MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
Zaporizhzhia Polytechnic National University

CALCULATION TASKS ON HIGHER MATHEMATICS (4th module)

for students majoring in
141 Power Engineering, Electrical Engineering
and Electrical Mechanics

2021

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CONTENTS

INTRODUCTION

One of the types of unaided work of students is the execution of calculation tasks during the semester. Their purpose is to develop practical skills of students. They are designed to help students capture theoretical material more deeply and learn how to apply the acquired knowledge to solve practical problems. The offered typical calculation tasks correspond to the course "Higher Mathematics" taught to students majoring in Power Engineering, Electrical Engineering and Electrical Mechanics full-time in the forth module of the two-semester course. This set of tasks can also be used for students in other technical majors who study higher mathematics in a two-semester course.

This task book covers virtually all major sections of the forth module of the course on higher mathematics; it contains assignments for all the topics of the course, and offers 15 options for unaided work. A list of recommended literature is also provided. Examples of solving such problems are contained in the manual [14]. For ease of use, in this manual the numbering of examples corresponds to the numbering of tasks in the given task book. In addition to the methodological instructions for solving problems and examples of solutions, manual [14] also contains the necessary theoretical information and reference material (tables, literature, etc.).

Calculation tasks are executed during the semester, which provides students with a systematic study of the course. While carrying out these tasks, students work with recommended textbooks and manuals, independently search for necessary literary sources and materials, analyze them and summarize, independently research and make written presentation of practical assignments.

The student chooses the option (the number of the variant) according to his number in the register list. The work is done in English in writing, preferably in a notebook in a cell. Note fields must be left blank. The name of the subject, major, group and course, surname, first name and patronymic of the student, name of the teacher who accepts the work should be indicated on the title page of the work.

While performing the work, the student should solve the offered tasks by the methods specified in the tasks, as well as make all the necessary drawings (graphical solutions). The student must show the acquired theoretical knowledge of the course.

When doing the work, the student can use both the lecture and practical material, as well as the supporting literature listed at the end of this manual. Before starting the work, it is recommended to study the relevant theoretical material, then to understand the solutions of the tasks that were performed in the practical classes, and only after that to start the actual calculation work.

When evaluating a work, the indicator of its quality is, first of all, how the student independently and correctly solved the tasks and understood the content of the obtained solutions. That is why additional questions may be asked to protect the student's work, including the theoretical material presented for the exam.

Completed work is submitted to the teacher for verification and subsequent protection in the form of an interview (usually during modular control). The student must be able to:

– present the content of the tasks and to prove their solutions;
– answer questions about the content of the solutions obtained;
– answer additional questions.

If the work is successfully protected, the student receives a certain number of rating points. If the specified requirements are not fulfilled, then the work is returned to the student for completion, indicating the term of re-protection.

1 THEORY OF PROBABILITY

Task 1.1. Solve the problem.

1) Of the 60 graduates of the school, 4 students received a gold medal, and 5 students received a silver medal. What is the probability that two of the three randomly selected graduates won a gold medal?

2) Three shooters make one shot at the target each. The probability that the first shooter will hit is 0.7. For the second and third shooters, these probabilities are equal to 0.8 and 0.9, respectively. What is the probability that only one shooter will hit the target?

3) In a batch of 1000 parts, there are 200 parts of the highest grade, 750 parts of the first grade and 50 defective parts. 10 parts were randomly selected for control. What is the probability that among the selected parts exactly 2 parts will be of the highest grade?

4) There are 20 balls in the box, 8 of them are white. Two balls were drawn at random. Find the probability that they are both white.

5) There are 30 students in the group, 20 of them are boys . Two students were summoned to the dean's office. Find the probability that they are both boys.

6) The cafe has 12 types of desserts, 8 of them are with nuts. Visitors ordered three different desserts. Find the probability that all these desserts are with nuts.

7) There are 20 oaks among 80 trees. Two trees were cut down. Find the probability that both felled trees are oaks.

8) Among 100 bolts 6 bolts are non-standard. 3 bolts were chosen at random. Find the probability that all selected bolts are non-standard.

9) There are 15 Christmas decorations in a box, 5 of them are star-shaped. 8 decorations are chosen at random Find the probability that 3 decorations will be in the form of stars.

10) There are 20 pencils in the box, 6 of which are red. 9 pencils were taken out of the box at random. What is the probability that among the selected pencils will be 4 red?

11) There are 12 books on the shelf in the library, 9 of them are written by one author. 8 books are chosen at random. Find the probability that among them there are exactly 6 books written by one author.

12) The repair shop received 20 laptops, among them 8 laptops need battery replacement. The master takes five laptops at random. Find the probability that 2 of them need to be replaced battery.

13) From a deck of 36 cards, 16 cards are taken at random. Find the probability that among them there are 9 cards of the same suit.

14) There are 20 packs of juice in a package. It is known that 15 of them with cherry juice, and others with pomegranate. 10 packs are chosen at random. Find the probability that among them there are exactly two packs which contain pomegranate juice.

15) There are 24 chocolates in a box, eight of them contain liqueur. 10 chocolates are chosen at random. Find the probability that exactly 5 of them contain liqueur.

Task 1.2. Solve the problem.

1) A "Pioneer" car radio fails with a probability of 0.05 per year of operation, and an "Alpine" car radio fails with a probability of 0.03. "Pioneer" with a probability of 60% or "Alpine" with a probability of 40% is installed at the car service. The radio installed at this car service failed after a year of operation. What is the probability that it was "Pioneer"?

2) In the taxi fleet there are 30 Lanos cars and 20 Slavuta cars. The probability that Lanos was late to the customer is 2%, and for Slavuta this probability is 5%. What is the probability that the called car is a Lanos, if it happened that it was late?

3) A grandmother, visiting her granddaughter, can buy sweets in "Squirrel" (where she could go with a probability of 40%) or in "Cream Rosette" (where she could go with a probability of 60%). Favorite candies are on sale in "Squirrel" with a probability of 70%, and in "Cream Rosette" with a probability of 80%. What is the probability that the grandmother chose "Squirrel", if it became known that she bought her favorite candy?

4) A tourist could travel by plane with a probability of 35% or by bus with a probability of 65%. The level of service satisfies the tourist in the airline with a probability of 90%, and in the company of the carrier with a probability of 75%. What is the probability that a tourist went on a plane trip, if it is known that the level of service satisfied him?

5) Some group contains 20 students, among them there are 4 excellent students The probability of solving the problem is 0.9 for the excellent student, and 0.6 for the rest of the students. What is the probability that a randomly selected student is an excellent student if it is known that he/she has solved the problem?

6) The manufacturer receives components from two suppliers, whose shares are 40% and 60%. 2% of parts of the first supplier are defective and 3% of parts of the second supplier are defective. What is the probability that a randomly selected part came from the first supplier, if it turned out to be defective?

7) On hot days, the refrigerator in the cafe does not have time to cool drinks, so "Fanta" can be cooled with a probability of 0.65, and "Sprite" can be cooled with a probability of 0.8. The client chooses "Fanta" with a probability of 30%, and "Sprite" with a probability of 70%. What is the probability that he chose "Sprite" if the ordered drink turned out to be chilled?

8) A mechanical watch goes with a suitable accuracy with a probability of 0.7, and an electronic watch goes with a suitable accuracy with a probability of 0.999. From a batch of 160 watches (there were 40 mechanical watches among them) one watch was chosen at random. What is the probability that you have chosen a mechanical watch, if you know that it goes with a suitable accuracy?

9) A powerful incandescent lamp can burn out during the year with a probability of 6%, and low-power can burn with a probability of 2%. There were 24 lamps in the box, including 16 powerful ones. One of them was chosen at random, which burned out during the year. What is the probability that a powerful lamp was chosen?

10) The student solves problems in geometry with a probability of 0.7, in algebra with a probability of 0.8. The geometry problem can appear on the exam with a probability of 0.35. What is the probability that the exam problem was actually a geometry problem if it is known that the student solved it?

11) In the first photo studio they make 70% of glossy photos and 30% of matte photos, and in the second photo studio these indicators are equal to 60% and 40% respectively. The client applies to the first photo studio with a probability of 80%, or to the second with a probability of 20%. What is the probability that he turned to the first photo studio, if it is known that the resulting photo is glossy?

12) The secretary must convey the message. The probability that he will send a message by letter is equal to 30%, by phone – 50%, by fax – 20%. The probabilities that the message will be obtained are equal to 0.7; 0.85 and 0.9, respectively. 1) What is the probability receiving a message? 2) The message is received. Which the probability that it was transmitted by fax?

13) Three daughters, Galya, Marichka and Oksanka, are responsible for washing-up. Galya performs 45% of the work; Marichka performs 35% of the work and Oksanka performs 20% of work. Washing dishes, Galya can break a plate with probability of 3%, Marichka can break a plate with probability of 4%, Oksanka can break a plate with probability of 5%.

Parents in the living room heard the rattle of a broken plate. What is the probability that Oksanka is washing the dishes?

14) The home appliance store has refrigerators of the following brands: 50 refrigerators Samsung, 100 Indesit, 70 Ardo and 30 Gorenje. Probability that during the warranty period of three years the refrigerator needs repair, respectively equal to 2% for the Samsung brand, 2.5% for Indesit, 1.5% for Ardo and 0.5% for Gorenje. The buyer who bought refrigerator in this store at random, under warranty term applied for repairs. What is the probability that it was a refrigerator of the brand Indesit?

15) In the first ballot box there were 2 white balls and 3 black balls, and in the second ballot box there were 2 white balls and 8 black ones. One ball was randomly removed from a randomly selected ballot box. What is the probability that the ball was taken out of the first ballot box if it turned out to be white?

Task 1.3. Solve the problem.

1) Inside the circle with the center at the origin of the Cartesian coordinate system randomly placed 9 points. What is the probability that 2 of them are in the first quadrant?

2) In 8 out of 10 cases violation of traffic rules on the route is a violation of the speed limit. Within an hour, 8 violators of traffic rules were recorded on the route. What is the probability that 6 persons violated the speed limit?

3) 45% of modern mobile phones have a touch screen. What is the probability that among the 9 randomly selected mobile phones 4 phones have a touch screen?

4) To operate, a GPS navigator, which can measure altitude without a barometric sensor, must receive signals from at least four satellites. 14 satellites broadcast their signals in this area. Signal reception from each of them occurs with a probability of 0.5. What is the probability that the navigator will work?

5) Music historians say that enemies had sawed three strings on Niccolo Paganini's violin before the concert, but the master was able to play the whole concert on the fourth string. Suppose the probability of the breaking string is 0.6. What is the probability that two strings out of four will break?

6) The researcher planted 10 seeds. Each seed can germinate with a probability of 0.4. What is the probability that 5 seeds will germinate in this experiment?

7) There are 7 colors in the rainbow. Each of the 5 girls chooses one color. What is the probability that two girls chose green color?

8) MiniTool Power Data Recovery can recover files on a flash drive, even if it has been formatted. Under certain conditions, the recovery process of each file is successful with a probability of 0.8. 14 files were placed on the drive before formatting. What is the probability that it will be possible to recover at least 12 files?

9) When compiling the thirteenth page of these manual, there was a need to call the Microsoft Equation 3.0 formula editor 13 times. Since the compiling took place on Friday, 13th, due to otherworldly factors the probability of a failure was 0.13 for each call. What is the probability that when compiling this page, the editor failed twice?

10) The athlete shoots at the target 4 times. The probability of hitting for each shot is equal to 0.2. Which is more likely: to hit once or never hit?

11) The dice was thrown 5 times. What is the probability that in two cases out of five we will get one point?

12) A girl makes a bouquet of five roses. There are white and red roses. Randomly selected rose is white with a probability of 0.2. What is the probability that there will be one white rose in the bouquet?

13) The coin was tossed 8 times. What is the probability that there are 5 heads?

14) There are four children in the family. What is the probability that two of them are boys? The probability of giving birth to a boy is 0.515.

15) What is more likely: to win in chess from an equal opponent three games out of four or five games out of eight?

Task 1.4. The textbook was published in n copies. A copy of a textbook can be incorrectly stitched with a probability p. Find the probability that the edition circulation contains a) exactly 2 defective books; b) no more than 2 defective books.

1) $n = 2000$, $p = 0.00075$.

2) $n = 3000$, $p = 0.00067$.

3) $n = 1500$, $p = 0.00087$.

4) $n = 2500$, $p = 0.00060$.

5) $n = 1800$, $p = 0.00092$.

6) $n = 1300$, $p = 0.00153$.

7) $n = 2200$, $p = 0.00114$.

8) $n = 2600$, $p = 0.00108$.

9) $n = 3200$, $p = 0.00045$.

10) $n = 1400$, $p = 0.00143$.

11) $n = 1500$, $p = 0.00100$.

12) $n = 2100$, $p = 0.00095$.

13) $n = 1700$, $p = 0.00148$.

14) $n = 2900$, $p = 0.00086$.

15) $n = 1600$, $p = 0.00094$.

Task 1.5. a) The probability that a tram passenger does not buy a ticket is p. The inspector checks n passengers for tickets. Find the probability that the inspector detects k stowaways.

b) In the situation described above, find the probability that the inspector finds not less than k' and not more than k'' stowaways.

1) a) $n = 200$, $p = 0.22$, $k = 48$;

   b) $n = 500$, $p = 0.4$, $k' = 186$, $k'' = 220$.

2) a) $n = 320$, $p = 0.15$, $k = 50$;

   b) $n = 500$, $p = 0.3$, $k' = 137$, $k'' = 168$.

3) a) $n = 300$, $p = 0.2$, $k = 62$;

   b) $n = 400$, $p = 0.35$, $k' = 125$, $k'' = 161$.

4) a) $n = 500$, $p = 0.25$, $k = 120$;

   b) $n = 450$, $p = 0.17$, $k' = 54$, $k'' = 80$.

5) a) $n = 380$, $p = 0.2$, $k = 77$;

   b) $n = 350$, $p = 0.25$, $k' = 81$, $k'' = 108$.

6) a) $n = 400$, $p = 0.3$, $k = 129$;

b) $n = 420$, $p = 0.36$, $k' = 140$, $k'' = 172$.

7) a) $n = 400$, $p = 0.3$, $k = 111$;

b) $n = 280$, $p = 0.24$, $k' = 65$, $k'' = 72$.

8) a) $n = 320$, $p = 0.35$, $k = 121$;

b) $n = 510$, $p = 0.25$, $k' = 124$, $k'' = 135$.

9) a) $n = 320$, $p = 0.35$, $k = 103$;

b) $n = 440$, $p = 0.15$, $k' = 65$, $k'' = 69$.

10) a) $n = 280$, $p = 0.45$, $k = 134$;

b) $n = 340$, $p = 0.35$, $k' = 107$, $k'' = 120$.

11) a) $n = 280$, $p = 0.45$, $k = 118$;

b) $n = 350$, $p = 0.25$, $k' = 80$, $k'' = 100$.

12) a) $n = 310$, $p = 0.36$, $k = 103$;

b) $n = 510$, $p = 0.45$, $k' = 225$, $k'' = 238$.

13) a) $n = 310$, $p = 0.36$, $k = 120$;

b) $n = 350$, $p = 0.45$, $k' = 151$, $k'' = 181$.

14) a) $n = 140$, $p = 0.4$, $k = 62$;

b) $n = 450$, $p = 0.17$, $k' = 60$, $k'' = 80$.

15) a) $n = 140$, $p = 0.4$, $k = 50$;

b) $n = 280$, $p = 0.24$, $k' = 64$, $k'' = 70$.

Task 1.6. The problem below describes the situation and names a discrete random variable. Write the distribution law for this random variable and find its expected value, variance and standard deviation.

1) The dice is thrown twice. Random value is the sum of points dropped.

2) The probability that the first shooter hits the target is 0.7, and for the second shooter this probability is 0.8. Each shooter makes one shot. Random value is the number of hits on the target.

3) The examination test contains two questions. The student can correctly answer the first question with a probability of 0.8, and for the second question this probability is 0.9. Random value is the number of correct answers.

4) The shooter, having one cartridge, shoots at the target. If he doesn't hit, he is given another cartridge. This happens until he hits, or until the cartridges run out (there are 5 cartridges). The probability of hitting the target with each shot is the same and equal to 0.4. Random value is the number of shots fired.

5) Each of the two boxes contains 8 chocolates and 2 caramels. Someone take out one candy from each box. Random value is the number of chocolates.

6) There are 10 tickets, and 2 of them are winning. 3 tickets were chosen at random. Random value is the number of winning tickets among the selected ones.

7) There are five traffic lights on the way of the car. Each of them with a probability of 0.5 either allows or prohibits further movement. Random value is the number of traffic lights passed by the car before the first stop.

8) Everyone entering the university must pass 3 exams. The probability of passing the first exam with excellent marks is 0.9, for the second and the third exams this probability is 0.8 and 0.7 respectively.

Random value is the number of exams passed perfectly with excellent marks.

9) A device consisting of three independently operating blocks is being tested. The block failure probabilities are as follows: 0.3; 0.4; 0.5. Random value is the number of failed blocks.

10) In a batch of 10 parts, there is one defective one. To find it, they randomly choose one part after another and check each one. Random value is the number of checked parts, including defective ones.

11) The worker operates 3 machines. The probabilities that within an hour each machine does not require adjustment are equal 0.85; 0.9; 0.95 respectively. Random value is the number of machines that does not require setup within an hour.

12) The store has 15 laptops in stock, 3 of them are defective. School purchases 8 laptops. Random value is the number of defective laptops purchased.

13) The company has signed 7 contracts, and 4 of them were concluded in violation of the law. 3 randomly selected contracts were audited. Random value is the number of detected contracts with violations among the checked ones.

14) A lottery is organized in the student group. There are three things to play, two of them worth $ 1,000 each and one worth $ 3,000. Random value is the amount of net winnings for a student who purchased one ticket for $ 100; a total of 50 tickets were sold.

15) There are three books on mathematics and three on computer science in a stack of six books. Three books are chosen at random. Random value is the number of books on mathematics among the selected ones.

Task 1.7. A continuous random variable is given by a distribution function $F(x)$. Find the probability density function $f(x)$. Plot graphs of

functions $F(x)$ and $f(x)$. Find the probability of the event $A=(X\in[a;b])$. For this random variable find its expected value, variance and standard deviation.

1) $F(x)=\begin{cases}0, & x<0;\\ \frac{1}{2}x, & 0\le x<2;\\ 1, & x\ge 2.\end{cases}$ $a=1,\ b=2$.

2) $F(x)=\begin{cases}0, & x<1;\\ \frac{1}{2}(x-1), & 1\le x<3;\\ 1, & x\ge 3.\end{cases}$ $a=2,\ b=13$.

3) $F(x)=\begin{cases}0, & x<-2;\\ \frac{1}{4}(x+2), & -2\le x<2;\\ 1, & x\ge 2.\end{cases}$ $a=-3,\ b=1$.

4) $F(x)=\begin{cases}0, & x<2;\\ \frac{1}{3}(x-2), & 2\le x<5;\\ 1, & x\ge 5.\end{cases}$ $a=3,\ b=4$.

5) $F(x)=\begin{cases}0, & x<0;\\ sin\,x, & 0\le x<\frac{\pi}{2};\\ 1, & x\ge\frac{\pi}{2}.\end{cases}$ $a=\frac{\pi}{6},\ b=\frac{\pi}{3}$.

6) $F(x)=\begin{cases} 0, & x<-\dfrac{\pi}{2}; \\ \cos x, & -\dfrac{\pi}{2}\le x<0; \\ 1, & x\ge 0. \end{cases}$ $a=-\dfrac{\pi}{6},\ b=0.$

7) $F(x)=\begin{cases} 0, & x<0; \\ \sin 2x, & 0\le x<\dfrac{\pi}{4}; \\ 1, & x\ge \dfrac{\pi}{4}. \end{cases}$ $a=\dfrac{\pi}{6},\ b=3.$

8) $F(x)=\begin{cases} 0, & x<0; \\ e^x-1, & 0\le x<\ln 2; \\ 1, & x\ge \ln 2. \end{cases}$ $a=\dfrac{1}{2},\ b=1.$

9) $F(x)=\begin{cases} 0, & x<0; \\ 2x-x^2, & 0\le x<1; \\ 1, & x\ge 1. \end{cases}$ $a=\dfrac{1}{2},\ b=1.$

10) $F(x)=\begin{cases} 0, & x<0; \\ x^2, & 0\le x<1; \\ 1, & x\ge 1. \end{cases}$ $a=\dfrac{1}{2},\ b=1.$

11) $F(x)=\begin{cases} 0, & x<0; \\ x^3, & 0\le x<1; \\ 1, & x\ge 1. \end{cases}$ $a=\dfrac{1}{2},\ b=1.$

12) $F(x)=\begin{cases}0, & x<0;\\ \sqrt{x}, & 0\le x<1;\\ 1, & x\ge 1.\end{cases}$ $a=\frac{1}{2},\ b=1.$

13) $F(x)=\begin{cases}0, & x<1;\\ (x-1)^2, & 1\le x<2;\\ 1, & x\ge 2.\end{cases}$ $a=-3,\ b=\frac{3}{2}.$

14) $F(x)=\begin{cases}0, & x<0;\\ \frac{1}{2}\sqrt{x}, & 0\le x<4;\\ 1, & x\ge 4.\end{cases}$ $a=1,\ b=4.$

15) $F(x)=\begin{cases}0, & x<0;\\ \frac{1}{4}x^2, & 0\le x<2;\\ 1, & x\ge 2.\end{cases}$ $a=\frac{1}{2},\ b=1.$

Task 1.8. The weight of parcels that are sent by "Nova Poshta" is a normally distributed random variable with a mean a kg and a standard deviation σ kg. Find the proportion of parcels whose weight is in the range from α kg to β kg.

1) $a=10,\ \sigma=4,\ \alpha=2,\ \beta=13.$

2) $a=9,\ \sigma=5,\ \alpha=5,\ \beta=14.$

3) $a=8,\ \sigma=4,\ \alpha=2,\ \beta=13.$

4) $a=7,\ \sigma=2,\ \alpha=3,\ \beta=10.$

5) $a=6,\ \sigma=3,\ \alpha=2,\ \beta=11.$

6) $a=5,\ \sigma=1,\ \alpha=1,\ \beta=12$

7) $a=4,\ \sigma=5,\ \alpha=2,\ \beta=11$

8) $a=2,\ \sigma=4,\ \alpha=6,\ \beta=10$

9) $a=2,\ \sigma=2,\ \alpha=1,\ \beta=6$.

10) $a=2,\ \sigma=2,\ \alpha=3,\ \beta=5$.

11) $a=2,\ \sigma=2,\ \alpha=1,\ \beta=6$.

12) $a=3,\ \sigma=4,\ \alpha=0,\ \beta=6$.

13) $a=2,\ \sigma=2,\ \alpha=0,\ \beta=5$.

14) $a=3,\ \sigma=3,\ \alpha=2,\ \beta=5$.

15) $a=1,\ \sigma=3,\ \alpha=3,\ \beta=7$.

2 ELEMENTS OF MATHEMATICAL STATISTICS

Task 2.1. On Black Friday, the store sets up discounts on all types of goods. Random value X is discount percentage. A sample of 15 objects was made from the general population.

a) Form a variation series for this sample.

b) Form a statistical distribution.

c) Find the empirical distribution function and plot it.

d) Construct a frequency polygon.

e) Calculate the numerical characteristics of the sample: the sample mean, estimates for the median and mode, the sample variance, the unbiased standard deviation, the coefficient of variation. Explain the practical meaning of the values found.

f) On the basis of the frequency polygon and the calculated point estimates of the characteristics of the variation series make a conclusion about the law of distribution of the general population.

1) x: 21, 34, 35, 24, 26, 26, 34, 29, 33, 29, 23, 34, 26, 26, 22.

2) x: 44, 54, 34, 43, 45, 56, 67, 23, 43, 56, 43, 54, 56, 56, 56.

3) x: 33, 34, 32, 33, 33, 32, 35, 34, 36, 31, 33, 32, 34, 32, 36.

4) x: 15, 11, 23, 45, 44, 23, 15, 22, 45, 33, 15, 22, 23, 30, 11.

5) x: 11, 24, 25, 34, 16, 16, 24, 39, 23, 19, 13, 24, 16, 16, 12.

6) x: 66, 54, 64, 73, 54, 66, 66, 73, 77, 66, 43, 54, 57, 66, 64.

7) x: 13, 14, 12, 13, 13, 12, 15, 14, 16, 11, 13, 12, 14, 12, 16.

8) x: 25, 10, 20, 25, 30, 35, 15, 25, 30, 25, 15, 25, 30, 30, 40.

9) x: 51, 54, 55, 54, 56, 56, 54, 59, 53, 59, 53, 54, 56, 56, 52.

10) x: 96, 74, 84, 84, 74, 96, 74, 84, 74, 84, 96, 74, 74, 84, 84.

11) x: 15, 17, 17, 19, 21, 23, 19, 19, 19, 19, 15, 17, 21, 21, 19.

12) x: 30, 15, 25, 30, 34, 40, 20, 30, 35, 30, 20, 30, 35, 35, 45.

13) x: 56, 59, 60, 59, 61, 61, 59, 64, 58, 64, 58, 59, 61, 61, 57.

14) x: 91, 69, 79, 79, 69, 91, 69, 79, 69, 79, 91, 69, 69, 79, 79.

15) x: 25, 27, 27, 29, 31, 33, 29, 29, 29, 29, 25, 27, 31, 31, 29.

Task 2.2. Let X be the price for a certain type of goods in different shops of the city. It is known that X is normally distributed. A sample of n elements was made from the general population. The sample mean $\bar{x}$ and the unbiased sample standard deviation s are found. Construct a γ% confidence interval for the unknown expected value if: a) the population standard deviation σ is known, b) the population standard deviation σ is unknown.

1) $n = 10;\ \bar{x} = 5.5;\ s = 2;\ \sigma = 2.2;\ \gamma = 0.95$.

2) $n = 12;\ \bar{x} = 67;\ s = 8;\ \sigma = 7;\ \gamma = 0.9$.

3) $n = 9;\ \bar{x} = 23.7;\ s = 6;\ \sigma = 5;\ \gamma = 0.95$.

4) $n = 6;\ \bar{x} = 80;\ s = 13;\ \sigma = 12;\ \gamma = 0.9$.

5) $n = 6;\ \bar{x} = 134;\ s = 7;\ \sigma = 5;\ \gamma = 0.95$.

6) $n = 12;\ \bar{x} = 3;\ s = 0.5;\ \sigma = 0.7;\ \gamma = 0.9$.

7) $n = 5;\ \bar{x} = 23;\ s = 5;\ \sigma = 4;\ \gamma = 0.95$.

8) $n = 11;\ \bar{x} = 90;\ s = 7;\ \sigma = 7.5;\ \gamma = 0.9$.

9) $n = 11;\ \bar{x} = 42;\ s = 5;\ \sigma = 7;\ \gamma = 0.95$.

10) $n = 9;\ \bar{x} = 37;\ s = 5.4;\ \sigma = 5;\ \gamma = 0.9$.

11) $n = 13;\ \bar{x} = 35;\ s = 4;\ \sigma = 4.5;\ \gamma = 0.95$.

12) $n = 7;\ \bar{x} = 14.5;\ s = 3;\ \sigma = 3.5;\ \gamma = 0.9$.

13) $n = 7;\ \bar{x} = 23;\ s = 7;\ \sigma = 7.2;\ \gamma = 0.95$.

14) $n = 15;\ \bar{x} = 120;\ s = 21;\ \sigma = 18;\ \gamma = 0.9$.

15) $n = 11;\ \bar{x} = 89;\ s = 14;\ \sigma = 12;\ \gamma = 0.95$.

Task 2.3. The following data represent the error (mm) when processing a part on a lathe.

a) Make a frequency table displaying class intervals, frequencies, relative frequencies, accumulated relative frequencies.

b) Draw a relative frequency histogram.

c) Draw a cumulative function.

d) Using the grouped data calculate the numerical characteristics of the sample: the sample mean, the sample variance, the unbiased standard deviation, the coefficient of variation.

e) Sustain or reject the null hypothesis of the normal distribution of the random variable using Pearson's χ^2 test with the significance level α.

1) $\alpha = 0.05$.

1,09	1,27	1,20	1,31	1,30	1,05	1,25	1,19	1,31	1,30
1,58	1,32	1,18	1,39	1,11	1,56	1,36	1,16	1,37	1,11
1,56	1,12	1,32	1,35	1,34	1,58	1,13	1,30	1,36	1,30
1,30	1,24	1,28	1,27	1,30	1,31	1,21	1,29	1,29	1,30
1,30	1,31	1,45	1,33	1,18	1,30	1,34	1,46	1,33	1,17
1,29	1,20	1,42	1,28	1,30	1,31	1,20	1,41	1,38	1,27
1,25	1,14	1,34	1,38	1,33	1,25	1,14	1,34	1,28	1,33
1,23	1,30	1,26	1,35	1,20	1,23	1,26	1,26	1,35	1,22
1,18	1,28	1,29	1,24	1,25	1,16	1,30	1,30	1,27	1,24
1,35	1,16	1,36	1,24	1,19	1,36	1,18	1,36	1,22	1,21

2) $\alpha = 0.025$.

1,03	1,24	1,19	1,32	1,30	1,01	1,24	1,18	1,33	1,15
1,53	1,39	1,15	1,37	1,13	1,50	1,14	1,14	1,35	1,26
1,59	1,13	1,28	1,36	1,30	1,59	1,14	1,27	1,38	1,37
1,33	1,18	1,31	1,31	1,34	1,35	1,15	1,33	1,33	1,17
1,30	1,36	1,45	1,31	1,17	1,30	1,40	1,44	1,30	1,25
1,32	1,21	1,40	1,28	1,25	1,33	1,21	1,39	1,29	1,34
1,25	1,14	1,37	1,36	1,34	1,25	1,16	1,40	1,34	1,26
1,24	1,23	1,28	1,35	1,24	1,25	1,21	1,28	1,35	1,23
1,36	1,19	1,35	1,21	1,22	1,40	1,35	1,31	1,20	1,23
1,15	1,33	1,30	1,31	1,24	1,34	1,20	1,34	1,30	1,35

3) $\alpha = 0.01$

1,26	1,07	1,44	0,84	1,32	1,31	1,09	1,45	0,87	1,27
1,21	1,28	0,82	1,10	1,32	1,20	1,26	0,85	1,09	1,36
1,15	1,49	1,06	1,41	1,13	1,11	1,45	0,99	1,42	1,19
1,19	1,30	1,34	1,30	1,28	1,15	1,31	1,39	1,24	1,27
1,31	1,20	1,19	1,50	1,15	1,29	1,22	1,22	1,53	1,14
1,26	1,09	0,96	1,56	1,41	1,26	1,09	0,98	1,55	1,44
1,10	1,02	0,89	1,16	1,40	1,09	1,03	0,87	1,16	1,40
1,29	0,84	1,28	1,15	0,97	1,30	0,83	1,20	1,14	0,95
1,33	1,31	1,10	1,22	1,21	1,30	1,26	115	1,20	1,20
1,29	1,07	0,84	1,30	0,76	1,29	1,10	0,86	1,32	0,80

4) $\alpha = 0.1$.

1,23	1,05	1,40	1,11	1,29	1,21	1,05	1,34	0,82	1,41
1,25	1,28	0,80	1,38	1,09	1,27	1,29	0,78	1,12	1,29
1,23	1,51	1,14	1,37	1,28	1,26	1,52	1,22	1,33	1,06
1,28	1,29	1,29	1,46	1,18	1,31	1,29	1,24	1,36	1,27
1,32	1,18	1,14	1,54	1,38	1,33	1,15	1,08	1,43	1,21
1,24	1,09	0,93	1,17	1,37	1,22	1,09	0,90	1,50	1,34
1,11	1,01	0,90	1,18	1,00	1,13	1,00	0,91	1,18	1,31
1,29	0,88	1,34	1,25	1,21	1,28	0,92	1,39	1,21	1,03
1,36	1,34	1,07	1,28	0,75	1,37	1,34	1,04	1,29	1,20
1,29	1,06	0,86	1,37	0,82	1,20	1,09	0,89	1,26	0,76

5) $\alpha = 0.05$.

1,20	1,07	1,26	0,84	1,45	1,20	1,09	1,17	0,88	1,45
1,28	1,30	0,77	1,15	1,28	1,26	1,31	0,77	1,19	1,29
1,27	1,50	1,29	1,27	1,05	1,23	1,46	1,36	1,22	1,0
1,32	1,29	1,20	1,43	1,24	1,32	1,29	1,18	1,49	1,22
1,34	1,14	1,03	1,41	1,25	1,33	1,12	1,00	1,41	1,30
1,19	1,09	0,87	1,44	1,32	1,15	1,08	0,84	1,38	1,31
1,15	0,98	0,91	1,17	1,23	1,19	0,96	0,92	1,15	1,14
1,27	0,97	1,41	1,22	1,06	1,26	1,03	1,41	1,23	1,08
1,37	1,33	1,02	1,32	1,17	1,36	1,30	1,00	1,34	1,13
1,27	1,12	0,92	1,24	0,81	1,25	1,18	0,95	1,22	0,90

6) $\alpha = 0.025$.

1,20	1,24	0,92	1,40	1,24	1,19	1,20	0,95	1,03	1,43
1,12	1,40	1,42	1,19	1,24	1,15	1,36	0,86	1,36	1,28
1,11	1,32	1,23	1,56	1,14	1,1?	1,35	1,44	1,17	1,20
1,28	1,24	0,99	1,52	1,44	1,28	1,26	1,21	1,57	1,15
1,27	1,05	0,85	1,17	1,40	1,28	1,10	0,98	1,49	1,43
1,09	0,84	1,12	1,12	0,94	1,08	1,07	0,83	1,19	1,38
1,30	1,26	1,20	1,18	1,19	1,29	0,86	1,05	1,10	0,85
1,27	1,13	0,88	1,34	0,87	1,25	1,21	1,27	1,18	1.17
1,31	1,43	0,90	1,24	1,30	1,32	1,17	0,92	1,35	0,94
1,12	0,89	1,06	1,40	1,21	1,15	1,39	0,93	1,22	1,21

7) $\alpha = 0.1$.

1,18	1,40	1,49	1,50	1,54	1,16	1,39	1,49	1,51	1,54
1,30	1,61	1,49	1,52	1,54	1,38	1,61	1,50	1,52	1,52
1,54	1,50	1,50	1,49	1,51	1,55	1,51	1,49	1,49	1,52
1,50	1,49	1,49	1,51	1,54	1,48	1,50	1,49	1,51	1,54
1,59	1,50	1,49	1,50	1,44	1,53	1,50	1,49	1,49	1,44
1,64	1,49	1,50	1,46	1,47	1,68	1,49	1,49	1,46	1,47
1,52	1,49	1,50	1,51	1,46	1,48	1,49	1,50	1,52	1,49
1,47	1,49	1,49	1,50	1,47	1,45	1,49	1,49	1,50	1,48
1,50	1,50	1,55	1,54	1,52	1,55	1,50	1,75	1,54	1,53
1,56	1,49	1,51	1,48	1,54	1,59	1,49	1,49	1,48	1,55

8) $\alpha = 0.05$.

1,20	1,42	1,49	1,51	1,55	1,16	1,39	1,49	1,51	1,56
1,49	1,60	1,49	1,52	1,52	1,38	1,59	1,50	1,52	1,52
1,27	1,49	1,50	1,48	1,53	1,54	1,50	1,49	1,48	1,53
1,53	1,42	1,49	1,52	1,52	1,47	1,49	1,49	1,52	1,51
1,61	1,50	1,49	1,49	1,45	1,51	1,50	1,49	1,49	1,45
1,63	1,49	1,50	1,47	1,47	1,71	1,49	1,49	1,47	1,46
1,54	1,49	1,50	1,52	1,50	1,48	1,49	1,49	1,52	1,50
1,47	1,49	1,49	1,50	1,48	1,45	1,50	1,49	1,51	1,49
1,47	1,50	1,55	1,52	1,53	1,56	1,50	1,74	1,52	1,53
1,55	1,49	1,49	1,49	1,56	1,60	1,50	1,49	1,49	1,56

9) $\alpha = 0.025$.

1,07	1,71	1,49	1,36	1,61	0,91	1,91	1,68	1,36	1 73
0,91	1,31	1,40	1,44	1,54	1,12	1,48	1,35	1,71	1,57
1,39	1,33	1,34	1,46	1,79	1,02	1,25	1,22	1,42	1,56
1,50	1,42	1,30	1,76	1,62	1,48	1,29	1,48	1,69	1,83
1,53	1,34	1,41	1,38	1,41	1,44	1,52	1,35	1,38	1,35
1,48	1,72	1,33	1,53	1,12	1,48	1,56	1,45	1,49	1,34
1,75	1,43	1,67	1,84	1,66	1,56	1,60	1,49	1,77	1,34
1,83	1,44	1,49	1,41	1,62	1,86	1,36	1,61	1,68	1 79
1,19	1,60	1,24	1,64	1,30	1,60	1,51	1,41	1,58	1,48
1,65	1,67	1,67	1,54	1,31	1,24	1,65	1,32	1,59	1,13

10) $\alpha = 0.01$.

1,03	1,76	1,54	1,35	1,64	0,87	1,94	1,71	1,35	1,72
0,96	1,35	1,40	1,51	1,55	1,16	1,52	1,34	1,76	1,56
1,30	1,30	1,27	1,45	1,74	0,94	1,24	1,25	1,40	1,52
1,51	1,35	1,33	1,78	1,68	1,46	1,32	1,52	1,65	1,86
1,50	1,37	1,39	1,36	1,38	1,45	1,57	1,34	1,45	1,37
1,47	1,68	1,35	1,51	1,16	1,50	1,52	1,47	1,49	1,39
1,70	1,46	1,63	1,82	1,58	1,53	1,65	1,44	1,73	1,29
1,82	1,43	1,51	1,46	1,67	1,87	1,36	1,65	1,75	1,81
1,28	1,58	1,30	1,64	1,37	1,69	1,47	1,42	1,54	1,49
1,54	1,66	1,60	1,53	1,27	1,16	1,64	1,24	1,60	1,11

REFERENCES

1. Англо-русский словарь математических терминов / [под ред. П.С. Александрова]. – 2-е, исправл. и дополн. изд. – М. : Мир, 1994. – 416с.
2. Вентцель Е.С. Задачи и упражнения по теории вероятностей: учеб. пособие для втузов / Е.С. Вентцель, Л.А. Овчаров. – 5-е изд. исправл. – М.: Академия, 2003. – 448 с.
3. Вентцель Е.С. Теории вероятностей и её инженерные приложения: учеб. пособие для втузов / Е.С. Вентцель, Л.А. Овчаров. – 3-е изд. перераб. и доп. – М.: Академия, 2003. – 464 с.
4. Гмурман В.Е. Теория вероятностей и математическая статистика: учеб. пособие для вузов / В.Е. Гмурман. – Изд. 9-е, стер. – М.: Высш. шк., 2003. – 479 с.
5. Гмурман В.Е. Руководство к решению задач по теории вероятностей и математической статистике: учеб. пособие для студентов вузов / В.Е. Гмурман. – Изд. 5-е, стер. – М.: Высш. шк., 2001. – 400 с.
6. Гурский Е.И. Сборник задач по теории вероятностей и математической статистике / Е.И. Гурский. – Минск, “Вышэйш. школа”, 1975. –272 с.
7. Краткий англо-русский технический словарь / Ю.А. Кузьмин, В.А. Владимиров, Я.Л. Гельман и др. – Издательство: Московская международная школа переводчиков, 1992. – 416 с.
8. Сборник задач по по высшей математике с контрольными работами: 2 курс / К.Н. Лунгу, В.П. Норин, Д.Т.Письменный и др., под ред. С.Н. Федина. – М.: Айрис-пресс, 2007. – 592 с.
9. Микулик Н.А. Руководство к решению технических задач по теории вероятностей и математической статистике / Н.А. Микулик, Г.Н. Рейзина, под ред. Е.И. Гурского. – Минск, “Вышэйш. школа”, 1977 – 143 с.

10. Письменный Д.Т. Конспект лекций по терии вероятностей, математической статистике и случайным процессам / Д.Т. Письменный – М.: Айрис-пресс, 2008. – 288 с.
11. Bird J. Higher Engineering Mathematics / J. Bird. – AmsterdamTokyo : Newnes, 2010. – 705 p.
12. Kuznetsov B. Russian-English Polytechnic Dictionary / B. Kuznetsov. – Moscow: Russian Language Publishers, 1980. – 723 p.
13. Smirnov V.I. A course of Higher Mathematics / V.I. Smirnov; transl. from Russian by D.E. Brown. – Pergamon Press, 1964. – 543 p.
14. Workbook on higher mathematics (4th module) for students majoring in 141 Power Engineering, Electrical Engineering and Electrical Mechanics / Compiler: Snizhko Nataliia. – Zaporizhzhia: Zaporizhzhia Polytechnic National University, 2021. – 46 p.

Appendix A

Values of the Gauss function $\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

	0	1	2	3	4	5	6	7	8
0,0	0,3989	3989	3989	3988	3986	3984	3982	3980	3977
0,1	3970	3965	3961	3956	3951	3945	3939	3932	3925
0,2	3910	3902	3894	3885	3876	3867	3857	3847	3836
0,3	3814	3802	3790	3778	3765	3752	3739	3726	3712
0,4	3683	3668	3652	3637	3621	3605	3589	3572	3555
0,5	3521	3503	3485	3467	3448	3429	3410	3391	3372
0,6	3332	3312	3292	3271	3251	3230	3209	3187	3166
0,7	3123	3101	3079	3056	3034	3011	2989	2966	2943
0,8	2897	2874	2850	2827	2803	2780	2756	2732	2709
0,9	2661	2637	2613	2589	2565	2541	2516	2492	2468
1,0	2420	2396	2371	2347	2323	2299	2275	2251	2227
1,1	2179	2155	2131	2107	2083	2059	2036	2012	1989
1,2	1942	1919	1895	1872	1849	1826	1804	1781	1758
1,3	1714	1691	1669	1647	1626	1604	1582	1561	1539
1,4	1497	1476	1456	1435	1415	1394	1374	1354	1334
1,5	1295	1276	1257	1238	1219	1200	1182	1163	1145
1,6	1109	1092	1074	1057	1040	1023	1006	0989	0973
1,7	0940	0925	0909	0893	0878	0863	0848	0833	0818

	0	1	2	3	4	5	6	7	8
1,8	0790	0775	0761	0748	0734	0721	0707	0694	0681
1,9	0656	0644	0632	0620	0608	0596	0584	0573	0562
2,0	0540	0529	0519	0508	0498	0488	0478	0468	0459
2.1	0440	0431	0422	0413	0404	0396	0387	0379	0371
2,2	0355	0347	0339	0332	0325	0317	0310	0303	0297
2,3	0283	0277	0270	0264	0258	0252	0246	0241	0235
2,4	0224	0219	0213	0208	0203	0198	0194	0189	0184
2,5	0175	0171	0167	0163	0158	0154	0151	0147	0143
2,6	0136	0132	0129	0126	0122	0119	0116	0113	0110
2,7	0104	0101	0099	0096	0093	0091	0088	0086	0084
2,8	0079	0077	0075	0073	0071	0069	0067	0065	0063
2,9	0060	0058	0056	0055	0053	0051	0050	0048	0047
3,0	0044	0043	0042	0040	0039	0038	0037	0036	0035
3,1	0033	0032	0031	0030	0029	0028	0027	0026	0025
3,2	0024	0023	0022	0022	0021	0020	0020	0019	0018
3,3	0017	0017	0016	0016	0015	0015	0014	0014	0013
3,4	0012	0012	0012	0011	0011	0010	0014	0010	0009
3,5	0009	0008	0008	0008	0008	0007	0007	0007	0007
3,6	0006	0006	0006	0005	0005	0005	0005	0005	0005
3,7	0004	0004	0004	0004	0004	0004	0003	0003	0003
3,8	0004	0004	0004	0004	0004	0004	0003	0003	0003
3,9	0002	0002	0002	0002	0002	0002	0002	0002	0001

Appendix B

Values of the Laplace function $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt$

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0,00	0,0000	0,19	0,0753	0,38	0,1480	0,57	0,2157
0,01	0,0040	0,20	0,0793	0,39	0,1517	0,58	0,2190
0,02	0,0080	0,21	0,0832	0,40	0,1554	0,59	0,2224
0,03	0,0120	0,22	0,0871	0,41	0,1591	0,60	0,2257
0,04	0,0160	0,23	0,0910	0,42	0,1628	0,61	0,2291
0,05	0,0199	0,24	0,0948	0,43	0,1664	0,62	0,2324
0,06	0,0239	0,25	0,0987	0,44	0,1700	0,63	0,2357
0,07	0,0279	0,26	0,1026	0,45	0,1736	0,64	0,2389
0,08	0,0319	0,27	0,1064	0,46	0,1772	0,65	0,2422
0,09	0,0359	0,28	0,1103	0,47	0,1808	0,66	0,2454
0,10	0,0398	0,29	0,1141	0,48	0,1844	0,67	0,2486
0,11	0,0438	0,30	0,1179	0,49	0,1879	0,68	0,2517
0,12	0,0478	0,31	0,1217	0,50	0,1915	0,69	0,2549
0,13	0,0517	0,32	0,1255	0,51	0,1950	0,70	0,2580
0,14	0,0557	0,33	0,1293	0,52	0,1985	0,71	0,2611
0,15	0,0596	0,34	0,1331	0,53	0,2019	0,72	0,2642
0,16	0,0636	0,35	0,1368	0,54	0,2054	0,73	0,2673
0,17	0,0675	0,36	0,1406	0,55	0,2088	0,74	0,2703
0,18	0,0714	0,37	0,1443	0,56	0,2123	0,75	0,2734

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0,76	0,2764	0,99	0,3389	1,22	0,3883	1,45	0,4265
0,77	0,2794	1,00	0,3413	1,23	0,3907	1,46	0,4279
0,78	0,2823	1,01	0,3438	1,24	0,3925	1,47	0,4292
0,79	0,2852	1,02	0,3461	1,25	0,3944	1,48	0,4306
0,80	0,2881	1,03	0,3485	1,26	0.3952	1,49	0,4319
0,81	0,2910	1,04	0,3508	1,27	0.3980	1,50	0,4332
0,82	0.2939	1,05	0,3531	1,28	0,3997	1,51	0,4345
0,83	0,2967	1,06	0,3554	1,29	0,4015	1,52	0,4357
0,84	0,2995	1,07	0,3577	1,30	0,4032	1,53	0,4370
0,85	0,3023	1,08	0,3599	1,31	0,4049	1,54	0,4382
0,86	0,3051	1,09	0,3621	1,32	0,4066	1,55	0,4394
0,87	0,3078	1,10	0,3643	1,33	0,4082	1,56	0,4406
0,88	0,3106	1,11	0,3665	1,34	0,4099	1,57	0,4418
0,89	0,3133	1,12	0,3686	1,35	0,4115	1,58	0,4429
0,90	0,3159	1,13	0,3708	1,36	0,4131	1,59	0,4441
0,91	0,3186	1,14	0,3729	1,37	0,4147	1,60	0,4452
0,92	0,3212	1,15	0,3749	1,38	0,4162	1,61	0,4463
0,93	0,3238	1,16	0,3770	1,39	0,4177	1,62	0,4474
0,94	0,3264	1,17	0,3790	1,40	0,4192	1,63	0,4484
0,95	0,3289	1,18	0,3810	1,41	0,4207	1,64	0,4495
0,96	0,3315	1,19	0,3830	1,42	0,4222	1,65	0,4505
0,97	0,3340	1,20	0,3849	1,43	0,4230	1,66	0,4515
0,98	0,3365	1,21	0,3869	1,44	0,4251	1,67	0,4525

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
1,68	0,4535	1,91	0,4719	2,28	0,4887	2,74	0,4969
1,69	0,4545	1,92	0,4726	2,30	0,4893	2,76	0,4971
1,70	0,4554	1,93	0,4732	2,32	0,4898	2,78	0,4973
1,71	0,4564	1,94	0,4738	2,34	0,4904	2,80	0,4974
1,72	0,4573	1,95	0,4744	2,36	0,4909	2,82	0,4976
1,73	0,4582	1,96	0,475	2,38	0,4913	2,84	0,4977
1,74	0,4591	1,97	0,4756	2,40	0,4918	2,86	0,4979
1,75	0,4599	1,98	0,4761	2,42	0,4922	2,88	0,4980
1,76	0,4608	1,99	0,4767	2,44	0,4927	2,90	0,4981
1,77	0,4616	2,00	0,4772	2,46	0,4931	2,92	0,4982
1,78	0,4525	2,02	0,4783	2,48	0,4934	2,94	0,4984
1,79	0,4633	2,04	0,4793	2,50	0,4938	2,96	0,4985
1,80	0,4641	2,06	0,4803	2,52	0,4941	2,98	0,4985
1,81	0,4649	2,08	0,4812	2,54	0,4945	3,00	0,49865
1,82	0,4656	2,10	0,4821	2,56	0,4948	3,20	0,49931
1,83	0,4664	2,12	0,4830	2,58	0,4951	3,40	0,49966
1,84	0,4671	2,14	0,4838	2,60	0,4953	3,60	0,499841
1,85	0,4678	2,16	0,4846	2,62	0,4956	3,80	0,499928
1,86	0,4686	2,18	0,4854	2,64	0,4959	4,00	0,499968
1,87	0,4693	2,20	0,4861	2,66	0,4961	4,50	0,499997
1,88	0,4699	2,22	0,4868	2,68	0,4963	5,00	0,499997
1,89	0.4706	2,24	0,4875	2,70	0,4965		
1,90	0,4713	2,26	0,4881	2,72	0,4967		

Appendix C

Values of $t_\gamma = t(\gamma; n)$

n, sample size	γ , confidence level				
	0,90	**0,95**	**0,98**	**0,99**	**0,999**
6	2,02	2,57	3,36	4,03	6,87
7	1,94	2,45	3,14	3,71	5,96
8	1,89	2,36	3,00	3,50	5,41
9	1,86	2,31	2,90	3,36	5,04
10	1,83	2,26	2,82	3,25	4,78
11	1,81	2,23	2,76	3,17	4,59
12	1,80	2,20	2,72	3,11	4,44
13	1,78	2,18	2,68	3,05	4,32
14	1,77	2,16	2,65	3,01	4,22
15	1,76	2,14	2,62	2,98	4,14
16	1,75	2,13	2,60	2,95	4,07
17	1,75	2,12	2,58	2,92	4,02
18	1,74	2,11	2,57	2,90	3,97
19	1,73	2,10	2,55	2,88	3,92
20	1,73	2,09	2,54	2,86	3,88
21	1,72	2,09	2,53	2,85	3,85
22	1,72	2,08	2,52	2,83	3,82
23	1,72	2,07	2,51	2,82	3,79
24	1,71	2,07	2,50	2,81	3,77
25	1,71	2,06	2,49	2,80	3,75
26	1,71	2,06	2,49	2,79	3,73
27	1,71	2,06	2,49	2,78	3,71
28	1,70	2,05	2,47	2,77	3,69
29	1,70	2,05	2,47	2,76	3,67
30	1,70	2,05	2,46	2,76	3,66
31	1,70	2,04	2,46	2,75	3,65
41	1,68	2,02	2,42	2,70	3,55
51	1,68	2,01	2,40	2,68	3,50
61	1,67	2,00	2,39	2,66	3,46
121	1,66	1,98	2,36	2,62	3,37
∞	1,64	1,96	2,33	2,58	3,29

Appendix D

Critical points of chi-square distribution

k, number of degrees of freedom	α, significance level								
	0,99	**0,95**	**0,90**	**0,10**	**0,05**	**0,025**	**0,01**	**0,005**	**0,001**
1	0,0002	0,004	0,02	2,71	3,84	5,02	6,63	7,88	10,8
2	0,002	0,10	0,21	4,61	5,99	7,38	9,21	10,6	13,8
3	0,12	0,35	0,58	6,25	7,81	9,35	11,3	12,8	16,3
4	0,30	0,71	1,06	7,78	9,49	11,1	13,3	14,9	18,5
5	0,55	1,15	1,61	9,24	11,1	12,8	15,1	16,7	20,5
6	0,87	1,64	2,20	10,6	12,6	14,4	16,8	18,5	22,5
7	1,24	2,17	2,83	12,0	14,1	16,0	18,5	20,3	24,3
8	1,65	2,73	3,49	13,4	15,5	17,5	20,1	22,0	26,1
9	2,09	3,33	4,17	14,7	16,9	19,0	21,7	23,6	27,9
10	2,56	3,94	4,87	16,0	18,3	20,5	23,2	25,2	29,6
11	3,05	4,57	5,58	17,3	19,7	21,9	24,7	26,8	31,3
12	3,57	5,23	6,30	18,5	21,0	23,3	26,2	28,3	32,9
13	4,11	5,89	1,04	19,8	22,4	24,7	27,7	29,8	34,5
14	4,66	6,57	7,79	21,1	23,7	26,1	29,1	31,3	36,1
15	5,23	7,26	8,55	22,3	25,0	27,5	30,6	32,8	37,7
16	5,81	7,96	9,31	23,5	26,3	28,8	32,0	34,3	39,3
17	6,41	8,67	10,1	24,8	27,6	30,2	33,4	35,7	40,8
18	7,01	9,39	10,9	26,0	28,9	31,5	34,8	37,2	42,3
19	7,63	10,1	11,7	27,2	30,1	32,9	36,2	38,6	43,8
20	8,26	10,9	12,4	28,4	31,4	34,2	37,6	40,0	45,3
21	8,90	11,6	13,2	29,6	32,7	35,5	38,9	41,4	46,8
22	9,54	12,3	14,0	30,8	33,9	36,8	40,3	42,8	48,3
23	10,2	13,1	14,8	32,0	35,2	38,1	41,6	44,2	49,7
24	10,9	13,8	15,7	33,2	36,4	39,4	43,0	45,6	51,2
25	11,5	14,6	16,5	34,4	37,7	40,6	44,3	46,9	52,6
26	12,2	15,4	17,3	35,6	38,9	41,9	45,6	48,3	54,1
27	12,9	16,2	18,1	36,7	40,1	43,2	47,0	49,6	55,5
28	13,6	16,9	18,9	37,9	41,3	44,5	48,3	51,0	56,9
29	14,3	17,7	19,8	39,1	42,6	45,7	49,6	52,3	58,3
30	15,0	18,5	20,6	40,3	43,8	47,0	50,9	53,7	59,7