МІНІСТЕРСТВО ОСВІТИ ТА НАУКИ УКРАЇНИ
НАЦІОНАЛЬНИЙ УНІВЕРСИТЕТ "ЗАПОРІЗЬКА ПОЛІТЕХНІКА

Кафедра "Електропривод та автоматизація промислових установок"

КОНСПЕКТ
з дисципліни "Теоретичні основи електротехніки"
для студентів спеціальності 141 "Електроенергетика, електротехніка та електромеханіка"
усіх форм навчання
"Трифазні кола" (англійською мовою)
Part 5

2023

Конспект з дисципліни "Теоретичні основи електротехніки" для студентів спеціальності 141 "Електроенергетика, електротехніка та електромеханіка" усіх форм навчання "Трифазні кола" (англійською мовою) Part 5. /Укл.: В.В. Козлов, О.В. Набокова. - Запоріжжя: НУ "Запорізька політехніка", 2023. - 23 с.

Укладачі: В.В. Козлов, доцент, к.т.н.
О.В. Набокова, доцент.

Відповідальний
за випуск: О.В. Набокова, доцент.

Затверджено
на засіданні кафедри ТЗЕ
Протокол №1 від 7.10.22 р.

Затверджено
на засіданні
науково-методичної комісії
Електротехнічного факультету
Протокол №2 від 17.10.22 р.

THREE-PHASE CIRCUITS

5.1 Three-phase System of Electromotive Forces

Suppose we have a system of three *ac* electromotive forces (e.m.f.s) of a certain frequency such that their amplitudes are equal but these e.m.f.s are displaced from one another by 120° in time. Such a set of three sinusoidal e.m.f.s make up a so-called *symmetrical three-phase system of electromotive forces*. The instantaneous values of such e.m.f.s are shown in Fig. 5.1 a , and their vector diagram is in Fig. 5.1 b .

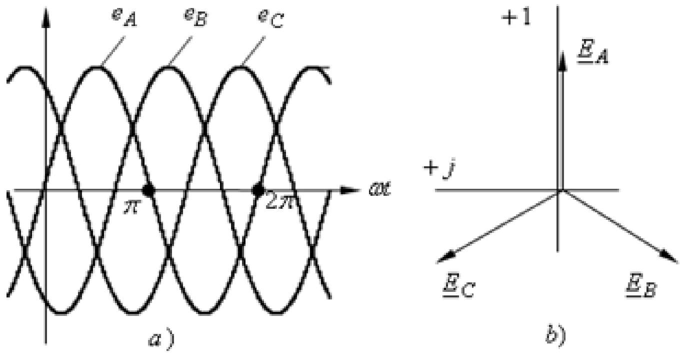

Fig. 5.1 Three-phase system of e.m.f.s.

The electromotive forces for a three-phase system are supplied by a three-phase generator (alternator). Such a generator has three identical (phase) coils rigidly attached to one another and rotating in a uniform magnetic field at a constant angular speed ω . The coils are displaced at 120° from one another, and the sine wave e.m.f.s induced in them, are also displaced at 120° in time phase.

The beginnings of coils (windings) can be marked out with the first letters of Latin alphabet *A*, *B* and *C*, and the ends - with last letters *X*, *Y*, *Z*. In particular, we call this system a *three-phase balanced system* - in contrast to an unbalanced system, in which the magnitudes may be unequal and/or the phase displacements may not by 120°.

For a balanced three-phase system, it follows from Eq. (5.1), that the phasor sum of these three electromotive forces is zero.

We may mathematically express this system of e.m.f.s as

$$e_A = E_m \sin \omega t = U_m \sin \omega t$$

$$e_B = E_m \sin (\omega t - 120^o) = U_m \sin (\omega t - 120^o) \qquad (5.1)$$

$$e_C = E_m \sin (\omega t + 120^o) = U_m \sin (\omega t + 120^o)$$

For the identification, the three electromotive forces generated by the alternator are marked as follows. One of them is marked as $\underline{E}_A$. Then the one leading it is $\underline{E}_C$ and the one lags behind it, $\underline{E}_B$.

The order in time in which they go through a zero value and begin to increase in an arbitrary positive sense is termed the *phase sequence*. The order *ABC* is taken as normal or *positive* phase sequence. If the voltages have been caused to reverse their sequence, making it *ACB*, this will be a *negative* phase sequence. If the voltages are in time phase, their vectors coincide, and the system is described as one of *zero* phase sequence.

5.2 A Three-phase Circuit. Phase and Line Quantities

A three-phase circuit is a combination of three-phase supply, a three-phase load (or loads), and connecting wires. The term "phase" may be applied to that part of three-phase system which carries the same current. Thus, each phase winding of a generator can be connected to the load by two wires. The most common types of interconnections are *wye connection* and *delta connection*, applicable to both the supply and the load of a system. The number of connecting wires in a system is three or four.

First, we consider "wye-wye" connection with a neutral wire (also referred to as a three-phase four-wire system) (see Fig 5.2).

Three-phase circuits may be symmetrical and asymmetrical. The circuit, in which a symmetrical three-phase system of e.m.f.s and symmetrical loading operate, is called symmetrical or uniform.

For wye connection the ends of three phases are connected together to form a node called a *neutral point N* or *O* (for the phases of generator) and *n* or *O'* (for the phases of the load), leaving the beginnings as terminals of three-phase star system. The wire connecting the neutral points of the alternator and the load is called the *neutral wire*. The neutral wire carries a neutral current, $\underline{I}_N$ or $\underline{I}_o$, the positive direction for which is from *n* to *N*.

The wires connecting the terminals *A*, *B* and *C* of the generator and the terminals *a*, *b* and *c* of the load are termed the *line wires*, or simply the *lines*. The currents in the line wires are *line currents*, $\underline{I}_A$, $\underline{I}_B$ and $\underline{I}_C$.

A positive direction for line currents is assumed to be from the supply to the load. When only the magnitudes of the line currents are involved, it is customary to use the symbol I_l, especially when all the line currents are equal in magnitude.

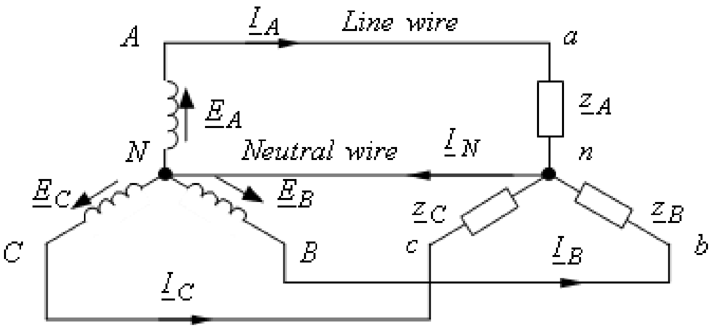

Fig. 5.2. Wye-wye connection with a neutral wire.

Fig. 5.2 shows a wye-to-wye (star-to-star) system with a neutral wire (also referred to as a *three-phase four-wire system*). If there is no a neutral wire between neutral points it will be *three-phase three-wire system*.

The voltage between two wires is termed the *line voltage*. It symbol has a two-letter subscript, for example, $\underline{U}_{AB}$, which is the line voltage between the terminals *A* and *B*. The symbol for line voltage is U_l.

The voltage between the beginning and the end of a phase, or the voltage from line to earth is called a *phase voltage*, U_p. The currents in the line wires are referred to as *line currents.* They are marked with one letter as $\underline{I}_A$, $\underline{I}_B$ and $\underline{I}_C$. When we need the magnitudes of line currents we use symbol I_l, especially when all the line currents are equal. The currents in the phases of either the supply or the load are called the *phase currents*, I_p.

Look at Fig. 5.3. $\underline{U}_{AB}$, $\underline{U}_{BC}$, $\underline{U}_{CA}$ are the line voltages across the generator terminals; $\underline{U}_{ab}$, $\underline{U}_{bc}$, $\underline{U}_{ca}$ are the line voltages across the load terminals; $\underline{U}_A$, $\underline{U}_B$, $\underline{U}_C$ are the phase generator voltages; $\underline{U}_a$, $\underline{U}_b$, $\underline{U}_c$ are the phase voltages across the load terminals.

If there are no impedances in connecting wires the line voltages across the generator terminals are equal to the line voltages across the load terminals, that is $\underline{U}_{AB} = \underline{U}_{ab}$, $\underline{U}_{BC} = \underline{U}_{bc}$, $\underline{U}_{CA} = \underline{U}_{ca}$.

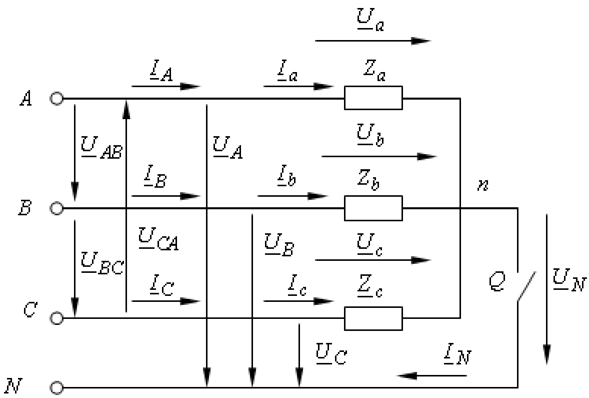

Fig. 5.3. A wye-connected load.

For the same reason, generator phase voltages and phase voltages across the load terminals are numerically equal to e.m.f.s:

$\underline{U}_A = \underline{U}a = \underline{E}_A$; $\underline{U}_B = \underline{U}_b = \underline{E}_B$; $\underline{U}_C = \underline{U}c = \underline{E}_C$.

$\underline{I}_A$, $\underline{I}_B$, $\underline{I}_C$ are the line currents in line wires; $\underline{I}_a$, $\underline{I}_b$, $\underline{I}_c$ are the phase currents through the load phases. As they are joined in series: $\underline{I}_A = \underline{I}_a$, $\underline{I}_B = \underline{I}_b$, $\underline{I}_C = \underline{I}_c$. $\underline{I}_N$ is the current in a neutral wire, and $\underline{U}_N$ is the neutral voltage shift (or displacement voltage or the bias neutral voltage). It is the voltage between two neutral junctions.

5.3 Relationship between Line and Phase Voltages and Currents

Between phase and line e.m.f.s, voltages and currents exist certain relationships for symmetrical circuits. For a wye connection the line voltages are related to the phase voltages such that

$$\underline{U}_{AB} = \underline{U}_A - \underline{U}_B$$
$$\underline{U}_{BC} = \underline{U}_B - \underline{U}_C \qquad (5.2)$$
$$\underline{U}_{CA} = \underline{U}_C - \underline{U}_A$$

where $\underline{U}_{AB}$, $\underline{U}_{BC}$ and $\underline{U}_{CA}$ are the line voltages in a generator; $\underline{U}_A$, $\underline{U}_B$ and $\underline{U}_C$ are the phase voltages in a generator.

When we have a symmetrical load that is $Z_A = Z_B = Z_C$, the line voltages are $\sqrt{3}$ times more than the phase voltages.

These relationships of the phase voltages and line voltages are illustrated in the phasor diagram of Fig. 5.4.

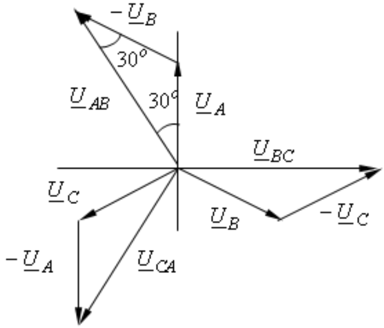

Fig. 5.4. The phasor diagram of line and phase voltages.

From the vector diagram, the line voltage may be visualized as the base of an equivalent triangle having acute angles of 30° (Fig.5.3)

$$U_l = U_{AB} = U_p \cdot 2\cos 30^o = \sqrt{3} \cdot U_p \qquad (5.3)$$

The line currents in a wye supply are equal to the phase currents as line and phase wires are connected in series: $I_l = I_{ph}$.

5.4 The Calculation of Wye-to-Wye with a Neutral Wire

For calculation such a circuit, we can use the symbolic method of calculation and some known laws: Ohm's law, Kirchhoff's laws, mesh-current method, node-analysis method, superposition theorem and so on.

If there is no impedance in a neutral wire in the circuit of Fig. 5.2, the potential at point *n* is the same as at point *N*, and two points are a single point. Then three separate meshes have the following currents:

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_A}; \quad \underline{I}_B = \frac{\underline{E}_B}{\underline{Z}_B}; \quad \underline{I}_C = \frac{\underline{E}_C}{\underline{Z}_C}. \qquad (5.4)$$

By Kirchhoff's current law the current in the neutral wire is the vector sum of the phase currents:

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C \qquad (5.5)$$

If load is balanced ($\underline{Z}_A = \underline{Z}_B = \underline{Z}_C$), the current $\underline{I}_N$ is zero. If load is unbalanced, the current $\underline{I}_N$ is other than zero. If the neutral wire has an impedance Z_o, the system should be calculated by the nodal-pairs method.

Example 5.1. The phase voltage of the alternator in Fig. 5.2 is 120 V. The phase impedances: $\underline{Z}_A = R = 30\Omega$; $\underline{Z}_B = j\omega L = 80\Omega$; $\underline{Z}_C = -j/\omega C = 80\Omega$. Find a current in a neutral wire.

Solution: Write phase voltages in a complex form:

$$\underline{U}_A = \underline{U_p} \cdot e^{j0^o} = 120\ V\ ;$$

$$\underline{U}_B = \underline{U_p} \cdot e^{-j120^o} = 120\, e^{-j120^o}\ V\ ;$$

$$\underline{U}_C = \underline{U_p} \cdot e^{j120^o} = 120\, e^{j120^o}\ V\ .$$

Calculate phase currents for each phase separately:

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_A} = \frac{120}{30} = 4\,A; \qquad \underline{I}_B = \frac{\underline{E}_B}{\underline{Z}_B} = \frac{120e^{-j120^o}}{80e^{j90^o}} = 1.5e^{-j210^o}\ A;$$

$$\underline{I}_C = \frac{\underline{E}_C}{\underline{Z}_C} = \frac{120e^{j120^o}}{80e^{-j90^0}} = 1.5\,e^{j210^o}\ A.$$

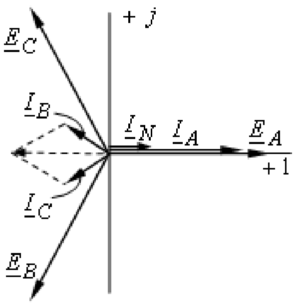

Fig. 5.5

The vector diagram is shown in Fig. 5.5.

Current $\underline{I}_A$ is in phase with $\underline{E}_A$, $\underline{I}_B$ lags behind $\underline{E}_B$, $\underline{I}_C$ leads $\underline{E}_C$. The algebraic sum $\underline{I}_A + \underline{I}_B + \underline{I}_C$ gives the current in a neutral wire:

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C = 4 + 1.5e^{-j210^o} + 1.5e^{j210^o} =$$
$$= 4 - 1.48 + j0.235 - 1.48 - j0.235 = 1.04\ \ A.$$

5.5 The Calculation of Wye-to-Wye without a Neutral Wire

When the load is balanced (or uniform) and there is no a neutral wire between nodes *N* and *n*, the voltage $\underline{U}_N = 0$.

If the load is unbalanced the voltage appearing between the neutral points of the load and the supply can be found by the formula

$$\underline{U}_N = \frac{\underline{E}_A \underline{Y}_A + \underline{E}_B \underline{Y}_B + \underline{E}_C \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} \qquad (5.6)$$

where $\underline{E}_A$, $\underline{E}_B$ and $\underline{E}_C$ are respective voltages at the generator end; $\underline{Y}_A, \underline{Y}_B$ and $\underline{Y}_C$ are the admittance of three phases. This voltage is called the *neutral voltage shifting* or the *bias neutral voltage*.

Now we can find the phase currents through the load

$$\underline{I}_a = \frac{\underline{U}_a}{\underline{Z}_a} = \frac{\underline{E}_A - \underline{U}_N}{\underline{Z}_a} = (\underline{E}_A - \underline{U}_N) \cdot \underline{Y}_a ; \qquad (5.7)$$

$$\underline{I}_b = \frac{\underline{U}_b}{\underline{Z}_b} = \frac{\underline{E}_B - \underline{U}_N}{\underline{Z}_a} = (\underline{E}_B - \underline{U}_N) \cdot \underline{Y}_b ; \qquad (5.8)$$

$$\underline{I}_c = \frac{\underline{U}_c}{\underline{Z}_c} = \frac{\underline{E}_C - \underline{U}_N}{\underline{Z}_c} = (\underline{E}_C - \underline{U}_N) \cdot \underline{Y}_c . \qquad (5.9)$$

where $\underline{U}_a, \underline{U}_b, \underline{U}_c$ are phase voltages at the load, $\underline{Y}_a, \underline{Y}_b, \underline{Y}_c$ are phase admittances, that is the values inverse to complex impedances.

For the numerical data see Example 5.2. (Fig.5.6.)

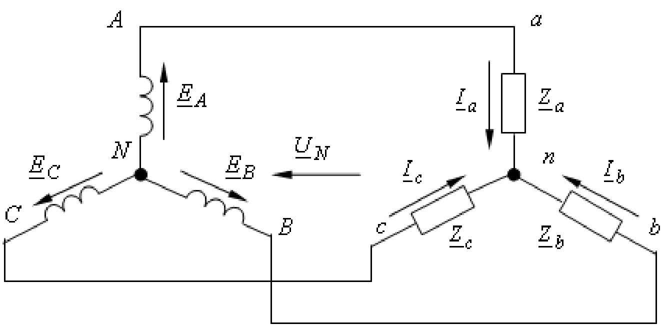

Fig.5.6

Example 5.2. Determine currents and voltages and draw a vector diagram in the circuit without a neutral wire, for $E_p = 50\ V$, $\underline{Z}_a = 20\ \Omega$, $\underline{Z}_b = 10\ \Omega$, $\underline{Z}_c = -j20\ \Omega$.

Write phase voltages in a complex form:

$$\underline{U}_A = \underline{E}_p \cdot e^{j0^o} = 50\ V,\ \underline{U}_B = \underline{E}_p \cdot e^{-j120^o} = 50\,e^{-j120^o}\ V$$

$$\underline{U}_C = \underline{E}_p \cdot e^{j120^o} = 50\,e^{j120^o}\ V.$$

Calculate complex phase admittances:

$$\underline{Y}_a = 1/\underline{Z}_a = 1/20 = 0.05\ \Omega^{-1};\quad \underline{Y}_b = 1/\underline{Z}_b = 1/10 = 0.1\ \Omega^{-1};$$

$$\underline{Y}_c = 1/\underline{Z}_c = 1/20e^{-j90^o} = 0.05e^{j90^o} = j0.05\ \Omega^{-1}.$$

For the asymmetrical load the neutral voltage shifting appears between neutral nodes. We define it, using Eq. (5.4).

$$\underline{U}_N = \frac{50 \cdot 0.05 + 50e^{-j120^o} \cdot 0.1 + 50e^{j120^o} \cdot 0.05e^{j90^o}}{0.05 + 0.1 + j0.05} =$$

$$= \frac{-2.165 - j5.58}{0.15 + j0.05} = -20.981 - j23.66 = 31.62e^{-j132^o}\ V.$$

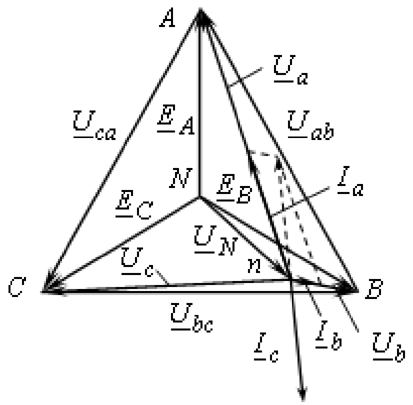

Fig. 5.7

Find the currents. using Eq. (5.7), (5.8), (5.9):

$$\underline{I}_a = (50 + 23{,}85 + j29.45)\cdot 0.05 = 3.55 + j1.183 = 3.74e^{j18^o}\ A;$$

$$\underline{I}_b = (-25 - j43.3 + 23{,}85 + j29.45)\cdot 0.1 = 1.002e^{-j102^o}\ A;$$

$$\underline{I}_c = (-25 + j43{,}3 + 23.85 + j29.45)\cdot j0.05 = 3.354e^{-j177^o}\ A.$$

The vector diagram for this problem one can see in Fig. 5.7.

5.6 The Calculation of Damage Conditions

There are two damage rates in three-phase circuit: a short circuit of any phase and an open circuit of a phase.

Example 5.3. Symmetrical load $\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = (240 + j100)$ is connected with line wires with line impedances $\underline{Z}_l = (30 + j40)\ \Omega$ to the generator with the phase e.m.f. $E_p = 220\ V$. The impedance of a neutral wire $\underline{Z}_N = 40 + j20$. Calculate damage conditions for two cases:

- a *short circuit* of the phase *A*;
- an *open circuit* of a line wire between nodes A and a.

When a short circuit in phase *A*, (Fig.5.8), it is asymmetrical load, $\varphi_n = \varphi_a$, i.e. "*n*" and "*a*" are the same node.

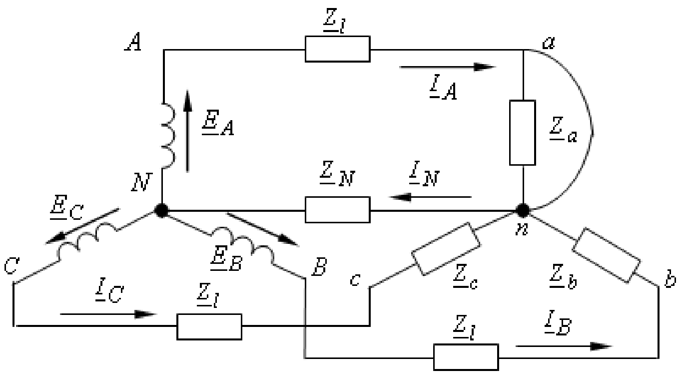

Fig.5.8. A short circuit of a load in the phase *A*

Define the neutral voltage shift appeared between neutral nodes:

$$\underline{U}_N = \frac{\underline{E}_A\underline{Y}_A + \underline{E}_B\underline{Y}_B + \underline{E}_C\underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C + \underline{Y}_N} \qquad (5.10)$$

At first, we calculate complex branch admittances:

$$\underline{Y}_A = \frac{1}{\underline{Z}_l} = \frac{1}{50e^{j53^o}} = 0.02e^{-j53^o} = (0.012 - j0.016)\ \Omega^{-1};$$

$$\underline{Y}_B = \frac{1}{\underline{Z}_b + \underline{Z}_l} = \frac{1}{304.14e^{j27^o}} = 0.0033e^{-j27} = (0.003 - j0.0015)\ \Omega^{-1};$$

$$\underline{Y}_C = \underline{Y}_B = 0.0033e^{-j27} = (0.003 - 0.0015)\ \Omega^{-1};$$

$$\underline{Y}_N = \frac{1}{\underline{Z}_N} = \frac{1}{44.72e^{j27^o}} = 0.022e^{-j27^o} = (0.02 - j0.01)\ \Omega^{-1}.$$

Determine the bias neutral voltage:

$$\underline{U}_N = \frac{127 \cdot 0.02e^{-j153^o} + 127e^{-j120^o} \cdot 0.0033e^{-j27^o} + 127e^{j120^o} \cdot 0.022e^{-j27^o}}{0.012 - j0.016 + 0.003 - j0.00115 + 0.003 - j0.0015 + 0.02 - j0.01} =$$

$$= \frac{3.761e^{-j57^o}}{0.048e^{-j37^o}} = 73.915 - j27.525 = 78.87e^{-j20^o}\ V.$$

Calculate the phase currents by Ohm's law:

$$\underline{I}_A = (\underline{E}_A - \underline{U}_N) \cdot \underline{Y}_a = \underline{U}_{an} \cdot \underline{Y}_a = 0.02e^{-j53^o} \cdot (127 - 78.87e^{-j20^o}) =$$

$$= 2.193 - j2.007 = 2.973e^{-j42^o}\ A;$$

$$\underline{I}_B = (\underline{E}_B - \underline{U}_N) \cdot \underline{Y}_b = \underline{U}_{bn}\underline{Y}_b = 0.0033e^{-j27^o}(127e^{-j120^o} - 59e^{-j5^o}) =$$

$$= -0.784 - j0.197 = 0.808e^{-j166^o}\ A;$$

$$\underline{I}_C = (\underline{E}_C - \underline{U}_N) \cdot \underline{Y}_c = \underline{U}_{cn} \cdot \underline{Y}_c = 0.0033e^{-j27^o}(127e^{j120^o} - 59e^{j5^o}) =$$

$$= -0.207 + j0.915 = 0.938e^{j103^o}\ A.$$

The current in a neutral wire

$$\underline{I}_N = \frac{\underline{U}_N}{\underline{Z}_N} = \frac{78.87e^{-j20^o}}{44.7e^{j27^o}} = 1.764e^{-j47^o}\ A.$$

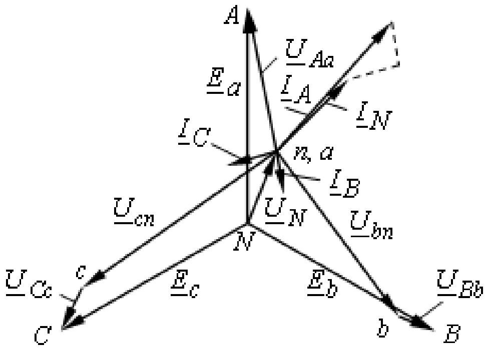

Fig.5.9. A vector diagram for a short circuit in the phase *A*.

Check up the calculation, using Kirchhoff's current law:

$$\underline{I}_N = 2.193 - j2.007 - 0.784 - j0.197 - 0.207 + j0.915 =$$

$$= 1.203 - j1.2 = 1.764e^{-j47^o}\ A.$$

Determine the phase voltage drops across the load and the voltage drops in line wires: $\underline{U}_{an} = 0$

$$\underline{U}_{bn} = \underline{Z}_b \cdot \underline{I}_b = 260e^{j23^o} \cdot 0.808e^{-j166^o} = 210.1e^{-j143^o}\ V$$

$$\underline{U}_{cn} = \underline{Z}_c \cdot \underline{I}_c = 260e^{j23^o} \cdot 0.938e^{j103^o} = 243.858e^{j125^o}\ V.$$

$$\underline{U}_{Aa} = \underline{Z}_l \cdot \underline{I}_a = 50e^{j53^o} \cdot 2.973e^{-j42^o} = 148.655e^{j11^o}\ V$$

$$\underline{U}_{Bb} = \underline{Z}_l \cdot \underline{I}_b = 50e^{j53^o} \cdot 0.808e^{-j166^o} = 40.40e^{-j113^o}\ V$$

$$\underline{U}_{Cc} = \underline{Z}_l \cdot \underline{I}_c = 50e^{j53^o} \cdot 0.938e^{j103^o} = 46.9e^{j156^o}\ V$$

A vector diagram for this condition is shown in Fig. 5.9.

When we have *an open circuit* of the phase *A*, (Fig.5.10) the working condition also becomes asymmetrical. Hence a neutral voltage shift appears between the neutral junctions of a generator and a load.

As $\underline{U}_A = 0$, Eq.(5.5) is used in the following condition:

$$\underline{U}_N = \frac{\underline{E}_B \underline{Y}_B + \underline{E}_C \underline{Y}_C}{\underline{Y}_B + \underline{Y}_C + \underline{Y}_N} \qquad (5.11)$$

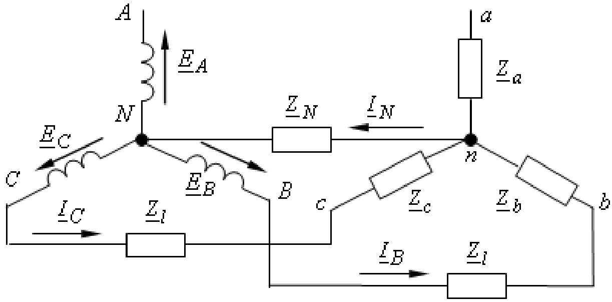

Fig.5.10. An open circuit in the line wire *Aa*.

$$\underline{U}_N = \frac{220e^{-j120^o}\cdot 0.0033e^{-j27^o} + 127e^{j120^o}\cdot 0.0033e^{-j27^o}}{0.003 - j0.0015 + 0.003 - j0.0015 + 0.02 - j0.01} =$$

$$= -24.997 - j0.284 = 24.998e^{-j179^o}\ V.$$

Calculate the phase currents and the current in the neutral wire:

$$\underline{I}_B = (\underline{E}_B - \underline{U}_N)\cdot \underline{Y}_b = \underline{U}_{bn}\cdot \underline{Y}_b = 0.0033e^{-j27^o}\cdot(127e^{-j120^O} -$$

$$- 24.998e^{j179^o}) = -0.537 - j0.43 = 0.69e^{-j141^o}\ A;$$

$$\underline{I}_C = (\underline{E}_C - \underline{U}_N)\cdot \underline{Y}_c = \underline{U}_{cn}\cdot \underline{Y}_c = 0.0033e^{-j27^o}\cdot(127e^{j120^O} -$$

$$- 24.998e^{j179^o}) = 0.04 + j0.684 = 0.69e^{j87^o}\ A$$

$$\underline{I}_N = \underline{U}_N\cdot \underline{Y}_N = 0.0022e^{-j27^o}\cdot 24.998e^{-j179^O} = 0.559e^{-j206^o}\ A.$$

Find the phase voltages and the voltage drops in line wires:

$$\underline{U}_{Bb} = \underline{Z}_l\cdot \underline{I}_b = 50e^{j53^o}\cdot 0.69e^{-j141^o} = 34.5\,e^{-j88^o}\ V$$

$$\underline{U}_{Cc} = \underline{Z}_l\cdot \underline{I}_c = 50e^{j53^o}\cdot 0.69e^{j87^o} = 34.5e^{j140^o}\ V$$

$$\underline{U}_{bn} = \underline{Z}_b\cdot \underline{I}_b = 0.69e^{-j141^o}\cdot 260e^{j23^o} = 179.4e^{-j118^o}\ V$$

$$\underline{U}_{cn} = \underline{Z}_c\cdot \underline{I}_c = 0.69e^{j87^o}\cdot 260e^{j23^o} = 179.4e^{j110^o}\ V$$

The vector diagram for this case is represented in Fig. 5.11.

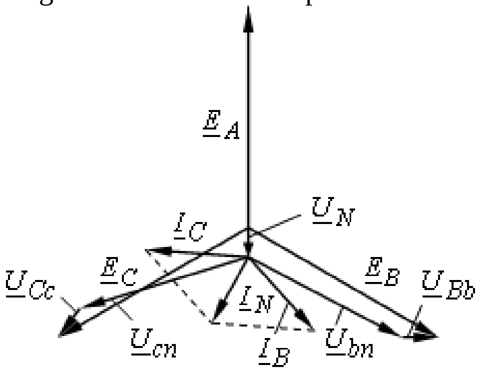

Fig. 5.11. A phasor diagram for the open circuit in the line wire *Aa*

5.7. Delta-Connected Load

For delta connection (Fig. 5.12), the end of the 1st load phase is connected to the beginning of the 2nd one, the end of the latter is connected to the beginning of the 3rd phase, and its end - to the beginning of the 1st, thus making symmetrical cyclic junctions. The vector sum of the e.m.f.s in a closed triangle is zero. In delta-connected load, phase currents I_{ab}, I_{bc}, I_{ca} flowing through phase impedances Z_{ab}, Z_{bc}, Z_{ca} are supplied with two-letter subscripts. The positive direction of a current flow is assumed to be clockwise. The sequence of letters corresponds to the direction of current flow, the first letter standing for the sending end, and the second letter for the receiving end of a given current.

Let's mark the voltages and currents in the delta-connection:

U_{AB}, U_{BC}, U_{CA} are the line voltages in a three-phase delta-connected supply; U_{ab}, U_{bc}, U_{ca} are phase voltages in the load;

I_A, I_B, I_C are the line currents in line wires connecting the supply and the load; I_{ab}, I_{bc}, I_{ca} are phase currents in a delta-connected load.

If $Z_L = 0$ in line wires, the line voltages in a delta-connected supply are equal to the phase voltages in a delta-connected load: $U_l = U_p$.

If the load is balanced, the line currents, however, $\sqrt{3}$ times more than the phase currents, as above for the voltages in a wye supply.

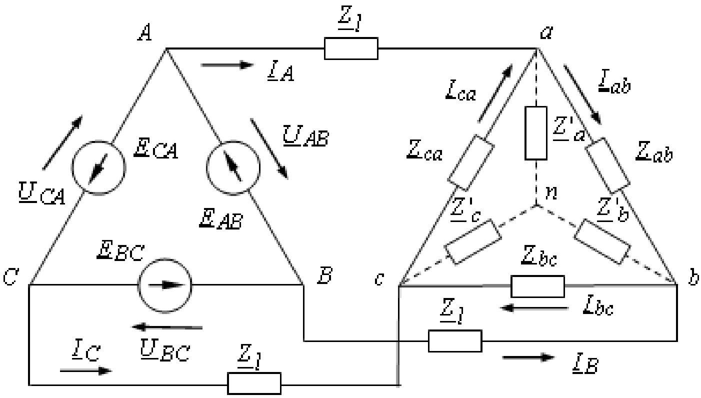

Fig.5.12. The delta connection.

If the load is unbalances, the line currents can be found in terms of the phase currents by Kirchhoff's current law:

$$\underline{I}_A = \underline{I}_{ab} - \underline{I}_{ca}, \quad \underline{I}_B = \underline{I}_{bc} - \underline{I}_{ab}, \quad \underline{I}_C = \underline{I}_{ca} - \underline{I}_{bc}. \qquad (5.12)$$

When there are line impedances in line wires, the line voltages across the load aren't equal to the generator line voltages.

That is why we must at first change delta-connected load into wye-connected one (see Fig. 5.12).

One can use the following formulas for transformation:

$$\underline{Z}'_a = \frac{\underline{Z}_{ab} \cdot \underline{Z}_{ca}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}; \quad \underline{Z}'_b = \frac{\underline{Z}_{bc} \cdot \underline{Z}_{ab}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}; \quad \underline{Z}'_c = \frac{\underline{Z}_{ca} \cdot \underline{Z}_{bc}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}. \qquad (5.13)$$

If there is a symmetrical load and the phase impedances are equal ($\underline{Z}_{ab} = \underline{Z}_{ba} = \underline{Z}_{ca}$), the neutral voltage shift (or the bias neutral voltage) $\underline{U}_N$ is equal to *0*. Then we can calculate currents:

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_l + \underline{Z}'_a}; \quad \underline{I}_B = \frac{\underline{E}_B}{\underline{Z}_l + \underline{Z}'_b}; \quad \underline{I}_C = \frac{\underline{E}_C}{\underline{Z}_l + \underline{Z}'_c}. \qquad (5.14)$$

The phase currents are $\sqrt{3}$ times less the line currents and angle shift between them is 30^{O} (phase current leads the corresponding line current).

$$\underline{I}_{ab} = \frac{\underline{I}_A}{\sqrt{3}} e^{j30^o}; \ \underline{I}_{bc} = \frac{\underline{I}_B}{\sqrt{3}} e^{j30^o}; \ \underline{I}_{ca} = \frac{\underline{I}_C}{\sqrt{3}} e^{j30^o} \qquad (5.15)$$

The vector diagram of currents and voltages (Fig. 5.13).

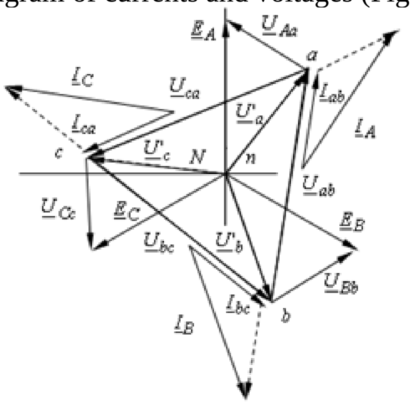

Fig. 5.13. A vector diagram for a symmetrical delta connection.

The description of the vector diagram:

$\underline{E}_A$, $\underline{E}_B$, $\underline{E}_C$ are the phase e.m.f.s of a generator;

$\underline{U}'_A$, $\underline{U}'_B$, $\underline{U}'_C$ are the phase voltages across a wye-connected load. They can be found by Ohm's law:

$$\underline{U}'_a = \underline{Z}'_a \cdot \underline{I}_A; \ \underline{U}'_b = \underline{Z}'_b \cdot \underline{I}_b; \ \underline{U}'_c = \underline{Z}'_c \cdot \underline{I}_c. \qquad (5.16)$$

$\underline{U}_{Aa}$, $\underline{U}_{Bb}$, $\underline{U}_{Cc}$ are the voltages across line impedances. We can determine them by the formulas:

$$\underline{U}_{Aa} = \underline{Z}_l \cdot \underline{I}_A; \ \underline{U}_{Bb} = \underline{Z}_l \cdot \underline{I}_B; \ \underline{U}_{Cc} = \underline{Z}_l \cdot \underline{I}_C. \qquad (5.17)$$

Calculate the phase voltages across a delta-connected load. They can be found as $\underline{U}_{ab} = \underline{U}'_a - \underline{U}'_b$; $\underline{U}_{ab} = \underline{U}'_a - \underline{U}'_b$. (5.18)

Example 5.4. Symmetrical load $\underline{Z}_{ab} = \underline{Z}_{bc} = \underline{Z}_{ca} = (500 - j450)\,\Omega$ is connected through line wires with line impedances $\underline{Z}_l = (35 + j45)\,\Omega$ to the symmetrical generator with the phase voltage equaled to $U_p = 380\ V$. Calculate phase and line currents and voltages. Draw a vector diagram.

*Solution: A*t first, we change a delta-connected load into a wye-connected one (see Fig. 5.12). As we have a balanced load, one can use the following equation:

$$\underline{Z}'_a = \underline{Z}'_b = \underline{Z}'_c = \frac{\underline{Z}_{ab}}{3} = 166{,}44 - j150 = 224.23 e^{-j42^o}\,\Omega\,.$$

Determine phase voltage across the generator and write them in a complex form:

$$\underline{U}_A = 380\ V;\ \underline{U}_B = 380 e^{-j120^o} V;\ \underline{U}_C = 380 e^{j120^o} V\,.$$

Define phase impedances:

$$\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = \underline{Z}_l + \underline{Z}_a = 35 + j45 + 166.44 - j150 = 227.36 e^{-j28^o}\,\Omega\,.$$

Calculate line currents by Ohm's law:

$$\underline{I}_A = \frac{\underline{U}_A}{\underline{Z}_a} = \frac{380}{227.36 e^{-j28^o}} = 1.67 e^{j28^o} A\,.$$

Then

$$\underline{I}_B = \frac{\underline{U}_B}{\underline{Z}_b} = 1.67 e^{-j92^o} A; \quad \underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_c} = 1.67 e^{j148^o} A\,.$$

Determine phase currents across the load by Eq. (5.15):

$$\underline{I}_{ab} = \frac{\underline{I}_A}{\sqrt{3}} e^{j30^o} = \frac{1.67 e^{j28^o}}{\sqrt{3}} e^{j30^o} = 0.965 e^{j58^o} A;$$

$$\underline{I}_{bc} = 0.965 e^{-j62^o} A; \quad \underline{I}_{ca} = 0.965 e^{j178^o}\ \text{A}.$$

Define line and phase voltages:

$$\underline{U}_{ab} = \underline{Z}_{ab} \cdot \underline{I}_{ab} = 672.68 e^{-j42^o} \cdot 0.965 e^{j57^o} = 649.14 e^{j58^o} V;$$

$$\underline{U}_{bc} = \underline{Z}_{bc} \cdot \underline{I}_{bc} = 649.14 e^{-j104^o} V;\ \underline{U}_{ca} = \underline{Z}_{ca} \cdot \underline{I}_{ca} = 649.14 e^{j136^o}\ V;$$

$$\underline{U}_{Aa} = \underline{Z}_l \cdot \underline{I}_A = 57 e^{j52^o} \cdot 1.67 e^{j27^o} = 92.27 e^{j80^o}\ V;$$

$$\underline{U}_{Bb} = \underline{Z}_l \cdot \underline{I}_B = 92.27 e^{-j40^o}\ V;\ \underline{U}_{Cc} = \underline{Z}_l \cdot \underline{I}_C = 92.27 e^{j200^o} V\,.$$

At last, we draw a vector diagram for this case (see Fig. 5.14).

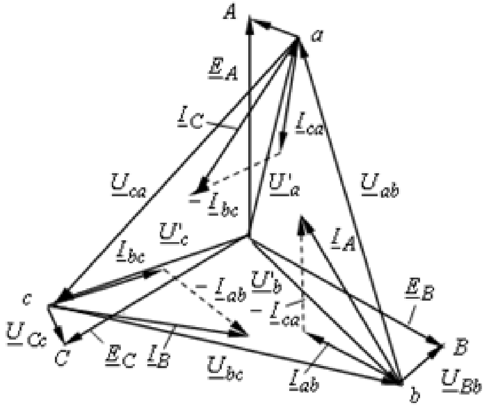

Fig. 5.14

5.8 Damage Conditions in a Delta-Connected Load

There are three damage rates in a delta-connected load:

an open circuit of any phase;

a short circuit of a phase;

an open circuit of a line wire.

The first damage condition we will consider for the case when the phase *bc* is broken. Then a delta-connected load changes into asymmetrical wye connection because we have got here an unbalanced load. Such a delta-connected circuit is depicted in Fig. 5.15.

So, it is the asymmetrical wye connection without a neutral wire. Hence, a neutral voltage shift appears between the two neutral nodes. We can find it by Eq. (5.6):

$$\underline{U}_N = \frac{\underline{E}_A \underline{Y}_A + \underline{E}_B \underline{Y}_B + \underline{E}_C \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C}$$

where $\underline{Y}_{A,}$ $\underline{Y}_{B,}$ $\underline{Y}_C$ are complex phase admittances.

The phase currents may be determined by Eq. (5.9) as

$\underline{I}_A = (\underline{E}_A - \underline{U}_N)\cdot\underline{Y}_A$, $\underline{I}_B = (\underline{E}_B - \underline{U}_N)\cdot\underline{Y}_B$, $\underline{I}_C = (\underline{E}_C - \underline{U}_N)\cdot\underline{Y}_C$ The voltages $\underline{U}_{Aa}, \underline{U}_{Bb}, \underline{U}_{Cc}$ in line wires and the phase voltages $\underline{U}_{ab}, \underline{U}_{ca}$ are determined in a common way.

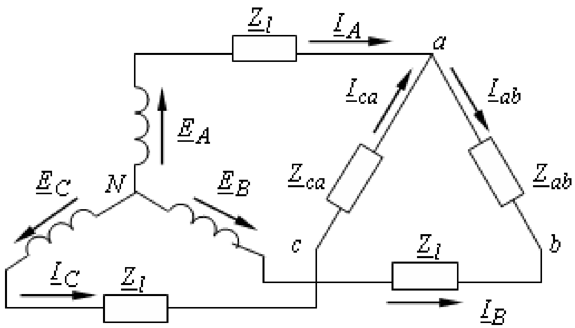

Fig. 5.15. An open circuit in the phase *bc*.

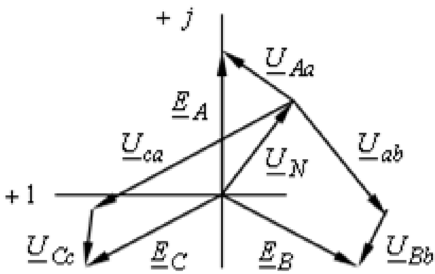

Fig. 5.16. A vector diagram for open circuit rate.

This diagram is drawn for the unbalanced case when $\underline{Z}_{ab} \neq \underline{Z}_{ca}$.

When we have a short circuit in the phase *bc*, a delta-connected circuit will change into asymmetrical wye connection without a neutral wire. Such a connection is depicted in Fig. 5.17. Hence, it appears the neutral voltage shift between the two neutral nodes *N* and *a*:

$$\underline{U}_N = \frac{\underline{E}_A\underline{Y}_A + \underline{E}_B\underline{Y}_B + \underline{E}_C\underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C},$$

where $\underline{Y}_A = \dfrac{1}{\underline{Z}_l + \dfrac{\underline{Z}_{ab} \cdot \underline{Z}_{ca}}{\underline{Z}_{ab} + \underline{Z}_{ca}}} = \dfrac{1}{\underline{Z}_l + \underline{Z}_{abc}}$; and $\underline{Y}_B = \underline{Y}_C = 1/\underline{Z}_l$ are phase complex admittances.

Knowing the neutral voltage shift, one can determine current.

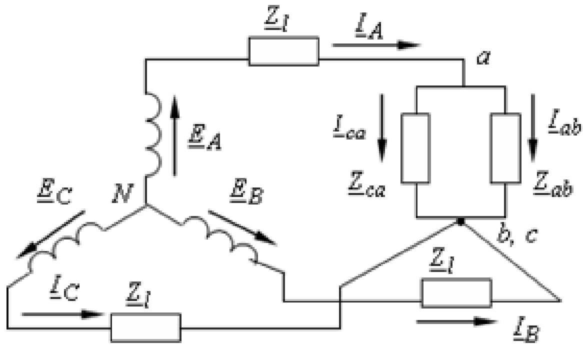

Fig. 5.17. The circuit with a short circuit in the phase *bc*.

Phase currents of the wye connection are found by Ohm's law:

$$\underline{I}_A = \frac{\underline{E}_A - \underline{U}_N}{\underline{Z}_l + \dfrac{\underline{Z}_{ab} \cdot \underline{Z}_{ca}}{\underline{Z}_{ab} + \underline{Z}_{ca}}} = (\underline{E}_A - \underline{U}_N) \cdot \underline{Y}_A$$

The currents in two parallel branches are determined by using so-called "resolving of the total current" (See Ch. 3, eq. 3.32):

$$\underline{I}_{ab} = \underline{I}_A \frac{\underline{Z}_{ca}}{\underline{Z}_{ca} + \underline{Z}_{ab}}; \qquad \underline{I}_{ca} = \underline{I}_A \frac{\underline{Z}_{ab}}{\underline{Z}_{ca} + \underline{Z}_{ab}}$$

When there is an open circuit in a line wire, a three-phase delta connection has become an usual one-phase circuit.

Let's consider the case when the line wire *Aa* is broken in the circuit with a balanced load. The circuit for this case is shown in Fig. 5.18, *a*. So, now we have the one-phase circuit with the input voltage $\underline{U}_{BC}$. For this case the currents across line impedances can be determined by the formula

$$\underline{I}_B = -\underline{I}_C = \frac{\underline{U}_{BC}}{\underline{Z}_{eq}}, \text{ where } \underline{Z}_{eq} = 2\underline{Z}_l + \frac{\underline{Z}_{bc} \cdot (\underline{Z}_{ab} + \underline{Z}_{ca})}{\underline{Z}_{bc} + \underline{Z}_{ab} + \underline{Z}_{ca}}.$$

Then we can calculate the currents through phase impedances:

$$\underline{I}_{bc} = \frac{\underline{U}_{bc}}{\underline{Z}_{bc}}; \quad \underline{I}_{ab} = \underline{I}_{ca} = \frac{\underline{U}_{cb}}{\underline{Z}_{ab} + \underline{Z}_{ca}}$$

where $\underline{U}_{bc} = -\underline{U}_{cb} = \underline{I}_B \cdot \dfrac{\underline{Z}_{bc} \cdot (\underline{Z}_{ab} + \underline{Z}_{ca})}{\underline{Z}_{bc} + \underline{Z}_{ab} + \underline{Z}_{ca}}$.

Now we can find voltages across series impedances $\underline{Z}_{ab}, \underline{Z}_{ca}$:

$$\underline{U}_{ca} = \underline{U}_{ab} = \frac{\underline{U}_{bc}}{2}.$$

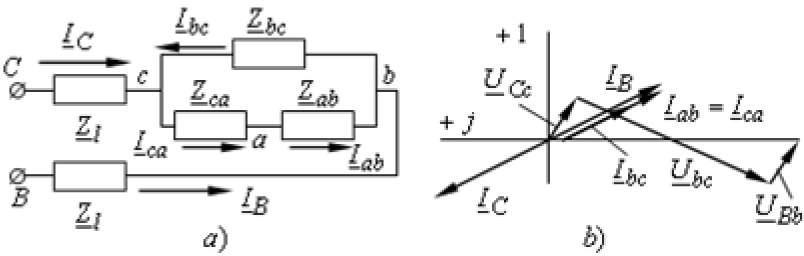

Fig. 5.18. An open circuit in the line wire *Aa*.

The vector diagram for this case is shown in Fig. 5.18, *b*.

5.9 Active, Reactive and Apparent Power in 3-phase System

The instantaneous power for a single-phase sinusoidal source varies itself sinusoidally at twice the frequency of the source. The expression for the single-phase sinusoidal source can be applied to each phase of the three-phase system.

The active power of a three-phase system is the sum of the active powers in each phase plus the active power dissipated across the resistance of a neutral wire (if it is not equal zero):

$$P = P_A + P_B + P_C + P_0 \tag{5.19}$$

where P_O is the active power in the resistance of a neutral wire.

The reactive power is the sum of the reactive powers in each phase plus the reactive power in the reactance of a neutral wire:

$$Q = Q_A + Q_B + Q_C + Q_0 \tag{5.20}$$

where Q_O is the reactive power in the reactance of a neutral wire.

The apparent (or total) power

$$S = \sqrt{P^2 + Q^2} \tag{5.21}$$

With a balanced load

$$P = 3U_p I_p \cos\varphi_p = \sqrt{3} U_L I_L \cos\varphi_p,$$
$$Q = 3U_p I_p \sin\varphi_p = \sqrt{3} U_L I_L \sin\varphi_p, \qquad (5.22)$$
$$S = 3U_p I_p = \sqrt{3} U_L I_L$$

where φ_p is the angle between phase voltage U_p and phase current I_p; U_L and I_L are line voltage and line current, respectively.

The equation for the power in a three-phase system is the same either for a wye or a delta connection when the power is expressed in terms of line quantities.

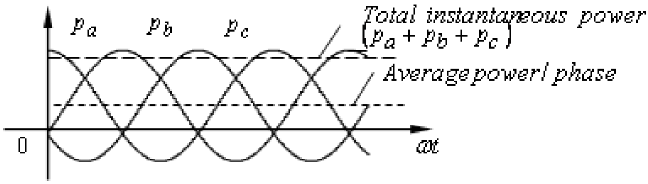

Fig. 5.19.

Before leaving this discussion on three-phase power, let us emphasize the fact that the total instantaneous power for the three-phase system is a constant as illustrated by Fig. 5.19. This stands in sharp contrast to the single-phase case where the single-phase power pulsates at twice the line frequency. Herein, then, lies another significant advantage of the three-phase system.

5.10 The Advantages of Three-phase System

The popularity of three-phase systems can be explained by the three principal advantages they offer, namely:

(a) over long distances it is more economical to transmit alternating current power three-phase then with any other number of phases;

(b) the components of three-phase systems, such as three-phase induction motors and three-phase transformers, are simple to manufacture and economical and reliable in service;

(c) given certain conditions, including a balanced load on the phases, the instantaneous power of system remains unchanged over a period of the sinusoid.