MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
National University «Zaporizhzhia Polytechnic»

WRITTEN TESTS ON HIGHER MATHEMATICS
and methodical instructions for their performance (3rd module)

for students majoring in
141 Power Engineering, Electrical Engineering
and Electrical Mechanics

2022

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INTRODUCTION

The offered written tests on higher mathematics and methodical instructions for their performance correspond to the course "Higher Mathematics" taught to students majoring in Power Engineering, Electrical Engineering and Electrical Mechanics full-time in the third module of the two-semester course. This set of tasks can also be used for students in other technical majors who study higher mathematics in a two-semester course.

The manual contains tasks for two written tests and methodical instructions for their performance (fully solved standard options with all the necessary explanations). Each test is composed of ten options.

Written Test №1 covers the following topics of the third module of the course, namely: complex numbers and arithmetic operations on them, the geometry of the complex plane, the functions of the complex variable, elements of the theory of analytical functions. Written Test №2 covers the following topics of the third module of the course, namely: elementary differential equations, first-order linear differential equations, linear differential equations with constant coefficients, higher-order differential equations that allow order reduction.

At the end there is a list of recommended literature for preparation. It includes author's textbooks, as well as manuals on the performance of calculation tasks executed by students during the semester as homework. In addition to the methodological instructions for solving problems and examples of solutions, manual [12] also contains the necessary theoretical information and reference material (tables, literature, etc.).

Students perform each written test at the end of the study of the relevant module of the course. Written test is performed during classroom classes, it is designed for two academic hours.

The student chooses the option (the number of the variant) according to his number in the register list. The work is done in English in writing, preferably in a notebook in a cell. Note fields must be left blank. The name of the subject, major, group and course, surname, first name and patronymic of the student, name of the teacher who accepts the work should be indicated on the title page of the work.

While performing the work, the student should solve the offered tasks by the methods specified in the tasks, as well as make all the necessary drawings (graphical solutions). The student must show the acquired theoretical knowledge of the course.

When evaluating a work, the indicator of its quality is, first of all, how the student independently and correctly solved the tasks and understood the content of the obtained solutions. After checking the work by teacher the student receives a certain number of rating points.

After checking and evaluating the work, solutions are analyzed and discussed. Thus, these tests are an important part of students' independent classroom work in practical classes; they help students assess their current level of knowledge. In addition, for the teacher these works is a method of diagnosis and control of learning material by students.

The materials of this manual can also be used for practical illustration of theoretical positions during the presentation of lecture material, during practical classes, as well as for modular control (its classroom part).

1 WRITTEN TEST №1

1.1 Methodical instructions for performance of Written Test №1 (solution of a standard option)

Option 0

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z = \frac{4-i}{i+4} + (i+1)^8$.

Solution. To reduce the fraction to the rectangular form, we multiply the numerator and denominator by the value conjugated to the denominator. In our case, this value is $4-i$:

$$\frac{4-i}{i+4} = \frac{(4-i)^2}{(i+4)(4-i)} = \frac{16-8i-1}{16+1} = \frac{15-8i}{17}.$$

Then

$$(i+1)^8 = \left(\left((i+1)^2\right)^2\right)^2 = \left((-1+2i+1)^2\right)^2 = \left((2i)^2\right)^2 = (-4)^2 = 16.$$

So, we have

$$z = \frac{4-i}{i+4} + (i+1)^8 = \frac{15-8i}{17} + 16 = \frac{287-8i}{17} = \frac{287}{17} - \frac{8i}{17}.$$

It’s the rectangular form of z.

We find the modulus and the argument of the number z. The number z belongs to the IV quadrant because $x = \frac{287}{17} > 0$, $y = -\frac{8}{17} < 0$;

$$r=\sqrt{\left(\frac{287}{17}\right)^2+\left(-\frac{8}{17}\right)^2}=\frac{\sqrt{82433}}{17}\approx 16.9\,;$$

$$\varphi=arctg\frac{-8/17}{287/17}=-arctg\frac{8}{287}\approx -2^\circ\,.$$

Answer: $z=\frac{287}{17}-\frac{8i}{17}$;

$$r=\frac{\sqrt{82433}}{17}\approx 16.9\,;\ \varphi=-arctg\frac{8}{287}\approx -2^\circ\,.$$

2) Solve the equation: $z^2-4z+40=0$.

Solution. It's a quadratic equation. Let's find its discriminant:

$$D=(-4)^2-4\cdot 40=-144\,.$$

We have the roots of this equation:

$$z_{1,2}=\frac{4\pm\sqrt{-144}}{2}=\frac{4\pm 12i}{2}=2\pm 6i\,.$$

Answer: $z_{1,2}=2\pm 6i$.

3) Find the set of points on the complex plane defined by the following condition: $|z-2|\ge\left|1-2\bar{z}\right|$. Make a graph.

Solution. Let $z=x+iy$. Then the inequality has the form:

$$|(x-2)+iy|\ge|(1-2x)+2iy|\,.$$

By definition of modulus we have:

$$\sqrt{(x-2)^2+y^2}\ge\sqrt{(1-2x)^2+4y^2}\,.$$

Solving the inequality we get:

$$(x-2)^2+y^2\ge(1-2x)^2+4y^2\,,$$

$$x^2-4x+4+y^2\ge 1-4x+4x^2+4y^2\,,$$

$$3x^2+3y^2\le 3\,,$$

$$x^2 + y^2 \le 1.$$

This is the inner domain of the unit circle, including the boundary (fig. 1.1).

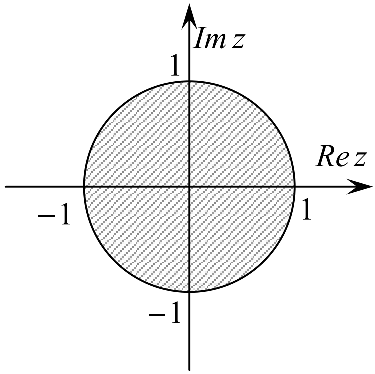

Figure 1.1

Answer: the unit circle $x^2 + y^2 \le 1$.

4) Find the analytic function $f(z)$ having its real part $u(x, y) = x^2 - y^2 + 5x$.

Solution. Since the function $f(z)$ is analytic, then its imaginary and real parts satisfy the Cauchy-Riemann conditions. Using the first condition, we obtain:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y},$$

$$\left(x^2 - y^2 + 5x\right)'_x = \frac{\partial v}{\partial y},$$

$$2x + 5 = \frac{\partial v}{\partial y},$$

$$v(x, y) = \int (2x + 5)dy + \psi(x) = 2xy + 5y + \psi(x).$$

From the second condition we obtain equality:

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x},$$

$$\left(x^2 - y^2 + 5x\right)'_y = -\left(2xy + 5y + \psi(x)\right)'_x,$$

$$-2y = -2y - \psi'(x),$$

from which we get $\psi'(x) = 0$ and $\psi(x) = C$. Consequently:

$$v(x, y) = 2xy + 5y + C;$$

$$f(z) = u(x, y) + iv(x, y) =$$

$$= \left(x^2 - y^2 + 5x\right) + i\left(2xy + 5y + C\right) =$$

$$= \left(x^2 + 2ixy - y^2\right) + \left(5x + 5iy\right) + iC =$$

$$= (x + iy)^2 + 5(x + iy) + iC =$$

$$= z^2 + 5z + iC.$$

We finally have:

$$f(z) = z^2 + 5z + iC.$$

Answer: $f(z) = z^2 + 5z + iC$.

1.2 Options of Written Test №1

Option 1

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z = \frac{2 - i}{i + 2} + (i - 1)^4$.

2) Solve the equation: $z^2 + 6z + 13 = 0$.

3) Find the set of points on the complex plane defined by the following condition: $Im\left(\bar{z}\right)^2 < 3$. Make a graph.

4) Find the analytic function $f(z)$ having its real part $u(x,y)=x^2-y^2+3x$.

Option 2

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z=\dfrac{2+i}{3i-1}+(2i-1)^3$.

2) Solve the equation: $z^2+4z+13=0$.

3) Find the set of points on the complex plane defined by the following condition: $|z|=Re\,z+3$. Make a graph.

4) Find the analytic function $f(z)$ having its real part $u(x,y)=x^2-y^2+2x$.

Option 3

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z=\dfrac{(2-i)^3}{(i+2)^2}+2i+3$.

2) Solve the equation: $z^2-6z+13=0$.

3) Find the set of points on the complex plane defined by the following condition: $3|z|-Re\,z=12$. Make a graph.

4) Find the analytic function $f(z)$ having its real part $u(x,y)=x^2-y^2-x$.

Option 4

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z = \frac{1-i}{i+1} - \frac{2-3i}{1-2i}$.

2) Solve the equation: $z^2 - 4z + 13 = 0$.

3) Find the set of points on the complex plane defined by the following condition: $|z| = Re\, z + 1$. Make a graph.

4) Find the analytic function $f(z)$ having its real part $u(x, y) = x^2 - y^2 - 3x$.

Option 5

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z = \frac{(1-i)(2+3i)}{2i+1} + i^{12}$.

2) Solve the equation: $z^2 + 2z + 10 = 0$.

3) Find the set of points on the complex plane defined by the following condition: $Im\left(\bar{z}\right)^2 > 2$. Make a graph.

4) Find the analytic function $f(z)$ having its real part $u(x, y) = x^2 - y^2 - 2x$.

Option 6

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z = \frac{(4-i)(2i+1)}{i} + i^{21}$.

2) Solve the equation: $z^2 + 4z + 8 = 0$.

3) Find the set of points on the complex plane defined by the following condition: $|z| = Im\, z + 3$. Make a graph.

4) Find the analytic function $f(z)$ having its imaginary part $v(x, y) = 2xy + 3y$.

Option 7

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z = \dfrac{(2+i)(3+5i)}{2i-1} + i^{32}$.

2) Solve the equation: $z^2 - 2z + 10 = 0$.

3) Find the set of points on the complex plane defined by the following condition: $3|z| - Im\, z = 12$. Make a graph.

4) Find the analytic function $f(z)$ having its imaginary part $v(x, y) = 2xy + 2y$.

Option 8

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z = \dfrac{2-i}{i+2} - \dfrac{3-4i}{2-3i}$.

2) Solve the equation: $z^2 - 4z + 8 = 0$.

3) Find the set of points on the complex plane defined by the following condition: $|z| = Im\, z + 1$. Make a graph.

4) Find the analytic function $f(z)$ having its imaginary part $v(x, y) = 2xy - 3y$.

Option 9

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z = \dfrac{4+i}{4i-1} + (i-2)^3$.

2) Solve the equation: $z^2 + 2z + 5 = 0$.

3) Find the set of points on the complex plane defined by the following condition: $|z| = Re\, z - 1$. Make a graph.

4) Find the analytic function $f(z)$ having its imaginary part $v(x,y) = 2xy - 2y$.

Option 10

1) Calculate (express the complex number z in a rectangular form). Find the modulus and the argument of the complex number: $z = \dfrac{(4-i)(2+5i)}{3i+1} + i^{51}$.

2) Solve the equation: $z^2 - 2z + 5 = 0$.

3) Find the set of points on the complex plane defined by the following condition: $|z| = Im\, z - 1$. Make a graph.

4) Find the analytic function $f(z)$ having its imaginary part $v(x,y) = 2xy - y$.

2 WRITTEN TEST №2

2.1 Methodical instructions for performance of Written Test №2 (solution of a standard option)

Option 0

1) Find the general solution of the differential equation:

$$(x-3)y'-y^2=25$$

Solution. It's an equations with separable variables. We separate the variables and integrate this equation:

$$(x-3)\frac{dy}{dx}=25+y^2,$$

$$\frac{dy}{y^2+25}=\frac{dx}{x-3},$$

$$\int\frac{dy}{y^2+25}=\int\frac{dx}{x-3},$$

$$\frac{1}{5}arctg\frac{y}{5}=ln(x-3)+C.$$

Answer: $\frac{1}{5}arctg\frac{y}{5}=ln(x-3)+C$.

2) Find the general solution of the differential equation:

$$xy'=y-\sqrt{y^2-9x^2}.$$

Solution. Write the equation in the normal form:

$$y'=\frac{y}{x}-\sqrt{\left(\frac{y}{x}\right)^2-9}.$$

This equation is homogeneous because the function $\frac{y}{x}-\sqrt{\left(\frac{y}{x}\right)^2-9}$ is a homogeneous function of zero degree.

Let's make the substitution $u=\frac{y}{x}$, then $y=ux$, $y'=u+xu'$. We have:

$$u+xu'=u-\sqrt{u^2-9},$$

$$\frac{du}{\sqrt{u^2-9}}=-\frac{dx}{x},$$

$$\int\frac{du}{\sqrt{u^2-9}}=-\int\frac{dx}{x},$$

$$ln\left|u+\sqrt{u^2-9}\right|=-ln|x|+ln\,C,$$

$$u+\sqrt{u^2-9}=\frac{C}{x}.$$

In the last expression, we return to the variable y:

$$y+\sqrt{y^2-9x^2}=C.$$

Answer: $y+\sqrt{y^2-9x^2}=C$.

3) Find the general solution of the differential equation:

$$y'-\frac{2}{x}y=y^2\left(cos\,x+x^2\right).$$

Solution. This is Bernoulli's equation. We make Bernoulli's substitution: $y=uv$, then

$$u'v+v'u-\frac{2}{x}uv=u^2v^2\left(cos\,x+x^2\right),$$

$$u'v+u\left(v'-\frac{2v}{x}\right)=u^2v^2\left(cos\,x+x^2\right).$$

Step 1: $v' - \frac{2v}{x} = 0 \Rightarrow \frac{dv}{dx} - \frac{2v}{x} = 0$,

$$\frac{dv}{v} = \frac{2}{x}dx,$$

$$\int \frac{dv}{v} = 2\int \frac{dx}{x},$$

and we get $v = x^2$.

Step 2: $u'x^2 = u^2x^4(\cos x + x^2) \Rightarrow \frac{du}{dx} = u^2x^2(\cos x + x^2)$,

$$\frac{du}{u^2} = x^2(\cos x + x^2)dx,$$

$$\int \frac{du}{u^2} = \int (x^2 \cos x + x^4)dx.$$

We integrate $\int x^2 \cos x dx$ twice by parts:

$$\int x^2 \cos x dx = (x^2 - 2)\sin x + 2x \cos x,$$

and we get $-\frac{1}{u} = (x^2 - 2)\sin x + 2x \cos x + \frac{1}{5}x^5 - \frac{C}{5}$, or:

$$u = \frac{5}{C - 5(x^2 - 2)\sin x - 10x \cos x - x^5}.$$

Thus, the general solution of this equation is:

$$y = uv = \frac{5x^2}{C - 5(x^2 - 2)\sin x - 10x \cos x - x^5}.$$

Answer: $y = \frac{5x^2}{C - 5(x^2 - 2)\sin x - 10x \cos x - x^5}$.

4) Find the general solution of the differential equation:

$$y'' - 3y' + 2y = 2x^2 - 6x + 4 + \sin 2x.$$

Solution. Find the roots of the characteristic equation:

$$\lambda^2 - 3\lambda + 2 = 0,$$

$$\lambda_1 = 1, \qquad \lambda_2 = 2.$$

Solution of homogeneous equation:

$$y_0 = C_1 e^x + C_2 e^{2x}.$$

For the given equation a particular solution looks like this:

$$y_{nh} = Ax^2 + Bx + C + D\cos 2x + E\sin 2x.$$

Let's find y'_{nh} and y''_{nh}:

$$y'_{nh} = 2Ax + B + 2E\cos 2x - 2D\sin 2x,$$

$$y''_{nh} = 2A - 4D\cos 2x - 4E\sin 2x.$$

We substitute y'_{nh} and y''_{nh} into the original equation:

$$(2A - 4D\cos 2x - 4E\sin 2x) - 3(2Ax + B + 2E\cos 2x - 2D\sin 2x) +$$
$$+ 2\left(Ax^2 + Bx + C + D\cos 2x + E\sin 2x\right) = 2x^2 - 6x + 4 + \sin 2x.$$

Equating the coefficients at $\cos 2x$, $\sin 2x$, x^2, x^1 and x^0 in the left-hand side and the right-hand side of the last equality, we obtain a system of algebraic equations with respect to A, B, C, D and E:

$\cos 2x$	$-2D - 6E = 0,$
$\sin 2x$	$-2E + 6D = 1,$
x^2	$2A = 2,$
x^1	$-6A + 2B = -6,$
x^0	$2A - 3B + 2C = 4.$

From the obtained system we find that $A = 1$, $B = 0$, $C = 1$, $D = \frac{3}{20}$, $E = -\frac{1}{20}$. So,

$$y_{nh} = x^2 + 1 + \frac{3}{20}\cos 2x - \frac{1}{20}\sin 2x.$$

The general solution of the equation:

$$y = y_0 + y_{nh} =$$

$$= C_1e^x + C_2e^{2x} + x^2 + 1 + \frac{3}{20}cos\,2x - \frac{1}{20}sin\,2x\,.$$

$$\textit{Answer}:\ y = C_1e^x + C_2e^{2x} + x^2 + 1 + \frac{3}{20}cos\,2x - \frac{1}{20}sin\,2x\,.$$

5) Solve the Cauchy problem using the operational method:

$$x'' + x = 6\,cos\,t\,;\quad x(0) = 0\,,\ x'(0) = 4\,;$$

Solution. Let $x(t) \leftrightarrow X(p)$, then by the differentiation theorem for the original we have:

$$x'(t) \leftrightarrow pX(p) - x(0) = pX(p),$$

$$x''(t) \leftrightarrow p^2X(p) - px(0) - x'(0) = p^2X(p) - 4\,.$$

In addition, we find in the table $cos\,t \leftrightarrow \frac{p}{p^2+1}$. We get the operator equation:

$$p^2X(p) - 4 + X(p) = \frac{6p}{p^2+1}\,.$$

We solve the equation and find the unknown function $X(p)$:

$$\left(p^2+1\right)X(p) = \frac{6p}{p^2+1} + 4\,,$$

$$X(p) = \frac{6p}{\left(p^2+1\right)^2} + \frac{4}{p^2+1}\,.$$

This is an operator solution. Let's find the original function for it. We find according to the table (see Appendix D):

$$\frac{1}{p^2+1} \leftrightarrow sin\,t\,,\qquad \frac{2p}{\left(p^2+1\right)^2} \leftrightarrow t\,sin\,t\,.$$

So, $X(p) \leftrightarrow 3t\,sin\,t + 4\,sin\,t = (3t+4)sin\,t = x(t)$.

Answer: $x(t) = (3t+4)sin\,t\,.$

2.2 Options of Written Test №2

Option 1

Find the general solution of the differential equation:

1) $\sqrt{y^2+9}dx = 2(x+3)ydy$;

2) $\left(x - y\cos\frac{y}{x}\right)dx + x\cos\frac{y}{x}dy = 0$;

3) $xy' + 2y + x^5y^3e^x = 0$;

4) $y'' - 9y = x$.

Solve the Cauchy problem using the operational method:

5) $x'' = 1$, $x(0) = 0$, $x'(0) = 1$.

Option 2

Find the general solution of the differential equation:

1) $\frac{dx}{y^3} - \frac{dy}{x^5} = 0$;

2) $xy + 2y^2 = (x^2 + xy)y'$;

3) $\frac{dy}{dx} - \frac{y}{x} = x^2$;

4) $y'' - 4y = x$.

Solve the Cauchy problem using the operational method:

5) $x'' = t$, $x(0) = 0$, $x'(0) = 1$.

Option 3

Find the general solution of the differential equation:

1) $ye^x dx = \ln y dy$;

2) $x(x-y)y' = xy - 2y^2$;
3) $2x(x^2 + y)dx = dy$;
4) $y'' - y = x$.

Solve the Cauchy problem using the operational method:
5) $x'' = t$, $x(0) = 1$, $x'(0) = 1$.

Option 4

Find the general solution of the differential equation:
1) $y' = 3\sqrt[3]{y^2}$;
2) $x^2 y' = y(x + y)$;
3) $y = x(y' - x\cos x)$;
4) $y'' - 25y = x$.

Solve the Cauchy problem using the operational method:
5) $x'' = t + 1$, $x(0) = 0$, $x'(0) = 1$.

Option 5

Find the general solution of the differential equation:
1) $ctgx \dfrac{dy}{dx} = y \ln y$;
2) $y'x^2 + xy + y^2 = 0$;
3) $(xy' - 1)\ln x = 2y$;
4) $y'' - 16y = x$.

Solve the Cauchy problem using the operational method:
5) $x'' = t + 2$, $x(0) = 0$, $x'(0) = 3$.

Option 6

Find the general solution of the differential equation:

1) $\frac{dy}{dx}-(5x-2)ctgy=0$;

2) $\left(x-ytg\frac{y}{x}\right)dx+xtg\frac{y}{x}dy=0$;

3) $xy^2y'=x^2+y^3$;

4) $y''-9y=-x$.

Solve the Cauchy problem using the operational method:

5) $x''=t+3$, $x(0)=2$, $x'(0)=0$.

Option 7

Find the general solution of the differential equation:

1) $\left(x^2+16\right)\frac{dy}{dx}+4y^2=0$;

2) $xyy'=5x^2+y^2$;

3) $x^2y'=2xy+3$;

4) $y''-4y=-x$.

Solve the Cauchy problem using the operational method:

5) $x''=t+3$, $x(0)=2$, $x'(0)=1$.

Option 8

Find the general solution of the differential equation:

1) $tgx\frac{dy}{dx}=y+3$;

2) $\left(x-y\sin\frac{y}{x}\right)dx+x\sin\frac{y}{x}dy=0$;

3) $\frac{dy}{dx} + 2xy = xe^{-x^2}$;

4) $y'' - y = -x$.

Solve the Cauchy problem using the operational method:

5) $x'' = t - 2$, $\quad x(0) = 1$, $x'(0) = 1$.

Option 9

Find the general solution of the differential equation:

1) $y'\sqrt{1-x^2} - tg\, y = 0$;

2) $xy' = y\, ln(y/x) + y$;

3) $y' + ytgx = \frac{1}{cos\, x}$;

4) $y'' - 16y = -x$.

Solve the Cauchy problem using the operational method:

5) $x'' = t - 3$, $\quad x(0) = 3$, $x'(0) = 0$.

Option 10

Find the general solution of the differential equation:

1) $xy' = y + 4$;

2) $(y^2 - 3x^2)dy + 2xydx = 0$;

3) $xy' - 2y = 2x^4$;

4) $y'' - 25y = -x$;

Solve the Cauchy problem using the operational method:

5) $x'' = t + 5$, $\quad x(0) = 0$, $x'(0) = 1$.

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Appendix A
Table of the derivatives

In the table $u = u(x)$ is a differentiable function

1	$(C)' = 0$, $C = const$	11	$(tgu)' = \frac{1}{cos^2 u} \cdot u'$
2	$(x)' = 1$	12	$(ctgu)' = -\frac{1}{sin^2 u} \cdot u'$
3	$(u^n)' = n \cdot u^{n-1} \cdot u'$	13	$(\arcsin u)' = \frac{1}{\sqrt{1-u^2}} \cdot u'$
4	$(\sqrt{u})' = \frac{1}{2\sqrt{u}} \cdot u'$	14	$(\arccos u)' = -\frac{1}{\sqrt{1-u^2}} \cdot u'$
5	$(a^u)' = a^u \cdot ln\, a \cdot u'$, $a = const$	15	$(arctgu)' = \frac{1}{1+u^2} \cdot u'$
6	$(e^u)' = e^u \cdot u'$	16	$(arcctgu)' = -\frac{1}{1+u^2} \cdot u'$
7	$(log_a u)' = \frac{1}{u \cdot ln\, a} \cdot u'$	17	$(sh\, u)' = ch\, u \cdot u'$
8	$(\ln u)' = \frac{1}{u} \cdot u'$	18	$(ch\, u)' = sh\, u \cdot u'$
9	$(sin\, u)' = cos\, u \cdot u'$	19	$(th\, u)' = \frac{1}{ch^2 u} \cdot u'$
10	$(cos\, u)' = -sin\, u \cdot u'$	20	$(cth\, u)' = -\frac{1}{sh^2 u} \cdot u'$

Appendix B
Table of the basic indefinite integrals

In the table $u = u(x)$ is a differentiable function

1	$\int du = u + C$	10	$\int ctgu\, du = ln\lvert sin\, u\rvert + C$
2	$\int u^{\alpha} du = \frac{u^{\alpha+1}}{\alpha+1} + C$, $\alpha \neq -1$	11	$\int \frac{du}{cos^2 u} = tg\, u + C$
3	$\int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$	12	$\int \frac{du}{sin^2 u} = -\, ctg\, u + C$
4	$\int \frac{du}{u} = ln\lvert u\rvert + C$	13	$\int \frac{du}{sin\, u} = ln\left\lvert tg\frac{u}{2}\right\rvert + C$
5	$\int a^u du = \frac{a^u}{ln\, a} + C$	14	$\int \frac{du}{cos\, u} = ln\left\lvert tg\left(\frac{u}{2} + \frac{\pi}{4}\right)\right\rvert + C$
6	$\int e^u du = e^u + C$	15	$\int sh\, u\, du = ch\, u + C$
7	$\int \sin u\, du = -\cos u + C$	16	$\int ch\, u\, du = sh\, u + C$
8	$\int cos\, u\, du = sin\, u + C$	17	$\int \frac{du}{ch^2 u} = th\, u + C$
9	$\int tgu\, du = -ln\lvert cos\, u\rvert + C$	18	$\int \frac{du}{sh^2 u} = -\, cth\, u + C$

19	$\int \frac{du}{u^2+a^2} = \frac{1}{a} arctg \frac{u}{a} + C$
20	$\int \frac{du}{u^2-a^2} = \frac{1}{2a} ln \left\| \frac{u-a}{u+a} \right\| + C$
21	$\int \frac{du}{\sqrt{a^2-u^2}} = arcsin \frac{u}{a} + C$
22	$\int \frac{du}{\sqrt{u^2 \pm a^2}} = ln \left\| u + \sqrt{u^2 \pm a^2} \right\| + C$
23	$\int \sqrt{a^2-u^2} du = \frac{1}{2} u \sqrt{a^2-u^2} + \frac{1}{2} a^2 arcsin \frac{u}{a} + C$
24	$\int \sqrt{u^2 \pm a^2} du = \frac{1}{2} u \sqrt{u^2 \pm a^2} \pm \frac{1}{2} a^2 ln \left\| u + \sqrt{u^2 \pm a^2} \right\| + C$

Appendix C
Properties of the Laplace transformation

Property	The calculation formula
Linearity	$\sum_{k=1}^{n}\lambda_k f_k(t) \leftrightarrow \sum_{k=1}^{n}\lambda_k F_k(p)$
Delay theorem	$f(t-\tau)\cdot\eta(t-\tau) \leftrightarrow e^{-\tau p}F(p)$
Shifting theorem	$e^{at}f(t) \leftrightarrow F(p-a)$
Similarity theorem	$f(at) \leftrightarrow \frac{1}{a}F\left(\frac{p}{a}\right)$
Differentiation of original	$f'(t) \leftrightarrow pF(p)-f(0)$, $f^{(n)}(t) \leftrightarrow p^n F(p)-\sum_{k=1}^{n}p^{n-k}f^{(k-1)}(0)$
Differentiation of image	$t^n f(t) \leftrightarrow (-1)^n \frac{d^n}{dp^n}F(p)$
Integration of original	$\int_0^t f(\tau)d\tau \leftrightarrow \frac{F(p)}{p}$
Integration of image	$\frac{f(t)}{t} \leftrightarrow \int_p^\infty F(p)dp$
Convolution theorem	$f_1(t)*f_2(t)=\int_0^t f_1(t)f_2(t-\tau)d\tau \leftrightarrow F_1(p)\cdot F_2(p)$

Appendix D
Table of Laplace transforms

Original function $f(t)$	Laplace transform $F(p)$	Original function $f(t)$	Laplace transform $F(p)$
$\eta(t)$	$\frac{1}{p}$	$\text{sh}\,\omega t$	$\frac{\omega}{p^2-\omega^2}$
$t^n, n\in N$	$\frac{n!}{p^{n+1}}$	$e^{\alpha t}\cdot\cos\omega t$	$\frac{p-\alpha}{(p-\alpha)^2+\omega^2}$
$t^{\beta}, \beta>-1$	$\frac{\Gamma(\beta+1)}{p^{\beta+1}}$	$e^{\alpha t}\cdot\sin\omega t$	$\frac{\omega}{(p-\alpha)^2+\omega^2}$
$e^{\alpha t}$	$\frac{1}{p-\alpha}$	$e^{\alpha t}\cdot\text{ch}\,\omega t$	$\frac{p-\alpha}{(p-\alpha)^2-\omega^2}$
$t^n e^{\alpha t},\ n\in N$	$\frac{n!}{(p-\alpha)^{n+1}}$	$e^{\alpha t}\cdot\text{sh}\,\omega t$	$\frac{\omega}{(p-\alpha)^2-\omega^2}$
$t^{\beta}e^{\alpha t},\ n\in N$	$\frac{\Gamma(\beta+1)!}{(p-\alpha)^{\beta+1}}$	$t\cdot\cos\omega t$	$\frac{p^2-\omega^2}{(p^2+\omega^2)^2}$
$\sin\omega t$	$\frac{\omega}{p^2+\omega^2}$	$t\cdot\sin\omega t$	$\frac{2p\omega}{(p^2+\omega^2)^2}$
$\cos\omega t$	$\frac{p}{p^2+\omega^2}$	$t\cdot\text{ch}\,\omega t$	$\frac{p^2+\omega^2}{(p^2-\omega^2)^2}$
$\text{ch}\,\omega t$	$\frac{p}{p^2-\omega^2}$	$t\cdot\text{sh}\,\omega t$	$\frac{2p\omega}{(p^2-\omega^2)^2}$