Ministry of Education and Science of Ukraine

ZAPORIZHZHA NATIONAL TECHNICAL UNIVERSITY

CONTACTS AND ELECTRODYNAMIC FORCES IN ELECTRICAL APPARATUS

Methodical instructions for the laboratory works on the subject:
“Fundamentals of the Theory of Electrical Apparatus”

for the students of the specialty 141:
"Power, Electrical and Electromechanical Engineering"
(educational program **“Electric and Electronic Apparatus”**

2019

Contacts and electrodynamic forces in electrical apparatus: Methodical instructions for the laboratory works on the subject: “Fundamentals of the Theory of Electrical Apparatus” for the students of the specialty 141:"Power, Electrical and Electromechanical Engineering" (educational program “Electric and Electronic Apparatus”) / Comp.: O.V. Blyzniakov. – Zaporihzhza: ZNTU, 2019. – 27 p.

The compiler: O. V. Blyzniakov, docent.

The reviewer: O. I. Afanasiev, docent.

The English text editor: Bondarenko O.M.

Responsible for the issue: R. E. Mokchnach.

Approved at the meeting of the department
"Electric and electronic apparatus".
Minutes No. 9 dated April 2, 2019

Recommended for publication by
educational-and-methodical commission
of the Electrical Engineering faculty.
Minutes No. 9 dated April 18, 2019

CONTENTS

1. LABORATORY WORK № 8.
STUDY OF TRANSITIONAL CONTACT RESISTANCE

Duration of the laboratory study is 4 hours

1.1 The purpose of the work

The purpose of the work is to experimentally determine the de-pendency transitional contact resistance vs pressing force for contacts of the same shape (half-sphere–half-sphere) made of different materials (copper-copper, brass–brass, aluminum–aluminum, copper–copper with silver electroplating), as well as to determine the experiment error.

1.2 Subject of the study

An electrical contact is the place where the current passes from one conductor to another. The presence of a contact always results in the appearance of an additional resistance called contact resistance. ГОСТ 14312-79 defines the contact resistance as the electrical resistance of the contact zone, which is determined by the real contact area and is equal to the ratio of the voltage drop across the contact to the current through it [13].

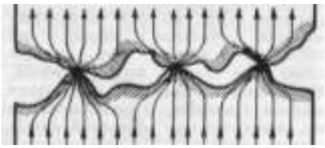

Figure 1.1 – Distribution of the current lines near the contact interface

The presence of contact resistance is caused by the fact that contact surfaces is not absolutely plane. The real surfaces, as a rule, have roughness, waviness and macro-unevenness. That is why the contact faces are always in contact only at separate points, as shown in Figure 1.1.

In the process of forming an electrical contact, the two surfaces first in contact in the places of large projections that deformed. Under increasing the force, more and more micro- roughness come into contact. This process is over when the total reaction of the deformed micro-roughness becomes equal to the contact load. The micro-roughness deformation results in formation small surfaces that focus the passage of the current (so-called *α-spots*). In the neighborhood of the contact interface, the current lines curve and meet at the contacting areas where the current density significantly increases that is the main cause of the presence of contact resistance. Beyond that, contact resistance will be determined by the presence of various types of films caused by the interaction between the contact material and the environment.

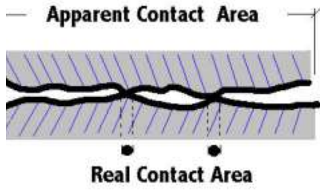

Figure 1.2

It is distinguished *geometric* or *apparent contact area* outlined by the dimensions of the contacting bodies, that is, the contour area of the touch formed by the volumetric bearing stress of the macro-roughness, and the *effective* or *real contact area*, which represents the total area of the actual contact and comprises only insignificant part of the geometric area, Figure 1.2 [1–9].

Consequently, the contact resistance in the general case has two components:

$$R_{к} = R_{с} + R_{пл}, \qquad (1.1)$$

where $R_{с}$ is the *constriction resistance* caused by the bend (meet) of the current lines at the region near the contact interface called *constriction region*;

$R_{пл}$ is the *film resistance* caused by the presence of pollutions and various films at the contact surfaces.

According to an elliptical contact model, the constriction resistance is evaluated by the Holm's formula [1–9]:

$$R_{с} = \frac{\rho}{2an}, \qquad (1.2)$$

where ρ is the resistivity of the contact's material;

n is the number of contact areas.

Equation (1.2) was derived by R. Holm under assumption that the contact areas are far from each other (that is, a mutual influence on the pattern of spreading out the current lines is absent), as well as in the absence of the skin and thermoelectric effects.

Under condition of the plastic deformation of the contact surface, the Holm formula has the following form:

$$R_{с} = \frac{\rho}{2}\sqrt{\frac{\pi[\sigma_{зм}]}{nP_{к}}}, \qquad (1.3)$$

where $[\sigma_{зм}]$ is the ultimate bearing strength under plastic deformation, N/m^2;

$P_{к}$ is the force of the contact pressure or the contact load, N.

The number of contacting areas depends on the contact load, as well as the shape of the contact surfaces. Theoretical and experimental researches show that the number of contact areas is a random value being defined by the probability of the encounter of the micro-roughnesses of two contact areas under action of the contact load. That is why naturally that the constriction resistance value after each closing will take different values at constant contact load. The spread of these values is characterized by the parameter, which is called the *static instability* of the constriction resistance. It is determined as the mean square deviation of the constriction resistance values. When the contact loads are sufficiently great, the distribution of its values for the same contact pair comply with normal law.

The nature of static instability is explained by the possibility to compensate the same contact load, when closing the contacts, by the reaction of different number of deformed micro-roughnesses. In the process, the minimal value of the resistance will be in the case when the most number of the micro-roughnesses are deformed by a value that is somewhat more that the film thickness, and vice versa, when the most number of micro-roughnesses are deformed by a value that is somewhat less than the film thickness.

Besides the notion of static instability, there is also the notion of the *dynamic instability* of contact resistance. It take place when the apparatus undergoes external exposures, mainly mechanic (impact, vibration) loads.

Another cause of the contact resistance existence is a variety of films available at the contact surfaces. First, so-called *tarnish films* that usually refer to dielectric films. There are two basic variations of tarnish films: oxide ones formed because of the compound of metal with air oxygen, and sulfide ones (the compound of metal and sulfur). Tarnish films have usually uneven thickness (of the order of 10^{-6} sm); their resistivity is in the range of 10^4–10^9 Ohm·sm.

Another variation of superficial films is so-called adhesive films that is formed due to absorption and adhesion, that is, taking up and deposition of substances, frequently having a high resistivity, on contact surfaces [7, 8]. Films of this type usually have a relatively small thickness (2.5–3 nm), but withstand significant mechanical loads and do not fail under closing the contacts. Due to the action of the tunnel effect, they are not isolator for an electric current circuit. However, if their thickness becomes larger, the tunnel effect stops, and the film becomes a dielectric.

There is one more variety of superficial films – this is the so-called passivating films. In terms of conductivity, they occupy an intermediate position between the tarnish and adhesive films. These films are often artificially formed to eliminate corrosion on the metal surface. Their thickness is more or less constant and is 1.0–1.5 nm, so they, like adhesive films, are not insulators due to the tunnel effect.

Consequently, the real contact area or α-spot in the general case contains three types of regions. First, these are the areas where the contact is carried out through organic films and tarnish films (oxide, sulfide, etc.), which refer to isolating (practically non-conductive) films. Secondly, these are areas where the contact is carried out through thin films (adhesion and passivation) having tunnel conductivity (regions of quasi-metallic contact). Thirdly, these are the regions of purely metallic contact. It should be noted that at the regions of purely metallic contact, the great forces exert because of intermolecular and interatomic bonds, therefore it is carried out only on small regions; otherwise, the forces that take place in the real apparatus could not break such a contact. Consequently, the presence of film on the metal surface plays a positive role, not allowing the development of large forces of intermolecular bonds. In the process of contact functioning this pattern is constantly changing, since the old conduction cells disappear, new ones are created, etc.

The presence of superficial films significantly affects only the functioning low-current contacts (less than 5 A) at small contact loads (less than 0.1 N). Therefore, such contacts are made of noble metals, which are weakly oxidized and have a small resistivity. Superficial films designed for high currents are destroyed either due to large contact load or self-cleaning due to sliding one contact relative to the other.

Wide variety of processes resulting to the formation of superficial films, as well as a rather complex structure of the contact spot, do not enable, with sufficient accuracy, to determine analytically the contact resistance, taking into account the resistance of the superficial films. Therefore, in practice, this is taken into account mainly by the way of experimental studies, methods of mathematical statistics etc.

Quite often, the following empirical formula is used to calculate the contact resistance [1–9]:

$$R_{к} = R_0 + \frac{\kappa}{\left(0{,}102 P_{к}\right)^{m}}, \qquad (1.4)$$

where R_0, κ are the constants that depend on the material and condition of the contact face;

m is the index of a power, which depends on the number of contact spots (i.e., the shape of the contact face) and ranges from 0.5 to 1.

It should be noted that the contact resistance also depends on the roughness of the contact face. For example, polishing of a contact face leads to the formation of more gently sloping micro-roughnesses of large sizes. They are squashed only with the great pressing forces, and the oxide film is weakly destroyed on the contact face. Therefore, the resistance of the contacts formed by the polished faces is significantly higher than the contacts formed by the surfaces with a rougher machining.

In the calculations of the contact resistance according to formulas (1.3) and (1.4), it is necessary to take into account the dependency the contact material resistance vs the temperature. In the process, the temperature distribution within the contact constriction region should be taken, but its determination is a rather complicated task. In approximate calculations, it is assumed that the average temperature in the constriction region is approximately equal to 2/3 of the maximum temperature, that is, the temperature at the contact spot $\vartheta_{\text{к.пл}}$ [1]. Thus, the resistivity will be determined by the following expression:

$$\rho = \rho_0 \left(1 + \frac{2}{3} \alpha \vartheta_{\text{к.пл}} \right), \tag{1.5}$$

where ρ_0 is the resistivity of the contact material at 0°C, Ohm·m

α is the temperature coefficient of resistance of the contact material, 1/K;

The constriction resistance is determined by the same manner [1]:

$$R_c = R_{c0} \left(1 + \frac{2}{3} \alpha \vartheta_{\text{к.пл}} \right), \tag{1.6}$$

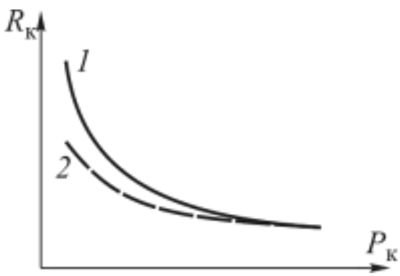

Figure 1.3 – Dependency the contact resistance vs the contact load

The dependency the contact resistance vs the contact load behaves usually as hyperbolic curve, Figure 1.3 [1]. At the small loads region (decimal fractions of Newton) causing only elastic deformations the contact conductance is carried out only due to tunnel effect, therefore the contact resistance is significant. Under increasing the contact load the contact resistance substantially reduces due to the plastic deformation that occurs with the isolating films are destructed and great number of contact spots are formed (curve 1, see Figure 3.1). When the number of contact spots increases, the load per one contact spot reduces, and under further increasing the contact load the contact resistance moderately depends upon the contact load. When the contact load decreases, the contact resistance increases according to curve 2 (see Figure 3.1), since under plastic deformation the micro-ridges are squashed.

1.3 Description of the laboratory plant

The dependency the contact resistance vs contact load is derived with the help of the experimental setup shown in Figure 1.4. The electrical circuit of the laboratory plant is shown in Figure 1.5.

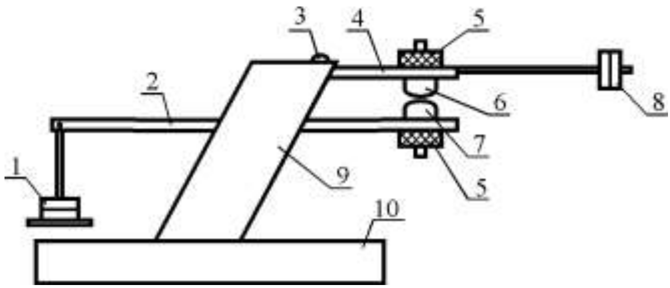

Figure 1.4 – Sketch of the experimental setup

The contacts to be studied 6 (fixed contact) and 7 (movable contact) are fastened on strips 4 and 2, respectively, which are fastened between side plates 9 mounted on isolating basement 10. Counterweight 8 serve to compensate the gravity force of the contacts. Desired contact load is produced with the help of balance weights 1. The relationship of the arms in the setup comprises 1:5. The fixed contact is substituted in the following order: screw 3 is unscrewed; the flexible connection of the contact is disabled so that isolating washer 5 would not be lost; strip 4 is taken out; contact 6 is unscrewed. Movable contact 7 is disabled from the conductor incoming the current and, at balance weights and lowered down, strip 2 the contact is substituted. After the substitution of contact 6 strip 4 is fastened by screw 3 between plates 9. The conductors incoming the current are connected after insulating washers 5 are mounted.

Contact resistance is measured by the method of comparing the voltage drop across the contacts to be tested BK and across the reference resistance (shunt) $R_{ш}$ under flowing the same in value current through them. The voltage drop is measured by millivoltmeter PV. Relays K1 and K2 provide protection of the millivoltmeter against sudden increase of voltage when the contacts are opened. The installation is connected to the power supply by the switch SF, the ammeter PA1 controls the current in the contacts being tested or in the shunt, depending on the position of switch S2. The purpose of resistor R is to limit the current through the contacts.

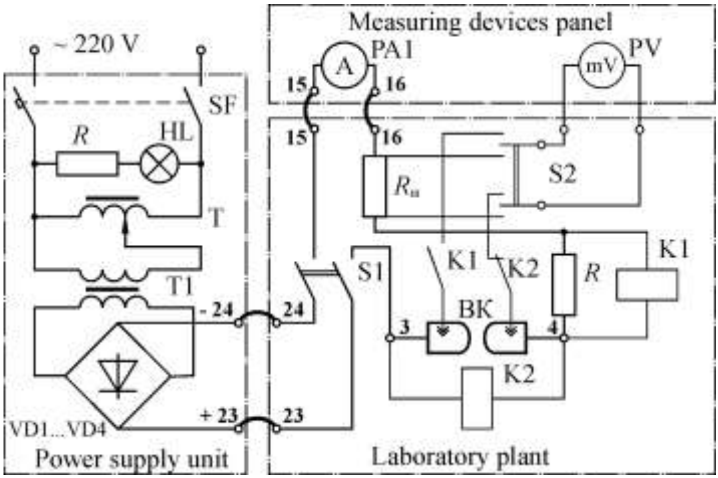

Figure 1.5 – Electric circuit for the experimental study

1.4 Task

1.4.1. Measure the dependency the contact resistance vs the contact load for several contact pairs from different materials (copper-copper, aluminum-aluminum, brass-brass, silver-silver, etc.).

1.4.2. Calculate the contact resistance according to formula (1.3) and empirical formula (1.4).

1.4.3. Construct the experimental and calculated dependencies the contact resistance vs the contact load on the one graphic and perform their comparative analysis.

1.4.4. Perform statistical processing of the experimental findings.

1.4.5. Draw the conclusions on the work.

1.5 Methodical instructions

1.5.1. The experimental part of the work is performed in the following order:

a) install one of the contact pairs according to item 1.4.1 (see subsection 1.2);

b) construct the test circuit according to Figure 1.5;

c) dress the contact faces with abrasive paper and wipe them off;

d) position the shunt to a given current limit;

e) close automatic circuit breaker SF and, using the autotransformer T and the indications of instrument PA1, adjust the current according to the instruction of the teacher;

f) in the absence of balance weights 1 (see Figure 1.4), with the help of the counterweight 8, adjust the zero pressure determined by the presence of the current through the contacts circuit (on the indications of instrument PA1);

g) install the load (using the trial-and-error method), which corresponds to the maximum voltage drop across the BK for this millivoltmeter (readings of millivoltmeter is determined under momentary placing S2 in the position BK);

h) consistently increase the load with the help of calibrated balance weight 1; for each value of the contact load, conduct at least 5 measurements of voltage drop at intervals of one to two seconds, removing and applying the voltage (excluding the load corresponding to the maximum $U_к$); the findings are included in the table. 1.1;

Table 1.1 – Experimental values of the parameters

$P_к$, N	№№ дослідів	Матеріал контактів											
		Cu-Cu			Al-Al			Ag-Ag			Латунь-Латунь		
		$U_к$, mV	$U_ш$, mV	$R_к$, Ω	$U_к$, mV	$U_ш$, mV	$R_к$, Ω	$U_к$, мВ	$U_ш$, мВ	$R_к$, Ω	$U_к$, мВ	$U_ш$, мВ	$R_к$, Ω
	1												
	2												
	3												
	4												
	5												
. . .													
	1												
	2												
	3												
	4												
	5												

j) if necessary, change to another measuring range, set switch S2 to neutral position;

k) open switch S1 and automatic circuit-breaker SF;

l) complete the task in the sequence described above.

1.5.2. Calculate the contact resistance with the following formula:

$$R_{к} = R_{ш} \frac{U_{к}}{U_{ш}}, \qquad (1.6)$$

where $U_{ш}$ is the voltage drop across the shunt, mV;

$R_{ш}$ is the resistance of the shunt.

1.5.3. The computed value $R_{к}$ is written in Table 1.1.

1.5.4. Process the experimental findings according to the procedure given in appendix A. Enter the derived findings into table. 1.2.

Table 1.2 – The results of processing the experimental findings

Quantities	$P_{к}$ =					$P_{к}$ =						$P_{к}$ =				
	1	2	3	4	5	1	2	3	4	5		1	2	3	4	5
$R_{кi}$																
$\overline{R}_{к}$																
$R_{кi} - \overline{R}_{к}$											…					
$\left(R_{кi} - \overline{R}_{к}\right)^2$																
s^2																
s																
t_{α}																
ε_0																
$R_{к}$																

1.5.5. Calculate contact resistance by the formula (1.4), using the values $P_{к}$ from Table. 1.1 and write the obtained findings into Table. 1.3. The magnitudes κ for contacts from different materials are given in Table. 1.4.

1.5.6 Experimental and calculated dependencies should be built on one graph.

1.5.7 The laboratory report should include:

a) electrical circuit of the study;

b) tables with results of experiments and calculations;

c) experimental and calculation graphs of the dependency the contact resistance vs the contact load;

d) conclusions on the work.

Table 1.3 – Calculated values of the contact resistance

$P_к$, N	Materials of the contacts							
	Cu-Cu		Al-Al		Ag-Ag		Латунь-Латунь	
	$R_{к.е}$, mΩ	$R_{к.р}$, mΩ	$R_{к.е}$, mΩ	$R_{к.р}$, mΩ	$R_{к.е}$, mΩ	$R_{к.р}$, mΩ	$R_{к.е}$, mΩ	$R_{к.р}$, mΩ

Table 1.4 – The magnitudes of coefficient κ for butt and flat contacts, $N^{1/2}\cdot\Omega$

Material of contacts				
Al-Al	Cu-Cu	Ag-Ag	Brass–brass	Steel–steel
$3\cdot10^{-4}$	$3{,}16\cdot10^{-4}$	$1{,}58\cdot10^{-4}$	$21{,}2\cdot10^{-4}$	$24\cdot10^{-4}$

1.6 Self-examining questions

1.6.1 What are the main causes of the transitional contact resistance?

1.6.2 What is the apparent (geometric) contact area?

1.6.3 What is the effective (real) contact area?

1.6.4 What components has the value of contact resistance?

1.6.5 How behave the dependencies the transient contact resistance vs such factors as contact pressure, material properties, contact area, contact shape, contact manufacturing technology, temperature.

1.6.6 What is the static and dynamic instability of the contact resistance?

1.6.7 How does the contact face roughness affect the contact resistance?

1.6.8 What types of films are formed on the contact surface?

1.6.9 How does the presence of superficial films affect the value of the contact resistance?

2. LABORATORY WORK № 11. **STUDY OF ELECTRODYNAMIC FORCES**

Duration of the laboratory study is 4 hours

2.1 The purpose of the work

The purpose of the work is to master the methods of theoretical and experimental research of electrodynamic forces produced in electric apparatus.

2.2. Subject of the study

Electrodynamic forces (EDF) are resulted from electromagnetic interaction of conductors with currents. These forces are caused by the Lorentz force. It is well known that a charge, moving with velocity V in a magnetic field with induction B, is exerted by the force defined by the following expression:

$$\overline{F} = q\left[\overline{V} \times \overline{B}\right], \tag{2.1}$$

where $\left[\overline{V} \times \overline{B}\right]$ is the vector product of velocity $\overline{V}$ into flux density $\overline{B}$.

An elemental conductor *dl* with current *I* includes a charge:

$$dq = Idt \text{ або } dq = I\frac{dl}{V}. \tag{2.2}$$

Consequently, the following equality will be valid:

$$dq \cdot V = I \cdot dl \tag{2.3}$$

Thus, the force exerting the elementary conductor will be determined by the following expression:

$$dF = I \cdot \overline{dl} \times \overline{B}. \tag{2.4}$$

An elemental conductor is in this case a vector $\overline{dl}$ whose direction coincides with the direction of the current in it. Consequently, in the simplest case, the straight-line conductor *l*, placed in a homogeneous magnetic field, is exerted by a force:

$$F = I \cdot l \cdot B \cdot \sin\alpha, \tag{2.5}$$

where α is the angle between the direction of the conductor current and the direction of the flux density vector.

An experience and calculations show that when the device is in nominal operation, that is, under a nominal load, EDF is very small. However, in the event of an emergency situation, namely a short circuit, when the current-carrying elements of the apparatus are under flowing currents comprising tens of kiloamperes, the magnitudes of EDF become so much that they can destruct the apparatus constructive elements. Consequently, the electric apparatus must have the appropriate *electrodynamic stability*, that is, the ability to withstand the action of the EDF under short-circuit conditions. Thus, the calculation of EDF is a very important practical task.

There are two basic methods to determine EDF: the first one is the application of the *Biot-Savart law*, and the second is using the change of electromagnetic energy store.

Let us consider the use of the Biot-Savart law by the example of electromagnetic interaction between two conductors of arbitrary shape of infinitely small cross-section l_1 and l_2 with currents i_1 and i_2 arranged arbitrarily relative to each other. The task in this case is to determine $F_{1/2}$, that is, the force exerting on conductor l_1 from conductor l_2, which produces the magnetic field.

According to the Biot-Savart law (formula 2.4), the elemental conductor dl_1 undergo the force:

$$dF_{1/2} = B_2 \cdot dl_1 \cdot i_1 \cdot \sin\alpha , \qquad (2.6)$$

where B_2 is the flux density produced by the conductor l_2, which exert to elemental conductor dl_1.

To determine the flux density B_2, we should first determine the elementary induction dB_2 produced by the elementary conductor dl_2 that is expressed as follows:

$$dB_2 = \frac{\mu_0}{4\pi} \cdot \frac{i_2 \cdot dl_2 \cdot \sin\beta}{r^2} , \qquad (2.7)$$

where β is the angle between the direction of the current in elemental conductor dl_2 and і променем r, що визначає відстань між елементарними провідниками dl_1 та dl_2.

Flux density B_2 will be determined by the line contour integral:

$$B_2 = \frac{\mu_0 \cdot i_2}{4\pi} \cdot \int_{l_2} \frac{dl_2 \cdot \sin\beta}{r^2} . \qquad (2.8)$$

Hence, EDF exerting all conductor l_1 will be expressed as follows:

$$F_{1/2} = \frac{\mu_0 \cdot i_1 \cdot i_2}{4\pi} \cdot \int_{l_1} \int_{l_2} \frac{dl_1 \cdot dl_2 \cdot \sin\alpha \cdot \sin\beta}{r^2} . \qquad (2.9)$$

As can be seen from this expression, the magnitude of EDF is determined by the product of the currents of the interacting conductors and the double line contour integral that depends on the geometric parameters, namely, the size, shape and the relative arrangement of the conductors. This integral is called the *geometric factor of electrodynamic forces*:

$$k_C = \int_{l_1}\int_{l_2} \frac{dl_1 \cdot dl_2 \cdot \sin\alpha \cdot \sin\beta}{r^2} \quad (2.10)$$

Consequently, the formula for determining the EDF will have the following form:

$$F_{1/2} = \frac{\mu_0 \cdot i_1 \cdot i_2}{4\pi} \cdot k_C = 10^{-7} i_1 \cdot i_2 \cdot k_C \quad (2.11)$$

In using this method (i.e., Biot-Savart law), the direction of action of the EDF is determined by the left-hand rule. In the process, the direction of the flux density vector is determined by the right-hand screw rule. Quite often, to determine the direction of the EDF, so-called the *Mithkevich's rule* is used. Its essence consists in the fact that EDF is always directed to the side of weakened magnetic field.

The second method for determining the EDF is based on using the energy balance equation for the conductor or conductor systems with current.

If the electrostatic energy of the conductor system is neglected, as well it is assumed that under its deformation or travel of its individual elements (under the EDF action), the currents remain invariable, then the magnitude of the EDZ will be determined by the partial derivative of the electromagnetic energy of the given conductor system with respect to the coordinate in the direction of EDF action:

$$F = \frac{\partial W}{\partial x}, \quad (2.12)$$

where W is the electromagnetic energy of the conductor system;

x is the possible movement in the direction of EDF action.

The electromagnetic energy of the conductor system consists of the sum of electromagnetic self-energies of the conductors of the system, and the electromagnetic energies caused by the magnetic coupling of the individual conductors among themselves. For example, for a system consisting of three magnetically coupled conductors with currents i_1, i_2, and i_3, the total electromagnetic energy will be determined by the following expression:

$$W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + \frac{1}{2}L_3 i_3^2 + M_{12} i_1 i_2 + M_{13} i_1 i_3 + M_{23} i_2 i_3, \quad (2.13)$$

where L_1, L_2, L_3, are the internal inductances of the conductors;

M_{12}, M_{13}, M_{23} are the mutual inductances between the corresponding conductors.

The first three terms determine their electromagnetic self-energy of the conductors, the rest – the electromagnetic energy caused by their magnetic coupling.

The equation represented enables us to calculate both the forces of interaction between the conductors and the forces acting within the conductor (i.e., the forces of interaction between individual its sections). Since the value of the current is invariable, the EDF exerting any conductor will be determined by the derivative of its self-inductance and mutual inductance in relation to other conductors with respect to corresponding coordinate. For example, for the above-discussed system of the three conductors, the EDF exerting the 1-st conductor will be defined by the following expression:

$$F = \frac{dW}{dx} = \frac{i_1^2}{2}\frac{dL_1}{dx} + i_1 i_2 \frac{dM_{12}}{dx} + i_1 i_3 \frac{dM_{13}}{dx} \qquad (2.14)$$

For a solitary conductor, that is, a conductor having not magnetic coupling with other conductors, the EDF will be determined only by the derivative of its self-inductance:

$$F = \frac{dW}{dx} = \frac{i^2}{2}\frac{dL}{dx}. \qquad (2.15)$$

This method is suitable to use in cases when the analytical dependency the inductance and mutual inductance vs the geometrical parameters of the considered conductor system is known.

2.3 Task

2.3.1 Experimentally determine the dependency the geometric factor of two parallel rectangular section buses vs the distance between them and construct its graph.

2.3.2 Calculate the dependency directed in the previous paragraph.

2.3.3 Experimentally determine the dependency the geometric factor of two circular rectangular section rings (coils) vs the distance between them and construct its graph.

2.3.4 Calculate the dependency directed in the previous paragraph.

2.4 Methodical instructions

2.4.1 Assemble the circuit according to Figure 2.1.

2.4.2 Close the circuit breaker SF and take the dependency the EDF (it is determined the value of the spring deformation) and the distance between the buses *a* vs the current load *I* that is adjusted with the help of transformer T1 (PHO). The initial distance between the buses is predetermined by the teacher. Enter the derived findings into table 2.1.

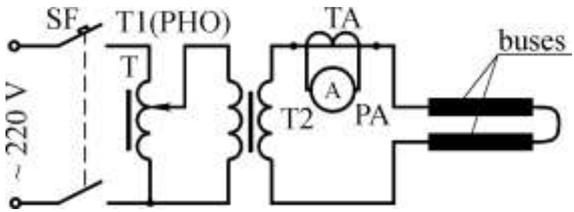

Figure 2.1 – Electric circuit for the study of EDF of flat buses

2.4.3 The experimental value of the geometric factor is determined by the following formula:

$$k_g = 10^7 F_{e/d} / (Iw)^2 ,$$

where w is the number of turns of the coils equal to 3500.

Table 2.1 – Dependency the geometric factor vs the distance between the buses

I, A						
$F_{e/d}$, H						
a, м						
k_g (experiment)						
k_g (calculation)						

2.4.4. The calculated value of the geometric factor is determined by the following formula [1–4]:

$$k_g = 2 \cdot 10^{-7} \frac{a \cdot l}{h^2} \left[2 \frac{h}{a} \operatorname{arctg} \frac{h}{a} - \ln\left(1 + \frac{h^2}{a^2}\right) \right],$$

where a is the distance between the buses axis;

l is the length of the buses that comprises 1025 mm;

b is width of the buses that comprises 80 mm.

2.4.5 Construct the experimental and calculated dependency the geometric factor vs the distance between the buses axis on one graph, compare them.

2.4.6 Assemble the circuit according to Figure 2.2. Connect the coils in parallel with each other and connect them to dc power supply (terminal 4 and 5 of the laboratory table.

2.4.7 Close the circuit breaker SF and take the dependency the EDF (it is determined the value of the spring deformation) and the distance between the buses vs the current load I that is adjusted with the help of transformer T1 (PHO). Enter the derived findings into table 2.1. The spring rate comprises 0,5 N/mm.

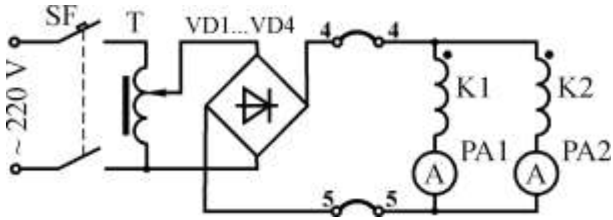

Figure 2.2 – Circuit for studying EDF of circular rings (coils)

2.4.8. Experimental value of the geometric factor of the coils should be determined by the following formula:

$$\kappa_{к} = 10^7 \frac{F_{е/д}}{(I_1 w_1)(I_2 w_2)},$$

where w_1, w_2 is the number of turns of the coils comprising 3500.

Table 2.2 – Dependency the geometric factor vs the distance between the coils

Current through 1-st coil I_1, A				
Current through 2-nd coil I_1, A				
Electrodynamic force $F_{е/д}$, N				
Distance between the coils x, m				
Geometric factor $\kappa_{к}$ (experimental)				
Geometric factor $\kappa_{к}$ (calculation)				

2.4.9. The calculated value of the geometric factor is determined by the following formula:

$$\kappa_{к} = 10^7 \frac{dM_{12}}{dx} \cdot \frac{1}{w^2},$$

where M_{12} is the mutual inductance of the coils that is determined according to the procedure represented in appendix B.

The derivative of the mutual inductance of the coils in respect to the distance between them should be determined using graphic differentiation.

2.4.10 The laboratory report should include:

a) the name and purpose of the work;

b) task and initial data for calculation;

c) the test circuit diagram;
d) calculations and calculation formulas;
c) graphs of experimental and calculated dependencies;
e) analysis of dependencies and conclusions from the work.

2.5 Self-examining questions

2.5.1 What are the calculation methods of electrodynamic forces exists?

2.5.2 In what cases and how is one or another calculation method of electrodynamic forces used?

2.5.3 How is the direction of action of electrodynamic forces determined?

2.5.4 What is the geometric factor of electrodynamic forces and what does it depend on?

2.5.5 How will the midvalue of electrodynamic force change if the DC source is substituted with an AC source?

2.5.6 What purpose the electrodynamic forces in electric apparatuses are calculated?

2.5.7 The notion of electrodynamic stability of an apparatus.

REFERENCE LIST

1. Клименко Б.В. Електричні апарати. Електромеханічна апаратура комутації, керування та захисту. Загальний курс: навч.посібник / Б.В. Клименко – Харків: Точка, 2012. – 340 с.

2. Основы теории электрических аппаратов: Учеб. для вузов/ [И.С. Таев, Б.К. Буль, А.Г. Годжелло и др.]; под ред. И.С. Таева. – М.: Высшая школа, 1987. – 352 с.

3. Теория электрических аппаратов/ [Г.Н Александров, В.В. Борисов, Г.С. Каплан и др.]; под ред. Г.Н. Александрова. – [2–е изд., перераб. и доп.] – СПб.: Изд-во СПбГТУ, 2000. – 540 с.

4. Справочник по расчету и конструированию контактных частей сильноточных электрических аппаратов / [Н.М. Адоньев, В.В. Афанасьев, В.Б. Борисов и др.]; под ред. В.В.Афанасьева. – Л.: Энергоатомиздат. – 384с.

5. Р. Хольм. Электрические контакты / Р. Хольм; пер. с англ. – М.: Издательство иностранной литературы, 1961. – 464 с.

6. Таев И.С. Электрические контакты и дугогасительные устройства аппаратов низкого напряжения / Таев И.С. – М.: Энергия, 1973. – 423 с.

7. Реут Е.К. Электрические контакты / Е.К. Реут, И.Н. Саксонов. – М.: Воениздат, 1971. – 160 с.

8. Касандрова О.Н. Обработка результатов наблюдений / О.Н. Касандрова, В.В. Лебедев. – М.: Наука, 1970. – 109 с.

9. Холявский Г.Б. Расчет электродинамических усилий в электрических аппаратах / Холявский Г.Б. – Л.: Энергия, 1971. – 156 с.

10. Калантаров П.Л. Расчет индуктивностей: Справочная книга / П.Л. Калантаров, Л.А. Цейтлин. – Л.: Энергоатомиздат, 1986. – 488 с.

11. Контакти електричні. Терміни та визначення. ДСТУ 2290-93: [Діє з 1995-01-01] – К.: Держстандарт України, 1993. – 32 с. (Державний стандарт України)

12. M. Electrical contacts. Fundamentals, Application and Technology / Braunovic, V.V. Konchits, N.K. Myshkin. CRC Press. Taylor&Francis Group. 2006.

13. Jean-Pierre Thierry, Christophe Kilindjian. Electrodynamic forces on busbars in LV systems // Cahier Technique Merlin Gerin n° 162, E/CT 162 first issued, October 1996.

Appendix A.
Statistical Processing Experimental Findings

As noted, contact resistance is a random quantity. By this why, each measurement under identical conditions results in different findings, that is, it gives an error caused not only by the measuring devices errors, but also by the physical phenomena in contacts. Measure error is considered as the deviation of the findings of the *i*-th measurement a_i from the true value of the measured value a, that is:

$$\Delta a_i^* = a - a_i\,. \tag{A.1}$$

It should be noted that the measurement error is usually unknown since we do not know the true value of the quantity being measured. Consequently, one of the tasks of mathematical (statistical) processing of the experimental findings is to determine the error of individual measurements (experiments) and to evaluate the true value of the quantity to be measured basing upon the derived findings.

As noted above, the random errors distribution of contact resistance corresponds to the so-called *normal distribution law*. According to this law, the density of the probability distribution of the existence of this or another value a_i is determined by the following dependency:

$$y(a) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(a_i - a)^2}{2\sigma^2}}, \tag{A.2}$$

where σ is the root mean square error of the measurement finings;

σ^2 is the general variance of the measurement finings, that is, the measure of the spread of the quantity to be measured about its true value.

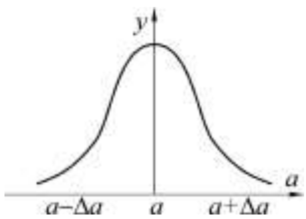

Figure A.1 – Gaussian curve

Figure A.1 shows the graphic representation of the distribution law for random values that in this case is the *normal distribution curve* or *Gaussian curve*. It should be noted that the function $y(a)$ is continuous and describes a general set containing an infinite set of measurements. However, in practical situation, the researcher has the opportunity to make a limited number (usually no more than 5–6) of measurements, that is, a sample of the general population. Therefore, numerical characteristics, determined on the basis of a limited number of measurements, are approximated and are called *sample estimates*.

Sample estimate of the true value of the quantity to be measured is defined as the arithmetic mean of n parallel measurements:

$$\bar{a} = \frac{1}{n}\sum_{i=1}^{n} a_i \qquad \text{(A.3)}$$

The arithmetic mean most often is closest to the true value of the quantity being measured and approaches it when the experiments number is increased.

Sample estimates of the general variance and the root mean square error of an individual measurement are calculated by the following equations:

$$s^2 = \frac{1}{f}\sum_{i=1}^{n}\left(a_i - \bar{a}\right)^2 \qquad s = \sqrt{\frac{1}{f}\sum_{i=1}^{n}\left(a_i - \bar{a}\right)^2}\,, \qquad \text{(A.4)}$$

where f is the number of degrees of freedom determined as $n-1$.

As noted above, the deviation magnitude $a - a_i$ cannot be determined, since a is not known. However, the mathematical statistics enable us to find the interval of values about the arithmetic mean $\bar{a} \pm \varepsilon_0$ that, with some probability P, will include the value a. The interval from $\bar{a} - \varepsilon_0$ to $\bar{a} + \varepsilon_0$ is called a *confidence interval*; the value of ε_0 is the error of the arithmetic mean, and P is called a *confidence probability*, expressed in fractions of a one or in percentage. In finding the confidence interval, the confidence interval is usually predetermined, which in this case (as well as for most calculations) can be chosen equal to 0.95 or 95%.

Consequently, the magnitude of the arithmetic mean error for a limited experiments number and the given confidence probability is determined by the following formula:

$$\varepsilon_0 = \frac{t_\alpha s}{\sqrt{n}}\,, \qquad \text{(A.6)}$$

where t_α is the so-called *Student t-test* accounting deviation of the value s^2 from normal distribution at small number of the trials.

Thus, the final result is given in the following form:

$$a = \bar{a} \pm \varepsilon_0\,.$$

This means that the true value of the measured quantity, with the confidence probability P, is contained in the confidence interval with the limits:

$$\bar{a} - \varepsilon_0 \le a \le \bar{a} + \varepsilon_0 .$$

The following example considers the evaluation of errors (confidential interval) in measuring the voltage drop for a pair of contacts. For the same value of contact pressure, 5 parallel experiments were performed. The findings of the experiments performed and their mathematical processing are represented in Table A.1.

Table A.1 – Findings of the experiment and their mathematical processing

№№ experiments	1	2	3	4	5
$U_{\text{к}i}$, mV	24,1	23,6	23,7	24,2	24,4
$\bar{U}_{\text{к}}$, mV	24,0				
$\left(U_{\text{к}i} - \bar{U}_{\text{к}}\right)$	0,1	–0,4	–0,3	0,2	0,4
$\left(U_{\text{к}i} - \bar{U}_{\text{к}}\right)^2$	0,01	0,16	0,09	0,04	0,16
s^2	0,115				
s	0,339				
t_α	2,776				
ε_0	0,42				
$U_{\text{к}}$	24,0±0,42				

Appendix B.
Calculation of the Mutual Inductance of Two Circular Circuits

The determination of the mutual inductance of two circular contours (coils) reduces to the determination of the internal inductances of the three coils shown in Figure B.1.

$$M_{12} = \frac{1}{2}\left(L_{123} + L_2 - 2L_{12}\right),$$

where L2 is the internal inductance of virtual coil 2 of length x with the same diameters d1 and d2 and the same density of turns as coils 1 and 3;

L_{123}, L_{12} are the internal inductances of the coils constructed of coils 1-2-3 and 1-2, respectively.

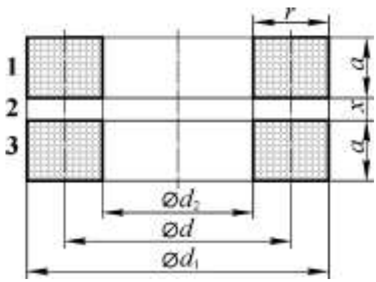

Figure B.1 – Determination of mutual inductance of the coils

Internal inductance of coils of a rectangular cross section is determined by the following formula:

$$L = 10^{-7} w_0^2 r^2 h^2 \Phi \cdot d \,,$$

where h is the coil heigth;

Φ is the quantity to be determined as the function of the winding height that is expressed as follows:

$$\rho = \frac{r}{d} = \frac{40}{200} = 0{,}2 \,.$$

For this value, function Φ is given in Table B.1.

Table B.1 – Function Φ

h, mm	20	40	60	80	100	120	140	160	180	200
Φ	13,2	11,5	10,2	9,1	8,3	7,6	7	6,5	6,05	5,7

Initial data for calculating the mutual inductance of coils:

$$h_2 = x;\ h_{123} = 2r + x;\ h_{12} = r + x;\ w_0 = 2{,}2\ 1/\text{mm}^2;\ r = 40\,\text{mm};\ d = 200\,\text{mm}.$$