}(i_{ab} - i_{ca}) - (u_{AB} + u_{CA})(i_{ca} - i_{bc}) = \qquad (8.12)$$

$$= u_{AB} i_{ab} + u_{B} i_{bc} + u_{CA} i_{ca} .$$

So, the power which is measured by both wattmeters is equal to the power of total three-phase load irrespective of the fact how phases are loaded. On the basis of the vector diagram (see fig. 8.2) we will get

$$P_1 = U_{AB} I_A \cos(\underline{U}_{AB} \wedge \underline{I}_A) = U_l I_l \cos(30^o + \varphi);$$

$$P_2 = U_{CB} I_B \cos(\underline{U}_{CB} \wedge \underline{I}_B) = U_l I_l \cos(60^o - 30^o - \varphi) = \qquad (8.13)$$

$$= U_l I_l \cos(30^o - \varphi) .$$

So, the readings of wattmeters will be equal at symmetrical load.

The reactive power is determined by the formula

$$Q = 3U_p I_p \sin\varphi_p = \sqrt{3} U_l I_l \sin\varphi_p . \qquad (8.14)$$

If we find the difference between the readings of wattmeters we'll get

$$P_1 - P_2 = U_l I_l \cos(30^o + \varphi) - U_l I_l \cos(30^o - \varphi) = ;$$

$$= U_l I_l (\cos(30^o + \varphi) - \cos(30^o - \varphi)) = \qquad (8.15)$$

$$= U_l I_l 2 \sin 30^o \sin\varphi = U_l I_l \sin\varphi .$$

Then the reactive power is equal.

$$Q = \sqrt{3}(P_1 - P_2) . \qquad (8.16)$$

8.2 The order of the working out of the spade-work

8.2.1 Prepare answers to control questions.

8.2.2 Prepare the report where must be the purpose of the work, the electric circuit (fig. 8.4) and tables 8.2, and a vector diagram (by 8.2.3).

8.2.3 Build a vector diagram corresponding to the load (Table 8.1). (Your variant is determined by last numeral in the academic register of your group. The values of active resistances and reactances are considered to be equal).

Table 8.1 - The character of the load in phases

	Variant									
Phase	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
ab	X_L	X_C	X_C	R	X_L	X_C	X_L	R	X_C	R
bc	R	R	R	X_C	R	X_L	X_C	X_L	R	R
ca	X_L	R	X_C	R	R	R	R	R	X_L	X_C

8.3 The description of the laboratory plant

For fulfilling of the laboratory work one must use:
- a three-phase sinusoidal voltage source (terminals ***A***, ***B***, ***C***);
- active resistances $R14$, $R15$, $R16$;
- the ampere-meters of electro-magnetic system $A5$ - $A10$;
- the wattmeters of electro-dynamic system $W1$, $W2$;
- capacitors $C3-C7$ and keys S_1, S_2.

8.4 The order of carrying out of the work

8.4.1 Make up an electric circuit (fig. 8.4).

8.4.2 A symmetrical load. Turn on keys S_1, S_2. Measure phase voltages U_{ab}, U_{bc}, U_{ca}, phase currents I_{ab}, I_{bc}, I_{ca}, line currents I_A, I_B, I_C active powers P_1, P_2. Write down the results into the table 8.2.

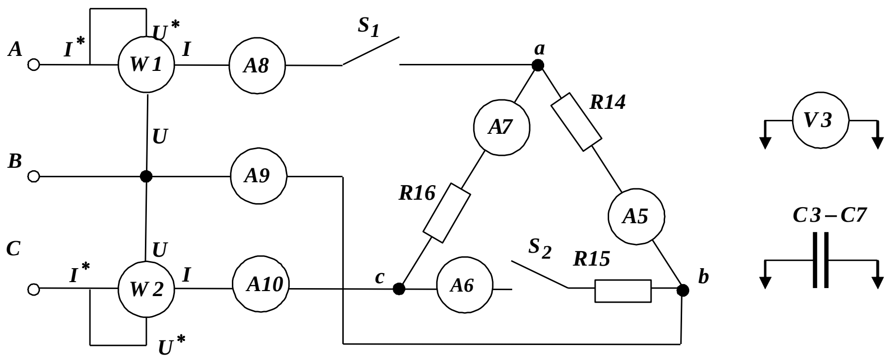


Figure 8.4 – A delta connection

8.4.3 An open circuit in "bc". Measure the same values as in case 8.4.2.

8.4.4 An open circuit in line "Aa". Turn off key S_1. Measure phase voltages U_{ab}, U_{bc}, U_{ca}, phase and line currents I_{ab}, I_{bc}, I_{ca}, I_A, I_B, I_C ,active powers P_1, P_2. Write down the results into the table 8.2.

8.4.5 Asymmetrical load. Connect a capacitor $C6$ instead of the resistor $R14$.. Measure voltages and currents, active powers. Switch off the stand. The results of measurements must be shown to a lecturer.

Table 8.2 – The result of examination of the three-phase circuit

Working rates	№ test	Measured											Calculated		
		U_{ab}, V	U_{bc}, V	U_{ca}, V	I_{ab}, V	I_{bc}, A	I_{ca}, A	I_A, A	I_B, A	I_C, A	P_1, Wt	P_2, Wt	P_{ab}, Wt	P_{bc}, Wt	P_{ca}, Wt
Symmetrical load	8.4.2														
An open circuit in phase "***bc***".	8.4.3.														
An open circuit in line "***Aa***"	8.4.4														
Asymmetrical load	8.4.5														

8.5 The processing of experimental results

8.5.1 Calculate active powers in each phase of the load for each rate and compare them with measured ones.

8.5.2 Using the result of experience 8.4.2 construct a vector diagram for symmetrical rate.

8.5.3 Using the results of experiences 8.4.3 construct a vector diagram for the rate "open circuit" in phase "$\boldsymbol{bc}$".

8.5.4 Using the results of experiences 8.4.4 construct a vector diagram for the rate "open circuit in line " $\boldsymbol{Aa}$ ".

8.5.5. Using the results of experiences 8.4.5 construct a vector diagram for asymmetrical rate.

8.5.6 Make some conclusions on the work.

8.6 Control questions

8.6.1 How are the receivers joined in "delta" connection?

8.6.2 What advantages has a "delta" connection? In which cases is it convenient to use such a connection?

8.6.3 Write down the relationships between phase and line currents in the cases of symmetrical and asymmetric load at such a connection?

8.6.4 How are active and reactive load powers calculated?

8.6.5 How are active and reactive powers measured in the cases of symmetrical and asymmetric loads?

8.6.6 How can we measure a reactive load power in the case of symmetrical load?

8.6.7 How do the values of line and phase currents change in the case of breakage in one of the phases when a load is joined in the "delta" connection?

8.6.8 Prove that by means of two wattmeters it is possible to measure the active power of a three-phase load.

8.6.9 Will the system of currents be three-phase one in the case of open circuit in some phase wire?

8.6.10. Will the system of currents be three-phase one in the case of open circuit in some line wire?

Laboratory work №9
THE RESEARCH OF ELECTRIC NON-SINUSOIDAL CURRENT CIRCUITS

The purpose is to investigate the non-sinusoidal electric circuit of non-sinusoidal current, and get skills of analysis of such circuits by means of decomposition of periodic functions into Fourier series and measure electrical values in circuits of non-sinusoidal current.

9.1 Short theoretical information
9.1.1 The main definitions

As a rule, even in sinusoidal circuits, a voltage and a current, maintaining periodicity are not absolutely sinusoidal for various reasons. It is characteristic for telecommunication circuits, electronic and semiconductor devices where it is related with their operation.

For example, in a single-phase circuit which contains a rectifier, voltage $\boldsymbol{u}$ across a rectifier is sinusoidal (fig. 9.1, a) through the input, but the voltage on its output and on a load $\boldsymbol{u_H}$ is already non-sinusoidal what is shown in fig. 9.1, b - for a case of a half-wave rectification and 9.1, c - for a full-wave rectification.

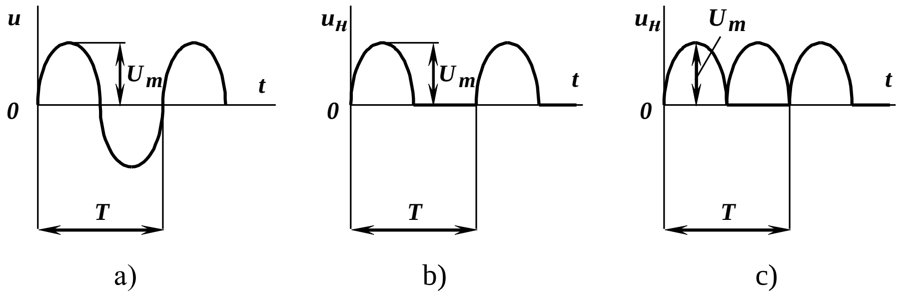


Figure 9.1 - Input voltage (a) and output voltage (b) - for a half-period rectification and (c) – for a full-wave rectification

As is known from mathematics, any function $f(x)$ which is periodic with period 2π and satisfies the conditions established by Dirichlet, can be represented by a series of cosine and sine terms, called a *Fourier series* and *harmonics*, respectively.

For example, a Fourier series for a voltage:

$$u(t)=U_0+\sum_{k=1}^{\infty}B_k\sin k\omega t+\sum_{k=1}^{\infty}C_k\cos k\omega t= \\ =\sum_{k=1}^{\infty}U_{m_k}\sin(k\omega t+\psi_{u_k}). \qquad (9.1)$$

where U_0 - a constant component of non-sinusoidal voltage; k - a number of a harmonic; B_k, C_k - coefficients of harmonic series; ω - an angular frequency of the 1st harmonic; U_{m_k}, ψ_{u_k} - an amplitude and an initial phase of k-th harmonic.

The constant component is equal to an average voltage value for the period of repetition T

$$U_0=\frac{1}{T}\int_0^T u(t)dt. \qquad (9.2)$$

The coefficients of harmonic series

$$B_k=\frac{2}{T}\int_0^T u(t)\sin k\omega t dt; \qquad C_k=\frac{2}{T}\int_0^T u(t)\cos k\omega t dt. \qquad (9.3)$$

Knowing the coefficients of harmonic series it is easy to determine the amplitude and the initial phase of k-th harmonic:

$$U_{m_k}=\sqrt{B_k^2+C_k^2}; \quad \psi_{u_k}=arctg\frac{C_k}{B_k}. \qquad (9.4)$$

When calculating the circuits with non-sinusoidal supplies one can apply a superposition method. Currents and voltage are found separately for a constant component and for each harmonic. It is necessary to take into account that inductive impedance for k-th harmonic in k times more, and capacitive impedance in k times less than for the first harmonic.

$$X_{L_k}=k\omega L=kX_{L_1}; \\ X_{C_k}=\frac{1}{k\omega C}=\frac{X_{C_1}}{k}. \qquad (9.5)$$

9.1.2 A half-wave rectification

For a half-wave rectification the voltage is the following (fig. 9.1, b):

$$u_н = \begin{cases} U_m \sin \omega t; & 0 \le \omega t < \pi \\ 0 & \pi \le \omega T < 2\pi \end{cases}. \qquad (9.6)$$

or it may be represented as the harmonic series

$$u_н = \frac{2U_m}{\pi}\left(\frac{1}{2} + \frac{\pi}{4}\sin \omega t - \frac{1}{1\cdot 3}\cos 2\omega t - \frac{1}{3\cdot 5}\cos 4\omega t - ...\right). \qquad (9.7)$$

A constant component of non-sinusoidal voltage

$$U_0 = \frac{1}{T}\int_0^T u_н dt = \frac{1}{T}\int_0^{T/2} U_m \sin \omega t dt = \frac{U_m}{\pi}. \qquad (9.8)$$

The effective value of non-sinusoidal voltage

$$U = \sqrt{\frac{1}{T}\int_0^T u_н^2 dt} = \sqrt{\frac{1}{T}\int_0^{T/2} U_m^2 \sin^2 \omega t dt} = \frac{U_m}{2}. \qquad (9.9)$$

The effective value of variable component of non-sinusoidal voltage

$$U_k = \sqrt{U^2 - U_0^2} = U_m\sqrt{\frac{1}{4} - \frac{1}{\pi^2}} \approx 0.386 U_m. \qquad (9.10)$$

Currents are calculated analogically.

9.1.3 A full-wave rectification

For this case the voltage is the following (fig. 9.1, c):

$$u_н = |U_m \sin \omega t| = \begin{cases} U_m \sin \omega t; & 0 \le \omega t < \pi \\ -U_m \sin \omega t; & \pi \le \omega T < 2\pi \end{cases}. \qquad (9.11)$$

or it may be represented as the harmonic series

$$u_н = \frac{4U_m}{\pi}\left(\frac{1}{2} - \frac{1}{1\cdot 3}\cos 2\omega t - \frac{1}{3\cdot 5}\cos 4\omega t - \frac{1}{5\cdot 7}\cos 6\omega t - ...\right). \qquad (9.12)$$

A constant component of non-sinusoidal voltage

$$U_0 = \frac{1}{T}\int_0^T u_н dt = \frac{2}{T}\int_0^{T/2} U_m \sin \omega t dt = \frac{2U_m}{\pi}. \qquad (9.13)$$

The affective value of non-sinusoidal voltage

$$U=\sqrt{\frac{1}{T}\int_0^T u_н^2(t)dt}=\sqrt{\frac{1}{T}\int_0^T U_m^2\sin^2\omega t dt}=\frac{U_m}{\sqrt{2}}. \quad (9.14)$$

The affective value of variable component of non-sinusoidal voltage

$$U_k=\sqrt{U^2-U_0^2}=U_m\sqrt{\frac{1}{2}-\frac{4}{\pi^2}}\approx 0.308U_m. \quad (9.15)$$

9.1.3. The power in a circuit of non-sinusoidal current

The instantaneous power for a single-phase sinusoidal source varies itself sinusoidally at twice the frequency of the source. The expression for the single-phase sinusoidal source can be applied to each phase of the three-phase system.

The active power of a periodic current is determined as an average power for a period

$$P=\frac{1}{T}\int_0^T pdt=\frac{1}{T}\int_0^T uidt. \quad (9.16)$$

If instantaneous values of voltage and current are expended in trigonometric series an active power in a circuit of non-sinusoidal current

$$P=P_0+\sum_{k=1}^{\infty}P_k=U_0I_0+\sum_{k=1}^{\infty}U_kI_k\cos\varphi_k, \quad (9.17)$$

where P_0 - an active power of a constant component, $\sum_{k=1}^{\infty}P_k$ - the sum of active powers of separate harmonics.

The reactive power in a circuit of non-sinusoidal current

$$Q=\sum_{k=1}^{\infty}Q_k=\sum_{k=1}^{\infty}U_kI_k\sin\varphi_k, \quad (9.18)$$

where $\sum_{k=1}^{\infty}Q_k$ - the sum of reactive powers of separate harmonics.

An apparent power in a circuit of non-sinusoidal current

$$S=UI, \quad (9.19)$$

where U, I - are effective values of non-sinusoidal current and voltage.

9.2 The order of the working out of the spade-work

9.2.1 Prepare answers to control questions.

9.2.2 Prepare the report where must be the purpose of the work, the electric circuits (fig. 9.3, 9.4), tables 9.2 и 9.3 and calculations by 9.2.3.

9.2.3 Determine the readings of the voltmeter of magneto-electrical system $V1$ and electromagnetic system $V2$, $V3$, $V4$, (fig. 9.2) when the reading of one of them (tab. 9.1) is known. The number is set by a lecturer.

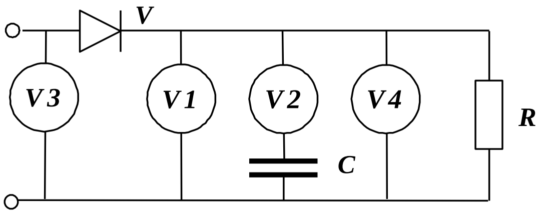


Figure 9.2 – The circuit for a calculation

Table 9.1 – The given readings of voltmeters

Voltmeters	Var. 1	Var. 2	Var. 3	Var. 4	Var. 5	Var. 6
$V1$	**100**					**80**
$V2$		**90**				
$V3$			**80**		**100**	
$V4$				**70**		

9.3 The description of the laboratory plant

For fulfilling of the laboratory work one must use:

- constant resistors $R_l 1$, $R_l 2$;
- the voltmeter of magneto-electrical system $V1$;
- the voltmeters of electro-magnetic system $V2$, $V3$, $V4$;
- the ampere-meter of magneto-electrical system $A4$;
- the ampere-meter of electro- magnetic system $A5$
- the wattmeters of electro-dynamic system $W1$, $W2$;
- capacitors $C3 - C7$; - keys S_1, S_2;
- a half-wave rectifier $1V$; - a full-wave rectifier $2V - 5V$.

9.4 The order of carrying out of the work

9.4.1 Make up an electric circuit (fig. 9.3).

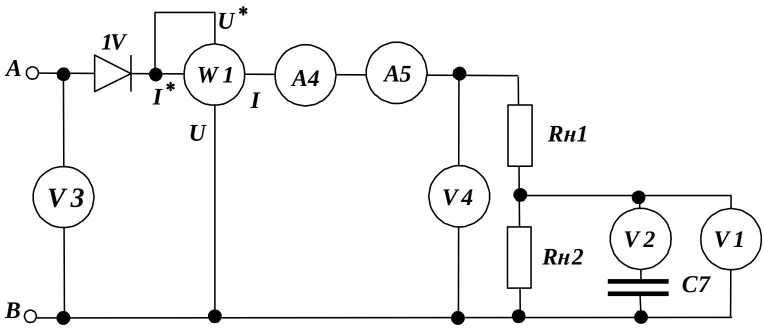


Figure 9.3 – The circuit of a half-wave rectification

9.4.2 Turn on a stand. Write down the results into the table 9.2. The indications of voltmeters ***V1*** и ***V2*** must be multiplied by ***3***..

Table 9.2 - The results of investigations for non-sinusoidal current circuits

A rectification	I_4, A	I_5, A	U_3, V	U_4, V	U_1, V	U_2, V	*P*, Wt
half-wave							
full-wave							

9.4.3 Compose electrical circuit (fig. 9.4).

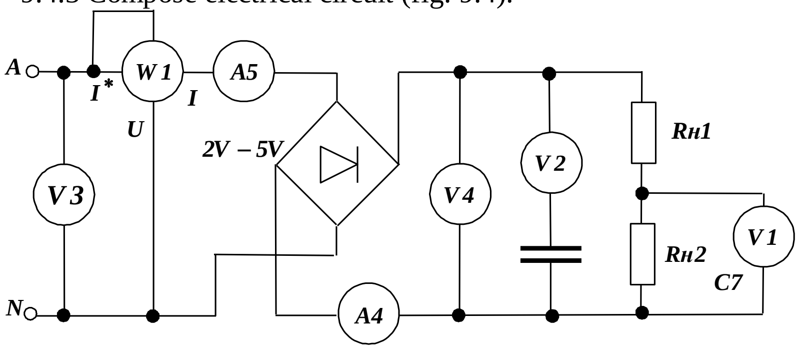


Figure 9.4 – The circuit of a full-wave rectification

9.4.4 Turn on a stand. Write down the results into the table 9.2.

9.4.5 The indications of voltmeters ***V1*** must be multiplied by ***3***. Turn off a stand. Show the results to a lecturer. After that the circuit can be disconnected.

9.5 The processing of experimental results

9.5.1 By the results of experience 9.4.1 one must calculate the ratio of the effective current value I_5 to a constant component I_4 and compare with the theoretical ratio of these values at a half-wave rectification.

9.5.2 By the results of experience 9.4.1 one must calculate effective voltage value across the load $U_н = \sqrt{U_1^2 + U_2^2}$ and compare with the measured reading of this value U_4.

9.5.3 By the results of experience 9.4.3 calculate the ratio of the effective current value I_5 to a constant component I_4 and compare with the theoretical ratio of these values at a full-wave rectification.

9.5.4 By the results of experience 9.4.1 calculate effective voltage value across the load $U_н = \sqrt{U_1^2 + U_2^2}$ and compare with the measured reading of this value U_4 at a full-wave rectification.

9.5.5 Make some conclusions on the work.

9.6 Control questions

9.6.1 What functions may be represented as a Fourier series?

9.6.2 Write down in a general state a trigonometric Fourier series and explain, how its components are determined.

9.6.3 The devices of which system may be used to measure the effective values of currents and voltages?

9.6.4 The devices of which system may be used to measure the constant components of currents and voltages?

9.6.5 How can the effective value of variable voltage component be measured?

9.6.6 How can active, reactive and apparent powers in a non-sinusoidal current circuit be calculated?

9.6.7 How do constant components and effective values of currents match up among themselves at a half-wave and full-wave rectification?

9.6.8 The reading of which voltmeter are the greatest in Figure 9.2?

Laboratory work №10
THE RESEARCH OF TRANSIENTS IN THE ELECTRICAL CIRCUIT WITH AN INDUCTOR

The purpose of the work is to investigate electrical circuits with an inductive coil in a transient rate.

10.1 Short theoretical information
10.1.1 The main definitions

Transients are the phenomena which occur between two permanent (or steady-state) conditions, as a rule periodic, differing from each other in, say, peak value, phase, wave-form or frequency of the electromotive force, the parameters or configuration of the circuit. Hence the changing process of any electrical circuit from one of the stationary conditions to another is called the transient. The stationary conditions are called such a rate when currents and voltages can exist infinitely long without changing of their character with given configuration of the circuit and its parameters. This rate depends on the configuration of the circuit and the value of energy sources. Normally, transient phenomena are brought about in electric circuits by switching operations, that is by closing or opening switches.

Physically, transient phenomena associated with switching operations are changes leading from the energy state existing prior to a switching operation to an energy state which exists after a switching operation.

Transients usually last for a few tenths, hundredths or even millionths of a second; it is seldom that they may last for several seconds.

There are two basic points, which may be called switching rules. The first rule of switching: *the inductor current immediately prior to and immediately after a switching operation must be the same.*

$$i_{L(0+)} = i_L = i_{L(0-)}, \tag{10.1}$$

The second rule of switching: the capacitor voltage immediately prior to and immediately after a switching operation must be the same.

$$u_{C(0+)} = u_C = u_{C(0-)}, \tag{10.2}$$

Transient currents and voltages are determined by means of solving the systems of integro-differential equations which are composed with the help of the method of immediate application of Kirchhoff's laws or mesh-current method. Transients usually last for a few tenths, hundredths or even millionths of a second; it is seldom that they may last for several seconds.

10.1.2 The connection of an inductive coil to a direct current source

Let's consider the electric circuit with a resistor and an inductor (fig.10.1).

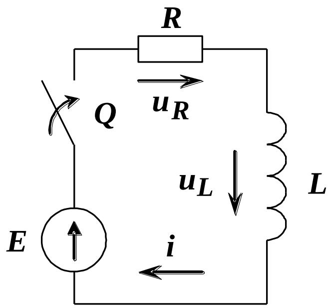


Figure 10.1 – The electric circuit with a resistor and an inductor

When before a switching key ***Q*** is open there is no a current in the circuit $i(0-)=0$.

After a switching the current flows through the contour. Write down the equation by the second Kirchhoff's law for the contour

$$u_R + u_L = E. \quad (10.3)$$

Express voltage through current ***i***

$$Ri + L\frac{di}{dt} = E. \quad (10.4)$$

The solution consists of two values: steady- state and transient or free components

$$i = i_{ss} + i_{tr}. \quad (10.5)$$

For a steady- state current we have an inhomogeneous differential equation of the first order

$$Ri_{ss} + L\frac{di_{ss}}{dt} = E. \quad (10.6)$$

As e.m.f. is direct, the steady- state current will be direct too. Then

$$\frac{di_{ss}}{dt} = 0. \quad (10.7)$$

So, an inductive element does not have an influence on a value of steady-state direct current

$$i_{ss} = I = \frac{E}{R}. \quad (10.8)$$

For a transient current we have a homogeneous differential equation of the first order

$$Ri_{tr} + L\frac{di_{tr}}{dt} = 0 . \qquad (10.9)$$

Pass from the differential equation to the algebraic one

$$R + Lp = 0 . \qquad (10.10)$$

Given algebraic equation is called a characteristic one because its roots determine a form of a transient component.

After solving the characteristic equation we have got

$$p = -\frac{R}{L} . \qquad (10.11)$$

So, a transient current component is

$$i_{tr} = Ae^{pt} = Ae^{-\frac{R}{L}t} , \qquad (10.12)$$

where A – a constant of integration, e – the base of a natural logarithm, p – a root of the characteristic equation.

Combine together the steady-state and the transient components

$$i = \frac{E}{R} + Ae^{pt} . \qquad (10.13)$$

Before a switching at $t = 0-$ the current was equal to zero. By the first Kirchhoff's law just after a switching the current in a circuit with inductance is the same

$$i(0-) = i(0+) = 0 . \qquad (10.14)$$

So, at $t = 0+$ we have got

$$0 = \frac{E}{R} + A . \qquad (10.15)$$

From here

$$A = -\frac{E}{R} . \qquad (10.16)$$

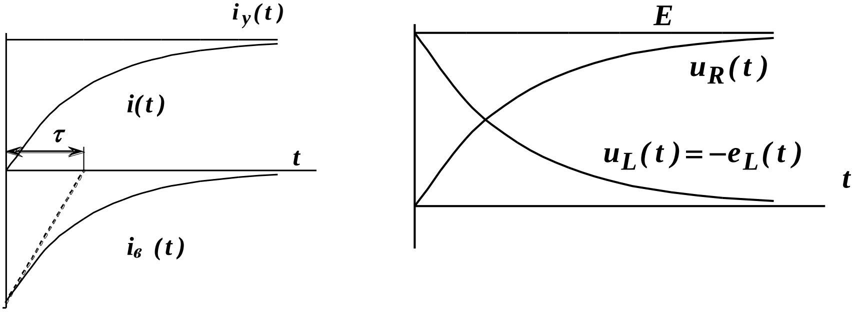
As

$$i = \frac{E}{R} - \frac{E}{R}e^{pt} = \frac{E}{R} - \frac{E}{R}e^{-\frac{R}{L}t} . \qquad (10.17)$$

E.m.f of self-induction

$$e = -L\frac{di}{dt} = -L\frac{d}{dt}\left(\frac{E}{R} - \frac{E}{R}e^{-\frac{R}{L}t}\right) = -Ee^{-\frac{R}{L}t}. \tag{10.18}$$

At initial moment when $t = 0+$, e.m.f. of self-induction completely compensate e.m.f. source because the current $i(0+) = 0$ and $u_R(0+) = Ri(0+) = 0$. Then e.m.f. of self-induction decreases but a voltage across a resistor and a current in the circuit increases by non-periodic law, approaching to a steady-state value (fig. 10.2, a, b).



a) a current in a circuit b) a voltage across resistive and inductive elements

Figure10.2 - The instantaneous currents and voltages in the circuit with resistive and inductive elements

The value $\tau = \frac{1}{|p|} = \frac{L}{R}$ is called a time constant of the circuit. $\tau = L/R$. Its physical sense: it is the time during which a free component decreases in $e \approx 2{,}718$ times in comparison with its initial value $i_t(0+)$.

$$i_{tr}(\tau) = i_{tr}(0+)å^{-1} = \frac{i_{tr}(0+)}{å} = \frac{A}{e}. \tag{10.19}$$

This value can be determined graphically. For this purpose it is necessary to draw a tangent to a curve $i_{tr}(t)$ in a point which answers the arbitrary time moment $t = t_1$ and prolong it to the intersection with a time axis in point $t = t_2$. Then a time constant $\tau = t_2 - t_1$. Usually the arbitrary time moment is the time $t = 0$.

The inverse value is termed as a damping factor

$$\alpha = \frac{1}{\tau}. \qquad (10.20)$$

The free component of a current damps more slowly and, therefore, new steady-state condition is not installed the longer, than more a time constant or than less a damping factor.

10.2 The order of the working out of the spade-work

10.2.1 Prepare answers to control questions.

10.2.2 Prepare the report where must be the purpose of the work, the electric circuit (fig. 10.3), table 10.2 and calculations by 10.2.3.

10.2.3 Calculate transient current and the voltage across an inductive element in the electric circuit (fig. 10.1), if $E = 100$ V, $R = 100 + 10n$ Ω, $L = 0.01n$ H, where n - the variant according to a group register. Draw diagrams of dependences $i = f(t)$, $u_L = f(t)$.

10.3 The description of the laboratory plant

We must use the generator - micropulser (GPI) with an internal resistance $R_{in} = 51$ Ω, which is turned on with key $S4$;

- active resistor $R14 = 120$ Ω;
- inductive coils $L1$ and $L2$;
- oscillograph.

10.4 The order of carrying out of the work

10.4.1 Make up an electric circuit (fig. 10.3). In fig. 10.3 rectangular pulse generator is markted as ГПІ (генератор прямокутних імпульсів). Invite a lecturer for checking up the correctness of the circuit.

10.4.2 Connect the oscillograph input to resistor $R14$, switch on GPI with key $S4$. Draw in scale the voltage diagram $u_{R14}(t)$ from the screen of the oscillograph. Determine the voltage amplitude U_m and steady-state voltage component u_{SS} from the diagram. Write the measured data into table 10.2.

10.4.3 Connect the oscillograph input to an inductive coil $L1$.

Draw in scale the voltage diagram $u_k(t)$ from the screen of the oscillograph. Determine the voltage amplitude U_m and steady-state voltage component u_{ss} from the diagram. Write down the measured data into table 10.2. Turn off GPI with key ***S4*** .

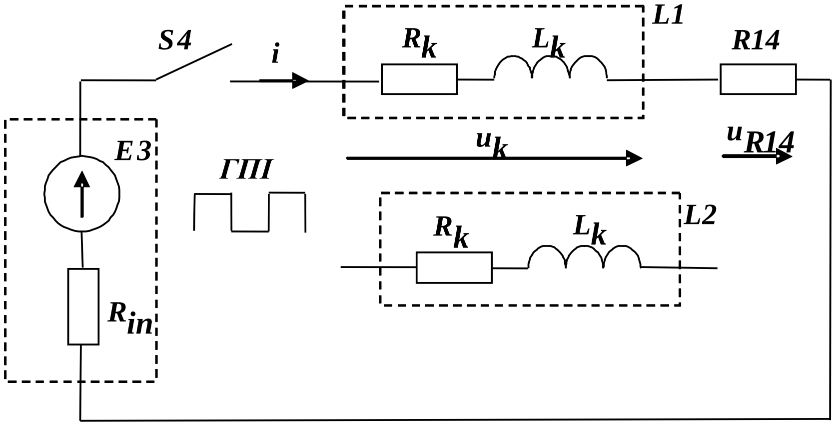


Figure 10.3 - The electric circuit for the investigation

Table 10.2 - The results of investigation

Test	Measured		Calculated				
	U_m, V	u_{ss}, V	τ, s	R_k, Ω	R_Σ, Ω	L, H	Transient function
10.4.2							
10.4.3							
10.4.4							
10.4.5							

10.4.4 Connect inductive coil ***L2*** in series with inductive coil ***L2***. Turn on GPI. Draw in scale the diagram for voltage across two coils (***L1*** and ***L2***) $u_k(t)$. Determine the voltage amplitude U_m and steady-state voltage component u_y. Write down the measured data into table 10.2.

10.4.5 Turn off GPI with key ***S4*** . Show the measured results to a lecturer. Turn off the oscillograph. Disconnect electric circuit.

10.5 The processing of experimental results

10.5.1 Determine time constants τ for each test from diagrams. For this purpose it is necessary to draw a tangent to a curve $i_{tr}(t)$ in a point which answers the arbitrary time moment $t=0$ and prolong it to the intersection with a diagram which matches to a steady-state voltage component u_{ss} in a point $t=\tau$. Write down the results into table 10.2.

10.5.2 Write down in algebraic form the transient function for the voltage across resistor ***R14*** by the result of experience 10.4.2, and write down the result into table 10.2.

$$u_{R14}=U_m(1-e^{-t/\tau}). \qquad (10.21)$$

10.5.3 Write down in algebraic form the transient function for the voltage $u_k(t)$ across inductive coils ***L1*** и ***L2*** by the result of experience 10.4.3 and 10.4.4. These functions contain the functions of two voltages (fig. 10.4). Write down the results into table 10.2.

$$u_k=u_{Rk}(t)+u_{Lk}(t). \qquad (10.22)$$

where $u_{R_k}(t)=u_{ss}(1-e^{-t/\tau})$ - an active component of the voltage across the inductive coil, and $u_{l_k}=U_m e^{-t/\tau}$ - a reactive component of the voltage across the inductive coil.

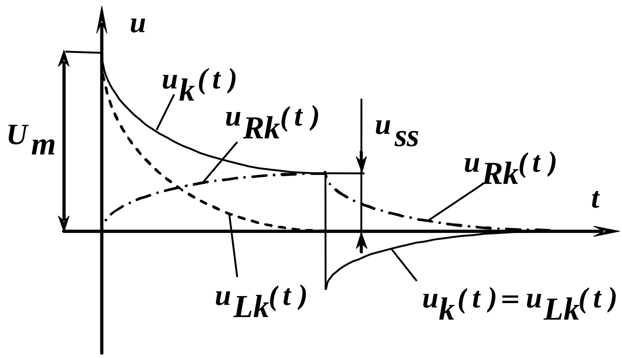


Figure 10.4 – The diagrams of the voltage across the inductive coil and its components

10.5.4 Calculate active resistance of the inductive coil R_k and write down it into table 10.2. You must take into account that the active resistance of the inductive coil R_k and active resistance of resistor $R14$ refer to each other as the matching steady-state voltage components

$$R_k = R14 \frac{u_{ss\,k}}{u_{ss\,R14}}. \qquad (10.23)$$

10.5.5 Calculate overall active resistance of the electrical circuit and write down it into table 10.2.

$$R_\Sigma = R_{in} + R_{14} + R_k. \qquad (10.24)$$

10.5.6 Calculate the inductance of the inductive coil and write down it into table 10.2.

$$L_k = \tau \cdot R_\Sigma. \qquad (10.25)$$

10.5.7 Make some conclusions on the work.

10.6 Control questions

10.6.1 Give the definitions of transient and steady-state working rates in electrical circuit.

10.6.2 In what electric circuits do transients occur?

10.6.3 What is called a transient component, a steady-state component? How are they defined?

10.6.4 Formulate the rules of switching. Give them the explanation.

10.6.5 What is called initial conditions? How are they defined?

10.6.6 Which initial conditions are called independent? Which ones are dependent?

10.6.7 How can the constants of integration determined in the expressions of transient currents and voltages?

10.6.8 What physical meaning have time constants? What do they depend on?

10.6.9 How long the transient process last? What does a transient time depend on?

10.6.10 How do transients pass when an inductive coil is connected to a source with constant t.m.f.?

Laboratory work №11
THE RESEARCH OF TRANSIENTS IN THE ELECTRICAL CIRCUIT WITH A CAPACITOR

The purpose of the work is to investigate electrical circuits with a capacitance in transient regime.

11.1 Short theoretical information
11.1.1 The main definitions

Hence the changing process of any electrical circuit from one of the stationary conditions to another is called the transient. Transients can occur only in the circuits with inductive coils and capacitors. These elements can accumulate the energy in the form of the energy of magnetic or electrical fields.

11.1.2 The connection of a capacitor to a direct current source

Let's consider that before a switching of key Q the capacitor has not been charged, that is we have zero independent initial conditions

$$q(0-)=0,\quad u_C(0-)=0. \tag{11.1}$$

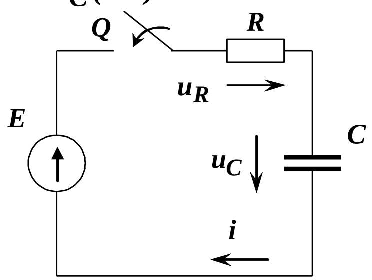


Figure 11.1 – The electrical circuit with a capacitor

At the moment of switching the key Q closes and it appears a closed contour. Compose the equation for it by the second Kirchhoff's law

$$u_R + u_C = E, \tag{11.2}$$

or taking into account that $u_R = Ri$ and $i = C\dfrac{du_C}{dt}$, we have got

$$RC\frac{du_C}{dt} + u_C = E. \tag{11.3}$$

The solution is looked for as the sum of two components

$$u_C = u_{C_{ss}} + u_{C_{tr}} . \qquad (11.4)$$

For a steady- state voltage component we have got an inhomogeneous differential equation of the first order

$$RC\frac{du_{C_{ss}}}{dt} + u_{C_{ss}} = E . \qquad (11.5)$$

As e.m.f. is direct all electrical elements (including a capacitive voltage) will be also direct. Then

$$\frac{du_{C\,ss}}{dt} = 0 . \qquad (11.6)$$

So, an active resistor does not have an influence on the value of a steady- state voltage across a capacitive device

$$u_{C_{ss}} = E . \qquad (11.7)$$

For a transient voltage component we have got a homogeneous differential equation of the first order

$$RC\frac{du_{C_{tr}}}{dt} + u_{C_{tr}} = 0 . \qquad (11.8)$$

Pass from the differential equation to the algebraic one

$$RC + 1 = 0 . \qquad (11.9)$$

After solving the characteristic equation we have got

$$p = -\frac{1}{RC} . \qquad (11.10)$$

So, a transient voltage component across a capacitor is

$$u_{C\,t} = Ae^{pt} = Ae^{-\frac{1}{RC}t} , \qquad (11.11)$$

where A – a constant of integration, p – a root of characteristic equation.

Combine together the steady-state and the transient components

$$u_C = E + Ae^{pt} . \qquad (11.12)$$

By the second rule of switching

$$u_C(0-) = u_C(0+) = 0 . \qquad (11.13)$$

So, at $t = 0+$, we have got

$$0 = E + A . \qquad (11.14)$$

From here

$$A = -E . \qquad (11.15)$$

Combining two solutions we've got a transient capacitive voltage

$$u_C = E - Ee^{pt} = E - Ee^{-\frac{1}{RC}t} . \tag{11.16}$$

The time constant

$$\tau = \frac{1}{|p|} = RC . \tag{11.17}$$

The current in a contour

$$i = C\frac{du_C}{dt} = C\frac{d}{dt}\left(E - Ee^{-\frac{1}{RC}t}\right) = \frac{E}{R}e^{-\frac{1}{RC}t} . \tag{11.18}$$

The voltage across a resistor

$$u_R = Ri = R\frac{E}{R}e^{-\frac{1}{RC}t} = Ee^{-\frac{1}{RC}t} . \tag{11.19}$$

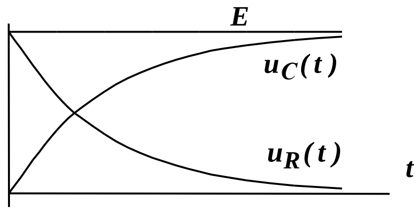


Figure 11.2 – The diagrams of capacitive and resistive voltages

So, the capacitive voltage increases and the resistive voltage decreases by non-periodic law, approaching to a steady-state value (fig. 11.2).

11.2 The order of the working out of the spade-work

11.2.1 Learn theoretical material and prepare control questions.

11.2.2 Draw the circuit (fig. 11.3), table 11.1 and calculations by 11.2.3.

11.2.3 Calculate a transient current and a capacitive voltage in the circuit (fig. 11.1), if $E = 100$ V, $R = 100 + 10n$ Ω, $C = n$ *mkF* where n - the variant by a group register. Draw diagrams of dependences $u_C = f(t)$, $i = f(t)$.

11.3 The description of the laboratory plant

We must use the generator - micropulser (GPI) with an internal resistance $R_{in} = 51$ Ω, which is turned on with key $S4$; active resistor $R14 = 120$ Ω; capacitors $C3$ and $C4$ and an oscillograph.

11.4 The order of carrying out of the work

11.4.1 Make up an electric circuit (fig. 11.3).

11.4.2 Connect the oscillograph input to resistor $R14$, switch on GPI. Draw in scale the voltage diagram $u_{R14}(t)$ from the screen. Determine the voltage amplitude U_m and steady-state component u_{ss} from the diagram.

11.4.3 Connect the oscillograph input to capacitor C_3, switch on GPI. Draw in scale the voltage diagram $u_C(t)$. Determine U_m and u_{ss}.

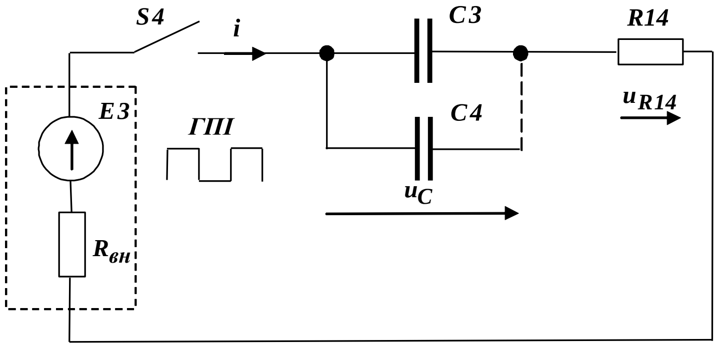


Figure 11.3 - The electric circuit for the investigation

11.4.4 Connect capacitors $C3$ and $C4$ in parallel. Connect the oscillograph to $R14$. Draw the voltage $u_{R14}(t)$. Determine U_m and u_{ss}.

11.4.5 Connect the oscillograph input to two capacitors. Draw $u_C(t)$ from the screen. Determine U_m and u_{ss} from the diagram.

Table 11.1 - The results of investigation

Test	Measured		Calculated				
	U_m, V	u_{SS}, V	τ, s	R_k, Ω	R_Σ, Ω	C, H	Transient function
11.4.2							
11.4.3							
11.4.4							
11.4.5							

11.4.5 Connect the oscillograph to parallel capacitors. Draw $u_C(t)$. Determine U_m and u_{ss}. Write all measured data into table 11.1.

11.4.5 Turn off GPI with key $S4$. Show all measured results to a lecturer. Turn off the oscillograph. Disconnect electric circuit.

11.5 The processing of experimental results

11.5.1 Determine time constants τ from diagrams. For this purpose draw a tangent to a curve $i_{tr}(t)$ in a point which answers the arbitrary time moment $t=0$ and prolong it to the intersection with a diagram which matches to a steady-state voltage component u_{ss} in a point $t=\tau$.

11.5.2 Write down in algebraic form the transient function for the voltage across resistor $R14$ by the result of experiences 11.4.2 and 11.4.4 and write down the result into table 11.1.

$$u_{R14} = U_m e^{-t/\tau} . \qquad (11.20)$$

11.5.3 Write down the transient function for the voltage across capacitors $C3$ and $C4$ into table 11.1.

$$u_C = U_m (1 - e^{-t/\tau}) . \qquad (11.21)$$

11.5.4 Calculate total active resistance of the circuit

$$R_\Sigma = R_{in} + R14 . \qquad (11.22)$$

11.5.5 Calculate the capacitance of the capacitor

$$C = \frac{\tau}{R_\Sigma} . \qquad (11.23)$$

11.5.6 Make some conclusions on the work.

11.6 Control questions

11.6.1 Give definitions of transient and steady-state working rates.

11.6.2 In what electric circuits do transients occur?

11.6.3 What is called a transient component, a steady-state component? How are they determined?

11.6.4 Formulate the rules of switching. Give them the explanation.

11.6.5 What is called initial conditions? How are they defined?

11.6.6 Independent and dependent conditions. Explain difference.

11.6.7 How can the constants of integration determined in the expressions of transient currents and voltages?

11.6.8 What do time constants depend on?

11.6.9 How long the transient process last? What does a transient time depend on?

11.6.10 How do transients pass when an a capacitor is connected to a source with direct e.m.f.?

Laboratory work №12
AN INDUCTIVE COIL IN ALTERNATING CURRENT ELECTRICAL CIRCUIT

The purpose of the work is to investigate the dependence of electrical parameters of inductive coil on the quantity of convolutions and its voltage, and build-up a magnetisation curve of a steel core.

12.1 Short theoretical information
12.1.1 The main definitions

If a current flows through the coil it creates a magnetic field in environmental space. If the current is direct the magnetic field is also direct. The coil magnetic linkage is equal to an algebraic sum of magnetic fluxes which pierce separate convolutions of a winding

$$\Psi = \Phi_1 + \Phi_2 + \Phi_3 + ... + \Phi_n = \sum_{k=1}^{n} \Phi_k \,. \quad (12.1)$$

If all convolutions of a coil are pierced by an equal magnetic flux

$$\Psi = w\Phi \,. \quad (12.2)$$

Coefficient of proportionality between a characteristic magnetic linkage of the coil and a coil current is termed as inductance (*H*)

$$L = \frac{\Psi}{i} \,. \quad (12.3)$$

The equivalent circuit of a coil consists of two sequentially joined elements: inductive and resistive. The impedance of such a coil:

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi f L)^2} \,. \quad (12.4)$$

The parameters of the coil is determined by the following equations:

- impedance

$$Z = \frac{U}{I}; \quad (12.5)$$

- active resistance

$$R = \frac{P}{I^2}; \quad (12.6)$$

- inductive reactance

$$X_L = \sqrt{Z^2 - R^2}\ ; \tag{12.7}$$

- inductance

$$L = \frac{X_L}{2\pi f}\ ; \tag{12.8}$$

- a power coefficient

$$\cos\varphi = \frac{P}{UI} = \frac{R}{Z}\,. \tag{12.9}$$

12.1.2 The coil with a steel core in alternating current circuit

From characteristic $B = f(H)$ by means of scale transformation it is possible to get the dependence $\psi = f(i)$. The coil magnetic linkage ψ in this case is not proportional to a current i , that is the capacitance of such a coil depends on a current. Distinguish a static nonlinear inductance

$$L(i) = \frac{\psi(i)}{i}\,. \tag{12.10}$$

and differential inductance

$$L_\partial(i) = \frac{d\psi(i)}{di}\,, \tag{12.11}$$

According to a Faraday law the voltage across a nonlinear inductance

$$u = \frac{d\psi}{di} = \frac{d\psi}{di}\frac{di}{dt} = \frac{d}{dt}\left[L(i)i\right]\cdot\frac{di}{dt} = \left[L(i) + i\frac{dL(i)}{di}\right]\cdot\frac{di}{dt}\,. \tag{12.12}$$

or

$$u = \frac{d\psi}{di}\frac{di}{dt} = L_\partial(i)\frac{di}{dt}\,. \tag{12.13}$$

At magnetisation of the ferromagnetic core a part of magnetic field energy is transformed into thermal energy. The power which matches to this part is termed as iron losses (P_{st}). These losses consist of two parts: hysteresis losses and eddy-current losses. The power of hysteresis losses:

$$P_h = \sigma_h f B_m^n G\,, \tag{12.14}$$

where σ_h - a hysteresis coefficient, depends on a steel brand; B_m - amplitude of magnetic induction, G - a core mass, n - an exponent, depends on the magnetic induction B_m .

The power of losses on eddy-current is determined by formula

$$P_{ed} = \sigma_{ed} f^2 \gamma B_m^2 G, \tag{12.15}$$

where σ_{ed} - an eddy-current coefficient which value depends on a brand of an electro-technical steel γ - a conductivity of an electro-technical steel.

Then the power of losses in a core

$$P_{st} = P_h + P_{ed} \tag{12.16}$$

The active resistance of a coil

$$R = \frac{P}{I^2} = \frac{P_{win}}{I^2} + \frac{P_{st}}{I^2} = R_{win} + R_{st}, \tag{12.17}$$

where R_{win}, R_{st} - active resistances which characterise power losses accordingly in a winding of a coil and in the core.

12.1.3 Dynamic magnetisation curve of a steel core

Magnetic properties of ferromagnetic materials are conveniently represented by curve of B_m against H_m at definite frequency. Such a curve is called dynamic magnetization curve (see fig. 12.1).

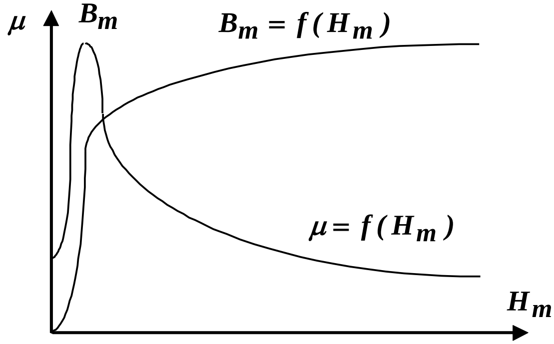


Figure 12.1 - The diagrams of magnetic induction dependence B_m and magnetic permeability μ against magnetic field intensity H_m

The amplitude of magnetic induction may be calculated by the equation

$$B_m = \frac{\Phi_m}{S} = \frac{E_L}{4.44\, fwS}, \tag{12.18}$$

where S - a cross-section of a core; $E_L = X_L I$ - effective value of e.m.f .

of coil self-induction; f - frequency; w - the quantity of turns.

The amplitude of magnetic intensity in a core

$$H_m = \frac{Iw}{l_{cp}}, \tag{12.19}$$

where I - current in a winding, l_{av} - average length of a core.

The static magnetic permeability

$$\mu_{cm} = \frac{B}{H} \cdot 10^{-6}, \tag{12.20}$$

12.2 The order of the working out of the spade-work

12.2.1 Prepare answers to control questions.

12.2.2 Prepare the report where must be the purpose of the work, the electric circuits (fig. 12.2 and 12.3), table 12.1, 12.2 and 12.3.

12.3 The description of the laboratory plant

For fulfilling of the laboratory work one must use:

- a laboratory autotransformer;
- a coil without a steel core $L2$;
- a coil with a steel core $L5$;
- the voltmeter of electro-magnetic system $V3$;
- the ampere-meters of electro-magnetic system $A5$, $A9$;
- the wattmeter of electro-dynamic system $W1$.

The parameters of the coil without a steel core: $w1 = 1800$, $w2 = 1600$, $w3 = 1400$, $w4 = 1200$. The parameters of a coil with a steel core $w = 200$; cross-section $S = 434$ *mm*^{2}; average length $l_{av} = 120$ *mm*.

12.4 The order of carrying out of the work

12.4.1 Make up an electric circuit (fig. 12.2). Switch on the stand. Fluently turning a handhold of a laboratory autotransformer one must change a voltage across the terminals of the coil and for the values of voltage, multiple 10 *V*, you should measure a current and a consumed active power. Write down the measured data in table 12.1. Turn off a stand.

12.4.2 Switch on the stand. At constant value of active power (≈ 20…40 Wt), one must measure a current and a voltage across the terminals of the coil at different quantity of coil turns (terminals ***w1***; ***w2***; ***w3***; ***w4***). Constant power is supported, controlling a voltage by means of a laboratory autotransformer. Write down the measured data in table 12.2. Turn off a stand. Don't disconnect the circuit.

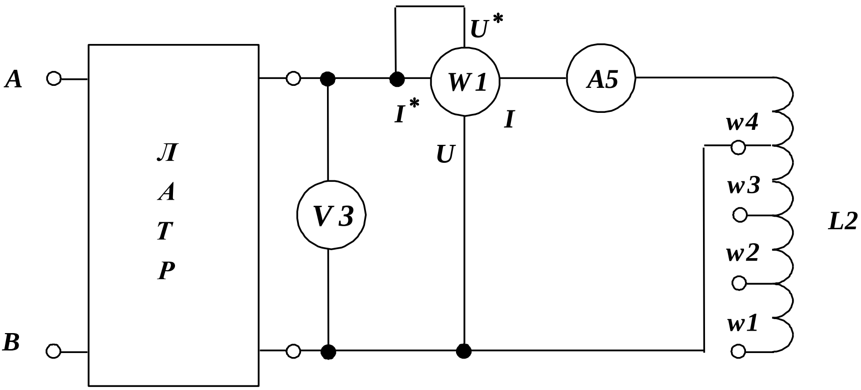


Figure 12.2 - The circuit for investigation of a coil without a core

Table 12.1 - The results of investigation for the coil without a core

U, V	0					
I, A	0					
P, Wt	0					

Table 12.2 - The results of investigation for the coil without a core

Terminals of the coil	Measured			Calculated				
	U, V	***I***, A	***P***, Wt	***Z***, Ω	***R***, Ω	X_L, Ω	$\cos\varphi$	***L***, mH
w1								
w2								
w3								
w4								

12.4.3 Compose electrical circuit (fig. 12.3).

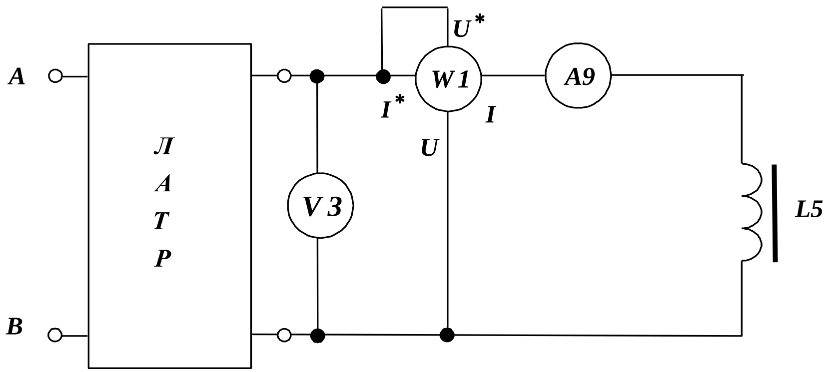


Figure 12.3 - The circuit for investigation of a coil with a steel core

Table 12.3 - The results of investigation for the coil with a steel core

№	Measured			Calculated						
	U, V	I, V	P, Wt	Z, Ω	R, Ω	X_L, Ω	$\cos\varphi$	L, mH	B_m, Tl	H_m, A/m
1										
2										
3										
4										

Connect a coil with a steel core ***L5*** to the circuit. Switch on the stand. Smoothly turning a handhold of a autotransformer one must change a voltage across the terminals of the coil, and for current values, multiple 0.5 *A*, you should measure a voltage across coil terminals and a consumed active power. Write down the measured data in table 12.3. Switch off the stand. Data must be shown to a lecturer. Disconnect the circuit.

12.5 The processing of experimental results

12.5.1 According to table 12.2 and table 12.3 in common axes one must build volt-ampere characteristics of the coils.

12.5.2 According to table 12.3 calculate coil parametres at different number of turns and build the diagrams of the following dependences: $Z = f(w)$, $R = f(w)$, $X_L = f(w)$, $\cos\varphi = f(w)$, $L = f(w)$.

12.5.3 By table 12.3 calculate the parametres of the coil with a steel core and build the diagrams of the following dependences: $\boldsymbol{Z = f(U)}$, $\boldsymbol{R = f(U)}$, $\boldsymbol{X_L = f(U)}$, $\boldsymbol{cos\varphi = f(U)}$, $\boldsymbol{L = f(U)}$.

12.5.4 By table 12.3 calculate the amplitude of magnetic induction and the magnetic field intensity and build the magnetization curve $\boldsymbol{B_m = f(H_m)}$. Make some conclusions on the work.

12.6 Control questions

12.6.1 How are inductive coils settled? Their singularities?

12.6.2 Make the definitions for terms: "inductance", "magnetic linkage", magnetic induction.

12.6.3 Which physical appearances originate in a coil with a steel core at an alternating current source?

12.6.4 What power losses are characteristic for a coil with a steel core at an alternating current source?

12.6.5 How does the impedance of a coil with a steel core change when the voltage across its terminals is varied?

12.6.6 What connection is the turns of a coil and its inductance?

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