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APPROXIMATION OF FUNCTIONS OF TWO VARIABLES ON THE UNIT BICIRCLE

The approximative properties of interpolational Lagrange polynomials, Fourier series of functions of two variables which are defined on the unit bicircle, is stated. For functions from generalized Hölder spaces H_ω the estimations are conducted for norms of $C,\ H_\omega,\ L_p,\ p>1$ spaces [1].

Let γ_{01} and γ_{02} denote the unit circles with centers in the origin; and $\gamma_0=\gamma_{01}\times\gamma_{02}$ is the unit bicircle. Let $\omega(\delta_1,\delta_2)$ be the some modulus of continuity, and $\Omega_1(\delta)$, $\Omega_2(\delta)$ are the simple moduli of continuity, which respect to that one; $\omega_{1,1}$ denotes the mixed modulus of continuity of the second order [2]. Let H_ω denotes the space of continuous functions $x(t,\tau)$ at γ_0, which satisfy the conditions:

$$H(x;\omega)=\sup_{\delta_1^2+\delta_2^2\neq 0}\frac{\omega(\delta_1,\delta_2;x)}{\omega(\delta_1,\delta_2)}\le c_1,\quad H^{t\tau}(x;\omega)=\sup_{\delta_1^2+\delta_2^2\neq 0}\frac{\omega_{1,1}(\delta_1,\delta_2;x)}{\Omega_1(\delta_1)\Omega_2(\delta_2)}\le c_2 .$$

In the space H_ω the norm is defined as:

$$\|x(t,\tau)\|_{H_\omega}=\|x(t,\tau)\|_C+H(x;\omega)+H^{t\tau}(x;\omega).$$

Let $(L_{mn}x)(t,\tau)=\sum_{p=0}^{2m}\sum_{q=0}^{2n}x(t_p,\tau_q)l_p(t)l_q(\tau)=\sum_{k=-m}^{m}\sum_{l=-n}^{n}\lambda_{kl}t^k\tau^l$,

$$l_p(t)=\prod_{\substack{k=0\\k\neq p}}^{2m}\frac{t-t_k}{t_p-t_k}\left(\frac{t_p}{t}\right)^m=\sum_{k=-m}^{m}\lambda_{1,k}^{(p)}t^k,\quad l_q(\tau)=\prod_{\substack{j=0\\j\neq q}}^{2n}\frac{\tau-\tau_j}{\tau_q-\tau_j}\left(\frac{\tau_q}{\tau}\right)^n=\sum_{j=-n}^{n}\lambda_{2,j}^{(q)}\tau^j,$$

denotes interpolational Lagrange polynomial of the function $x(t,\tau)$ with respect to the system of equidistant points at γ_0:

$$t_p=exp\left\{\frac{2\pi i}{2m+1}(p-m)\right\},\ i^2=-1,\ p=0,\dots,2m;$$

$$\tau_q = exp\left\{\frac{2\pi i}{2n+1}(q-n)\right\},\ i^2 = -1,\ q = 0,\dots,2n$$

Let operator Φ_{mn} assignes to any function $x(t,\tau)\in H_\omega$ the mn-th partial sum of its Fourier series, that is,

$$(\Phi_{mn}x)(t,\tau) = \sum_{k=-m}^{m}\sum_{l=-n}^{n} x_{kl}t^k\tau^l\ ;\ \ x_{kl} = \frac{1}{4\pi^2}\int_{\gamma_{01}}\int_{\gamma_{02}} \frac{x(t,\tau)}{t^{k+1}\tau^{l+1}}dtd\tau\ ,\ k,l = 0,\pm 1,\pm 2,\dots.$$

Theorem 1. For any function $x(t,\tau)\in H_\omega$ and for arbitrary natural m, n the unequality is fulfilled:

$$\|x - L_{mn}x\|_C \le clnmlnnH(x;\omega)\omega\left(\frac{1}{m},\frac{1}{n}\right);$$

$$\|x - \Phi_{mn}x\|_C \le (1 + cln(2m+1)ln(2n+1))H(x;\omega)\omega\left(\frac{1}{m},\frac{1}{n}\right).$$

Theorem 2. Let $x(t,\tau)\in H_{\omega^{(1)}}$; $\omega^{(2)}(\delta_1,\delta_2)$ is such that $H_{\omega^{(1)}} \subset H_{\omega^{(2)}}$ and $\Omega_1^{(1)}(\delta)/\Omega_1^{(2)}(\delta)$, $\Omega_2^{(1)}(\delta)/\Omega_2^{(2)}(\delta)$ are increasing functions [3].Then:

$$\|x - L_{mn}x\|_{H_{\omega^{(2)}}} \le clnmlnn\frac{\omega^{(1)}\left(\frac{1}{m},\frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)}\|x\|_{H_{\omega^{(1)}}};$$

$$\|x - \Phi_{mn}x\|_{H_{\omega^{(2)}}} \le clnmlnn\frac{\omega^{(1)}\left(\frac{1}{m},\frac{1}{n}\right)}{\Omega_1^{(2)}\left(\frac{1}{m}\right)\Omega_2^{(2)}\left(\frac{1}{n}\right)}\|x\|_{H_{\omega^{(1)}}}.$$

Theorem 3. Let $x(t,\tau)\in H_\omega$. Then the estimation is fulfilled ($p > 1$):

$$\|x - L_{mn}x\|_{L_p} \le (1 + c(p))H(x;\omega)\omega\left(\frac{1}{m},\frac{1}{n}\right),$$

where $\|L_{mn}\|_{C\to L_p} \le c(p)$

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