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APPROXIMATION OF FUNCTIONS OF TWO VARIABLES ON THE UNIT BICIRCLE IN GENERALIZED HÖLDER SPACES

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The approximative properties of interpolational Lagrange polynomials, Fourier series of functions of two variables which are defined on the unit bicircle, is stated. For functions from generalized Hölder spaces H_ω the estimations are conducted for norms of C , H_ω , L_p , $p > 1$ spaces.

Let γ_{01} and γ_{02} denote the unit circles with centers in the origin; and $\gamma_0 = \gamma_{01} \times \gamma_{02}$ is the unit bicircle. Let $\omega(\delta_1, \delta_2)$ be the some modulus of continuity, and $\Omega_1(\delta)$, $\Omega_2(\delta)$ are the simple moduli of continuity, which respect to that one. Let H_ω denotes the space of continuous functions $x(t, \tau)$ at γ_0 , which satisfy the conditions:

$$H(x; \omega) = \sup_{\delta_1^2 + \delta_2^2 \neq 0} \frac{\omega(\delta_1, \delta_2; x)}{\omega(\delta_1, \delta_2)} \leq c_1,$$

$$H^{t\tau}(x; \omega) = \sup_{\delta_1^2 + \delta_2^2 \neq 0} \frac{\omega_{1,1}(\delta_1, \delta_2; x)}{\Omega_1(\delta_1)\Omega_2(\delta_2)} \leq c_2.$$

Here c_1 and c_2 are the some constants; $\omega_{1,1}$ denotes the mixed modulus of continuity of the second order [1]. In the space H_ω the norm is defined as:

$$\|x(t, \tau)\|_{H_\omega} = \|x(t, \tau)\|_C + H(x; \omega) + H^{t\tau}(x; \omega).$$

Besides, we'll consider the other kinds of moduli of continuity (such as full, mixed and

partial ones) and constants $H^t(x; \Omega_1) = \sup_{\delta_1 > 0} \frac{\omega(\delta_1, 0; x)}{\Omega_1(\delta_1)},$

$H^\tau(x; \Omega_2) = \sup_{\delta_2 > 0} \frac{\omega(0, \delta_2; x)}{\Omega_2(\delta_2)},$ which are defined in [2]. Let X_{mn} be the set of all

polynomials of the form $x_{mn}(t, \tau) = \sum_{k=-m}^m \sum_{l=-n}^n c_{kl} t^k \tau^l$; $E_{mn}(x)$ is the best uniform approximation of $x(t, \tau)$ by the polynomials from X_{mn} . Let

$$(L_{mn}g)(t, \tau) = \sum_{p=0}^{2m} \sum_{q=0}^{2n} g(t_p, \tau_q) l_p(t) l_q(\tau) = \sum_{k=-m}^m \sum_{l=-n}^n \lambda_{kl} t^k \tau^l,$$

$$l_p(t) = \prod_{\substack{k=0 \\ k \neq p}}^{2m} \frac{t - t_k}{t_p - t_k} \left(\frac{t_p}{t} \right)^m = \sum_{k=-m}^m \lambda_{1,k}^{(p)} t^k,$$

$$l_q(\tau) = \prod_{\substack{j=0 \\ j \neq q}}^{2n} \frac{\tau - \tau_j}{\tau_q - \tau_j} \left(\frac{\tau_q}{\tau} \right)^n = \sum_{j=-n}^n \lambda_{2,j}^{(q)} \tau^j,$$

denotes interpolational Lagrange polynomial of the function $g(t, \tau)$ with respect to the system of equidistant points:

$$t_p = \exp\left\{ \frac{2\pi i}{2m+1} (p-m) \right\}, \quad i^2 = -1, \quad p = 0, \dots, 2m;$$

$$\tau_q = \exp\left\{ \frac{2\pi i}{2n+1} (q-n) \right\}, \quad i^2 = -1, \quad q = 0, \dots, 2n$$

at γ_0 . Beside that we use the following estimations [1] for the functions $x(t, \tau) \in H_\omega$:

$$E_{mn}(x) \leq c \left[H^t(x; \Omega_1) \Omega_1\left(\frac{1}{m}\right) + H^\tau(x; \Omega_2) \Omega_2\left(\frac{1}{n}\right) \right] \leq c H(x; \omega) \omega\left(\frac{1}{m}, \frac{1}{n}\right)$$

and for the derivatives of the polynomials $P_{mn}(t, \tau) \in X_{mn}$:

$$\text{a) } \left\| (P_{mn})'_t \right\|_C \leq cm \|P_{mn}\|_C;$$

$$\text{b) } \left\| (P_{mn})'_\tau \right\|_C \leq cn \|P_{mn}\|_C.$$

Theorem 1. Let $x(t, \tau) \in H_\omega$. Then:

$$\|x - L_{mn}x\|_C \leq c \ln m \ln n H(x; \omega) \omega\left(\frac{1}{m}, \frac{1}{n}\right).$$

Let operator Φ_{mn} assigns to any function $g(t, \tau) \in H_\omega$ the mn -th partial sum of its Fourier series, that is

$$(\Phi_{mn}g)(t, \tau) = \sum_{k=-m}^m \sum_{l=-n}^n g_{kl} t^k \tau^l,$$

where

$$g_{kl} = \frac{1}{4\pi^2} \int_{\gamma_{01}} \int_{\gamma_{02}} \frac{g(t, \tau)}{t^{k+1} \tau^{l+1}} dt d\tau, \quad k, l = 0, \pm 1, \pm 2, \dots$$

Theorem 2. For any function $g(t, \tau) \in H_\omega$ and for arbitrary natural m, n the inequality is fulfilled:

$$\|g - \Phi_{mn} g\|_C \leq (1 + c \ln(2m+1) \ln(2n+1)) H(g; \omega) \omega\left(\frac{1}{m}, \frac{1}{n}\right).$$

Lemma 1. Let $P_{mn} \in X_{mn}$ at γ_0 . Then for arbitrary pair $\Omega_1(\delta)$ and $\Omega_2(\delta)$ the estimations are true:

$$a) H^t(P_{mn}; \Omega_1) \leq c \frac{\|P_{mn}\|_C}{\Omega_1\left(\frac{1}{m}\right)},$$

$$b) H^\tau(P_{mn}; \Omega_2) \leq c \frac{\|P_{mn}\|_C}{\Omega_2\left(\frac{1}{n}\right)}.$$

In the future we'll assume that m, n satisfy the conditions:

$$M_1 \leq \frac{m}{n} \leq M_2,$$

where M_1, M_2 are constants, $0 < M_1, M_2 < \infty$.

Lemma 2. Let $x_{mn}^*(t, \tau)$ be the polynomial of the best uniform approximation of the function $x(t, \tau) \in H_{\omega(1)}$. Then for arbitrary $\omega^{(2)}(\delta_1, \delta_2)$ such that $H_{\omega(1)} \subset H_{\omega(2)}$ and $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta), \Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ are increasing functions, the estimations are true:

$$a) H^t(x - x_{mn}^*; \Omega_1^{(2)}) \leq c \left[H^t(x; \Omega_1^{(1)}) \frac{\Omega_1^{(1)}\left(\frac{1}{m}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)} + H^\tau(x; \Omega_2^{(1)}) \frac{\Omega_2^{(1)}\left(\frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{n}\right)} \right];$$

$$b) H^\tau(x - x_{mn}^*; \Omega_2^{(2)}) \leq c \left[H^t(x; \Omega_1^{(1)}) \frac{\Omega_1^{(1)}\left(\frac{1}{m}\right)}{\Omega_2^{(2)}\left(\frac{1}{m}\right)} + H^\tau(x; \Omega_2^{(1)}) \frac{\Omega_2^{(1)}\left(\frac{1}{n}\right)}{\Omega_2^{(2)}\left(\frac{1}{n}\right)} \right].$$

Lemma 3. The inequality is true:

$$\|L_{mn}\|_{H_{\omega} \rightarrow H_{\omega}} \leq c \ln m \ln n.$$

The proof of this statement follows from [4] on the basis of the estimation:

$$\|L_{mn}\|_{H_{\omega}} \leq c \|L_m\|_{H_{\Omega_1}} \cdot \|L_n\|_{H_{\Omega_2}} \leq c \ln m \ln n.$$

Lemma 4. Under the conditions of Lemma 2 the estimations are true:

$$\begin{aligned} a) \quad H^t(x - L_{mn}x; \Omega_1^{(2)}) &\leq c \ln m \left[H^t(x; \Omega_1^{(1)}) \frac{\Omega_1^{(1)}\left(\frac{1}{m}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)} + H^{\tau}(x; \Omega_2^{(1)}) \frac{\Omega_2^{(1)}\left(\frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{n}\right)} \right]; \\ b) \quad H^{\tau}(x - L_{mn}x; \Omega_2^{(2)}) &\leq c \ln n \left[H^t(x; \Omega_1^{(1)}) \frac{\Omega_1^{(1)}\left(\frac{1}{m}\right)}{\Omega_2^{(2)}\left(\frac{1}{m}\right)} + H^{\tau}(x; \Omega_2^{(1)}) \frac{\Omega_2^{(1)}\left(\frac{1}{n}\right)}{\Omega_2^{(2)}\left(\frac{1}{n}\right)} \right]. \end{aligned}$$

Lemma 5. Under the conditions of Lemma 2 the estimation is fulfilled:

$$H^{t\tau}(x - x_{mn}^*; \omega^{(2)}) \leq c \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right) \Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}}.$$

Lemma 6. Under the conditions of Lemma 2 the estimation is true:

$$H^{t\tau}(x - L_{mn}x; \omega^{(2)}) \leq c \ln m \ln n \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right) \Omega_2^{(2)}\left(\frac{1}{n}\right)}.$$

Theorem 3. Let $x(t, \tau) \in H_{\omega^{(1)}}$; $\omega^{(2)}(\delta_1, \delta_2)$ is such that $H_{\omega^{(1)}} \subset H_{\omega^{(2)}}$ and $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ are increasing functions. Then:

$$\|x - L_{mn}x\|_{H_{\omega^{(2)}}} \leq c \ln m \ln n \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right) \Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega^{(1)}}}.$$

The proof of this statement follows directly from Lemmas 4, 6 and Theorem 1.

Theorem 4. Let $x(t, \tau) \in H_{\omega(1)}$; $\omega^{(2)}(\delta_1, \delta_2)$ is such that $H_{\omega(1)} \subset H_{\omega(2)}$ and $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ are increasing functions. Then:

$$\|x - \Phi_{mn}x\|_{H_{\omega(2)}} \leq c \ln m \ln n \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x\|_{H_{\omega(1)}}.$$

Let us now consider the space L_p , $p > 1$. If put that Lagrange operator works from the space C to the space L_p , then respectively to [3] the estimation $\|L_{mn}\|_{C \rightarrow L_p} \leq c(p)$ is true, where $c(p)$ is completely defined constant, which depends on p and doesn't depend on m, n .

Theorem 5. Let $g(t, \tau) \in H_{\omega}$. Then the estimation is fulfilled:

$$\|g - L_{mn}g\|_{L_p} \leq (1 + c(p))H(g; \omega)\omega\left(\frac{1}{m}, \frac{1}{n}\right).$$

Remark 1. If under the conditions of Theorems 1 – 5 the partial derivatives of the approximated functions of orders p and q with respect to t and τ belong to the space H_{ω} (it means $x(t, \tau) \in H_{\omega}^{(p, q)}$), that the estimations in the mentioned theorems can be considerably improved, namely by the value $\frac{1}{m^p} + \frac{1}{n^q}$. This conclusion follows directly from the inequalities [1]:

$$\begin{aligned} E_{mn}(x) &\leq c \left[\frac{1}{m^p} H^t \left(\frac{\partial^p x}{\partial t^p}; \Omega_1 \right) \Omega_1 \left(\frac{1}{m} \right) + \frac{1}{n^q} H^{\tau} \left(\frac{\partial^q x}{\partial \tau^q}; \Omega_2 \right) \Omega_2 \left(\frac{1}{n} \right) \right] \leq \\ &\leq c \left(\frac{1}{m^p} + \frac{1}{n^q} \right) H \left(\frac{\partial^{p+q} x}{\partial t^p \partial \tau^q}; \omega \right) \omega \left(\frac{1}{m}, \frac{1}{n} \right). \end{aligned}$$

For example, in this case the theorem which is respected to Theorem 3 can be formulated as follows:

Theorem 6. Let $x(t, \tau) \in H_{\omega(1)}^{(p, q)}$; $\omega^{(2)}(\delta_1, \delta_2)$ is such that $H_{\omega(1)} \subset H_{\omega(2)}$ and $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ are increasing functions. Then the estimation is true

$$\|x - L_{mn}x\|_{H_{\omega(2)}} \leq c \ln m \ln n \left(\frac{1}{m^p} + \frac{1}{n^q} \right) \frac{\omega^{(1)}\left(\frac{1}{m}, \frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right) \Omega_2^{(2)}\left(\frac{1}{n}\right)} \|x^{(p,q)}\|_{H_{\omega(1)}}.$$

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