DEPARTMENT OF EDUCATION AND SCIENCE OF UKRAINE

NU “ZAPORIZHZHYA POLYTECHNIC”

METHODICAL INSTRUCTIONS

for independent work and implementation of Calculation-Graphic Work in Theoretical Electrical Engineering "Three-phase circuits" for the students of specialty 141

2020

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THE CONTENT

1 GENERAL INSTRUCTIONS

1.1 The aim and the task of the calculation-graphic work

One of the most important stages of independent student’s work in the studying of Theoretical Electrical Engineering (TEE) is the fulfilling of calculation-graphic work, which helps to master the material of discipline.

The CGW on the example of electric circuits, which are similar to existing ones (three-phase electrical circuits with different schemes of load connection in steady-state modes), provides the practical application of impotant section of the Theoretical Electrical Engineering course: three-phase electrical circuits in nominal and damage modes. This CG work is designed to help solving this problem. Students have the opportunity to apply the knowledge gained after studying TEE course, for simulating and calculating real electrical circuits.

The tasks assume the obtaining of mathematical models of three-phase circuits, the calculation and research of several steady-state modes and their analysis.

1.2 Contents and the volume of the work

This course work consists of two different parts: "The calculation of Wye-wye connection in nominal and damage modes" and "The calculation of Wye-delta connection also in nominal and damage modes". The explanatory report must include calculations of different conditions, figures, diagrams and graphs.

The contents of the explanatory report: - a title-page; - the technical task to the calculation-graphic work; - an abstract; - contents. The essence of the explanatory report: - explanations to computations and independent calculations; - checking up of rightness of the calculations; - conclusions; - the list of used original sources; - appendixes.

1.3 The organization and the leadership of the CGW

1.3.1 The lecturer is a manager (a leader) of the course work. He conducts practical lessons on theoretical electrical engineering in the group.

1.3.2 All students' questions on using, checking up and designing this course work must be decided at the weekly lecturer's consultations according to his time-table.

2. THREE-PHASE CIRCUITS

2.1 Three-phase System of Electromotive Forces

Suppose we have a system of three *ac* electromotive forces (e.m.f.s) of a certain frequency such that their amplitudes are equal but these e.m.f.s are displaced from one another by 120^o in time. Such a set of three sinusoidal e.m.f.s make up a so-called *symmetrical three-phase system of electromotive forces*. The instantaneous values of such e.m.f.s are shown in Fig. 2.1 a , and their vector diagram is in Fig. 2.1 b .

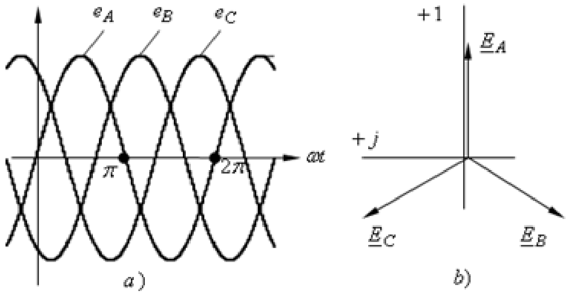


Fig. 2.1 Three-phase system of e.m.f.s.

The electromotive forces for a three-phase system are supplied by a three-phase generator (alternator). Such a generator has three identical (phase) coils displaced at 120^o from one another, and the sine wave e.m.f.s induced in them, also displaced at 120^o in time phase. The beginnings of coils (windings) can be marked out with *A, B* and *C*.

In particular, we call this system a *three-phase balanced system*^{o}. For a balanced three-phase system, it follows from Eq. (2.1), that the phasor sum of these three electromotive forces is zero.

We may mathematically express this system of e.m.f.s as

$$e_A = E_m \sin \omega t = U_m \sin \omega t$$

$$e_B = E_m \sin (\omega t - 120^o) = U_m \sin (\omega t - 120^o) \qquad (2.1)$$

$$e_C = E_m \sin (\omega t + 120^o) = U_m \sin (\omega t + 120^o)$$

These electromotive forces are marked as follows. One of them is marked as $\underline{E}_A$. The one leading it is $\underline{E}_C$ and the one lags behind it, $\underline{E}_B$.

2.2 A Three-phase Circuit. Phase and Line Quantities

A three-phase circuit is a combination of three-phase supply, a three-phase load, and connecting wires. The most common types of interconnections are *wye connection* and *delta connection*. Consider "wye-wye" connection with a neutral wire (see Fig. 2.2). Three-phase circuits may be symmetrical (or uniform) and asymmetrical.

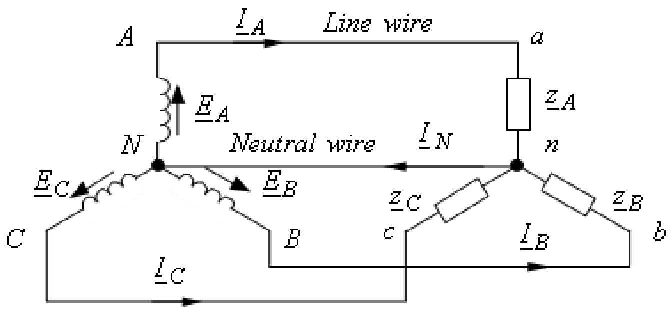


Fig. 2.2. Wye-wye connection with a neutral wire.

For star connection the ends of three phases are connected together to form a node called a *neutral point N* or *O* (for the phases of generator) and *n* or *O'* (for the phases of load). The wire connecting the neutral points of the alternator and the load is referred to as the *neutral wire*. The neutral wire carries a neutral current, $\underline{I}_N$ or $\underline{I}_o$, the positive direction for which is from *n* to *N***.** The wires connecting the terminals *A*, *B* and *C* of the generator and the terminals *a*, *b* and *c* of the load are termed the *line wires*. The currents in them are referred to as *line currents*, $\underline{I}_A$, $\underline{I}_B$, $\underline{I}_C$.

The voltage between two wires is termed the *line voltage*. As with any voltage, it symbol has a two-letter subscript, for example, $\underline{U}_{AB}$, which is the line voltage between the terminals *A* and *B*. The symbol for the magnitude of the line voltage is U_l.

The voltage between the beginning and the end of a phase, or the voltage from line to earth is called a *phase voltage*, U_p. The currents in the line wires are referred to as the *line current.*

2.3 Relationship between Line and Phase Voltages and Currents

Between phase and line e.m.f.s, voltages and currents exist certain relationships for symmetrical circuits. For a wye (star) connection the line voltages are related to the phase voltages such that

$$\underline{U}_{AB} = \underline{U}_A - \underline{U}_B$$
$$\underline{U}_{BC} = \underline{U}_B - \underline{U}_C \qquad (2.2)$$
$$\underline{U}_{CA} = \underline{U}_C - \underline{U}_A$$

where $\underline{U}_{AB}$, $\underline{U}_{BC}$ and $\underline{U}_{CA}$ are the line voltages in a wye-connected three-phase generator; $\underline{U}_A$, $\underline{U}_B$ and $\underline{U}_C$ are the phase voltages in a wye-connected generator. When we have a symmetrical load in a wye connection, that is $Z_A = Z_B = Z_C$, the line voltages are $\sqrt{3}$ times more than the phase voltages. These relationships of the phase voltages and line voltages are illustrated in the phasor diagram of Fig. 2.3.

From the vector diagram, the line voltage may be visualized as the base of an equivalent triangle having acute angles of 30° (Fig. 2.3)

$$U_l = U_{AB} = U_p \cdot 2\cos 30^o = \sqrt{3} \cdot U_p \qquad (2.3)$$

The line currents in a wye supply are equal to the phase currents as line and phase wires are connected in series: $I_l = I_{ph}$.

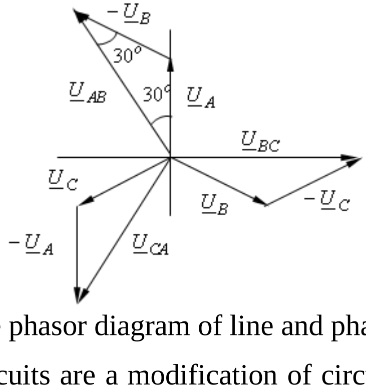


Fig. 2.3. The phasor diagram of line and phase voltages.

Three-phase circuits are a modification of circuits carrying currents, and as such they can be solved by the same methods as single-phase sinusoidal circuits. Symbolic notation and vector diagrams are also applicable to three-phase circuits.

2.4 The Calculation of Wye-to-Wye with a Neutral Wire

For calculation such a circuit we can use the symbolic method of calculation and some known laws: Ohm's law, Kirchhoff's laws, mesh-current method, node-analysis method, superposition theorem and so on.

If there is no impedance in a neutral wire in the circuit of Fig. 2.2, the potential at point *n* is the same as at point *N*, and two points are actually a single point Then three branches carry the following currents:

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_A}; \quad \underline{I}_B = \frac{\underline{E}_B}{\underline{Z}_B}; \quad \underline{I}_C = \frac{\underline{E}_C}{\underline{Z}_C}. \tag{2.4}$$

By Kirchhoff's current law the current in the neutral wire is the vector sum of the phase currents:

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C \tag{2.5}$$

If the load is balanced ($\underline{Z}_A = \underline{Z}_B = \underline{Z}_C$), the current $\underline{I}_N$ is zero. If the load is unbalanced, then the current $\underline{I}_N$ is other than zero.

Example 2.1. Consider unbalanced load. Phase voltage of the alternator in Fig. 2.2 is 120V. The phase impedances: $\underline{Z}_A = R = 30\Omega$; $\underline{Z}_B = j\omega L = 80\Omega$; $\underline{Z}_C = -j/\omega C = 80\Omega$. Find a current in a neutral wire.

Solution: Write phase voltages in a complex form:

$$\underline{U}_A = \underline{U_p} \cdot e^{j0^o} = 120\ V;$$

$$\underline{U}_B = \underline{U_p} \cdot e^{-j120^o} = 120\, e^{-j120^o}\ V;$$

$$\underline{U}_C = \underline{U_p} \cdot e^{j120^o} = 120\, e^{j120^o}\ V.$$

Calculate phase currents for each phase separately:

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_A} = \frac{120}{30} = 4\,A; \qquad \underline{I}_B = \frac{\underline{E}_B}{\underline{Z}_B} = \frac{120e^{-j120^o}}{80e^{j90^o}} = 1.5\,e^{-j210^o}\ A;$$

$$\underline{I}_C = \frac{\underline{E}_C}{\underline{Z}_C} = \frac{120e^{j120^o}}{80e^{-j90^0}} = 1.5\,e^{j210^o}\ A.$$

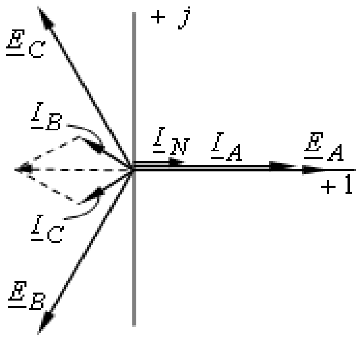


Fig. 2.4 The vector diagram

Current $\underline{I}_A$ is in phase with $\underline{E}_A$. Current $\underline{I}_B$ is lagging with $\underline{E}_B$. Current $\underline{I}_C$ is in leading with $\underline{E}_C$. Their algebraic sum gives the current in a neutral wire $\underline{I}_N$ the value of which may be found as following:

$$\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C = 4 + 1.5e^{-j210^o} + 1.5e^{j210^o} =$$
$$= 4 - 1.48 + j0.235 - 1.48 - j0.235 = 1.04 \quad A.$$

2.5 The Calculation of Wye-to-Wye without a Neutral Wire

If the load is unbalanced the voltage appearing between the neutral points - the bias neutral voltage - can be found by the formula

$$\underline{U}_N = \frac{\underline{E}_A\underline{Y}_A + \underline{E}_B\underline{Y}_B + \underline{E}_C\underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} \qquad (2.6)$$

Now we can find the phase currents through the load

$$\underline{I}_a = \frac{\underline{U}_a}{\underline{Z}_a} = \frac{\underline{E}_A - \underline{U}_N}{\underline{Z}_a} = (\underline{E}_A - \underline{U}_N)\cdot\underline{Y}_a; \qquad (2.7)$$

$$\underline{I}_b = \frac{\underline{U}_b}{\underline{Z}_b} = \frac{\underline{E}_B - \underline{U}_N}{\underline{Z}_a} = (\underline{E}_B - \underline{U}_N)\cdot\underline{Y}_b; \qquad (2.8)$$

$$\underline{I}_c = \frac{\underline{U}_c}{\underline{Z}_c} = \frac{\underline{E}_C - \underline{U}_N}{\underline{Z}_c} = (\underline{E}_C - \underline{U}_N)\cdot\underline{Y}_c. \qquad (2.9)$$

where $\underline{U}_a, \underline{U}_b, \underline{U}_c$ are phase voltages at the load, $\underline{Y}_a, \underline{Y}_b, \underline{Y}_c$ are phase admittances, that is the values inverse to complex impedances.

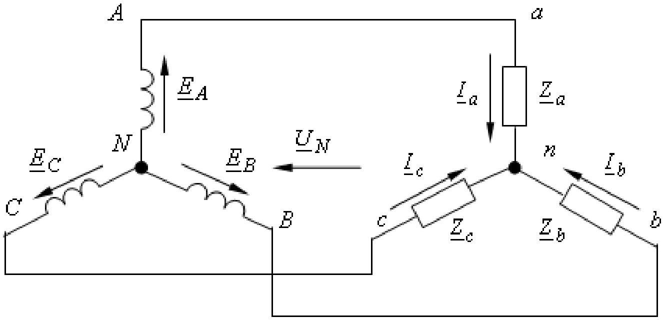


Fig.2.5

Example 2.2 (Fig.2.5.). Determine currents and voltages and draw a vector diagram in the circuit without a neutral wire, for $E_p = 50\ V$, $\underline{Z}_a = 20\ \Omega$, $\underline{Z}_b = 10\ \Omega$, $\underline{Z}_c = -j20\ \Omega$.

Write phase voltages in a complex form:

$$\underline{U}_A = \underline{E}_p \cdot e^{j0^o} = 50\ V\ ,\ \underline{U}_B = \underline{E}_p \cdot e^{-j120^o} = 50\,e^{-j120^o}\ V$$

$$\underline{U}_C = \underline{E}_p \cdot e^{j120^o} = 50\,e^{j120^o}\ V\ .$$

Calculate complex phase admittances:

$$\underline{Y}_a = 1/\underline{Z}_a = 1/20 = 0.05\ \Omega^{-1};\quad \underline{Y}_b = 1/\underline{Z}_b = 1/10 = 0.1\ \Omega^{-1};$$

$$\underline{Y}_c = 1/\underline{Z}_c = 1/20e^{-j90^o} = 0.05e^{j90^o} = j0.05\ \Omega^{-1}.$$

For the asymmetrical load the neutral voltage shifting appears between neutral nodes. We define it, using Eq. (2.4).

$$\underline{U}_N = \frac{50 \cdot 0.05 + 50e^{-j120^o} \cdot 0.1 + 50e^{j120^o} \cdot 0.05e^{j90^o}}{0.05 + 0.1 + j0.05} =$$

$$= \frac{-2.165 - j5.58}{0.15 + j0.05} = -20.981 - j23.66 = 31.62e^{-j132^o}\ V\ .$$

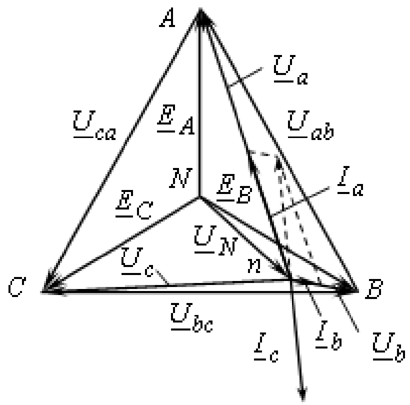


Fig. 2.6

Find the currents. using Eq. (2.7), (2.8), (2.9):

$$\underline{I}_a = (50 + 23{,}85 + j29.45) \cdot 0.05 = 3.55 + j1.183 = 3.74 e^{j18^o}\ A;$$

$$\underline{I}_b = (-25 - j43.3 + 23{,}85 + j29.45) \cdot 0.1 = 1.002 e^{-j102^o}\ A;$$

$$\underline{I}_c = (-25 + j43{,}3 + 23.85 + j29.45) \cdot j0.05 = 3.354 e^{-j177^o}\ A.$$

The vector diagram for this problem one can see in Fig. 2.6.

2.6 The Calculation of Damage Conditions

There are two damage rates in three-phase circuit: a short circuit of any phase and an open circuit of a phase.

Example 2.3. Symmetrical load $\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = (240 + j100)$ is connected up by way of line wires with line impedances $\underline{Z}_l = (30 + j40)\ \Omega$ to the generator with the phase e.m.f. $E_p = 220\ V$. The impedance of a neutral wire $\underline{Z}_N = 40 + j20$.

Calculate damage conditions for three cases:

- a ***short circuit*** in phase *A* (there are impedances Z_i in line wires);

- a ***short circuit*** in phase *A* (without impedances Z_i in line wires and a neutral wire);

- an ***open circuit*** of a line wire between nodes A and a.

Consider the first case: ***short circuit in phase A with impedances Z$_i$ in line wires.***

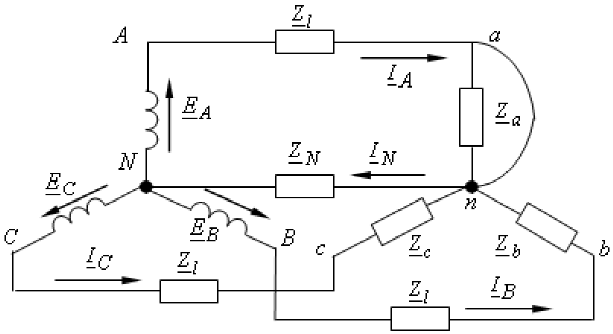


Fig.2.7. A short circuit of a load in the phase *A*

When a short circuit in phase *A*, (Fig. 2.7.), it is asymmetrical load, $\varphi_n = \varphi_a$, and bias neutral voltage appears between nodes “a” and “N”.

$$\underline{U}_N = \frac{\underline{E}_A \underline{Y}_A + \underline{E}_B \underline{Y}_B + \underline{E}_C \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C + \underline{Y}_N} \qquad (2.10)$$

At first we calculate complex branch admittances:

$$\underline{Y}_A = \frac{1}{\underline{Z}_l} = \frac{1}{50e^{j53^o}} = 0.02e^{-j53^o} = (0.012 - j0.016)\ \Omega^{-1};$$

$$\underline{Y}_B = \frac{1}{\underline{Z}_b + \underline{Z}_l} = \frac{1}{304.14e^{j27^o}} = 0.0033e^{-j27} = (0.003 - j0.0015)\ \Omega^{-1};$$

$$\underline{Y}_C = \underline{Y}_B = 0.0033e^{-j27} = (0.003 - 0.0015)\ \Omega^{-1};$$

$$\underline{Y}_N = \frac{1}{\underline{Z}_N} = \frac{1}{44.72e^{j27^o}} = 0.022e^{-j27^o} = (0.02 - j0.01)\ \Omega^{-1}.$$

Determine the bias neutral voltage:

$$\underline{U}_N = \frac{127 \cdot 0.02e^{-j153^o} + 127e^{-j120^o} \cdot 0.0033e^{-j27^o} + 127e^{j120^o} \cdot 0.022e^{-j27^o}}{0.012 - j0.016 + 0.003 - j0.00115 + 0.003 - j0.0015 + 0.02 - j0.01} =$$

$$= \frac{3.761e^{-j57^o}}{0.048e^{-j37^o}} = 73.915 - j27.525 \ = 78.87e^{-j20^o} V \ .$$

Calculate the phase currents by Ohm's law:

$$\underline{I}_A = (\underline{E}_A - \underline{U}_N) \cdot \underline{Y}_a = \underline{U}_{an} \cdot \underline{Y}_a = 0.02e^{-j53^o} \cdot (127 - 78.87e^{-j20^o}) =$$

$$= 2.193 - j2.007 = 2.973e^{-j42^o} A;$$

$$\underline{I}_B = (\underline{E}_B - \underline{U}_N) \cdot \underline{Y}_b = \underline{U}_{bn} \underline{Y}_b = 0.0033e^{-j27^o} (127e^{-j120^o} - 59e^{-j5^o}) =$$

$$= -0.784 - j0.197 = 0.808e^{-j166^o} \ A;$$

$$\underline{I}_C = (\underline{E}_C - \underline{U}_N) \cdot \underline{Y}_c = \underline{U}_{cn} \cdot \underline{Y}_c = 0.0033e^{-j27^o} (127e^{j120^o} - 59e^{j5^o}) =$$

$$= -0.207 + j0.915 = 0.938e^{j103^o} A.$$

The current in a neutral wire

$$\underline{I}_N = \frac{\underline{U}_N}{\underline{Z}_N} = \frac{78.87e^{-j20^o}}{44.7e^{j27^o}} = 1.764e^{-j47^o} \ A.$$

Check up the calculation, using Kirchhoff's current law:

$$\underline{I}_N = 2.193 - j2.007 - 0.784 - j0.197 - 0.207 + j0.915 =$$

$$= 1.203 - j1.2 = 1.764e^{-j47^o} A.$$

Determine the phase voltage drops across the load and the voltage drops in line wires: $\underline{U}_{an} = 0$

$$\underline{U}_{bn} = \underline{Z}_b \cdot \underline{I}_b = 260e^{j23^o} \cdot 0.808e^{-j166^o} = 210.1e^{-j143^o} \ V \ .$$

$$\underline{U}_{cn} = \underline{Z}_c \cdot \underline{I}_c = 260e^{j23^o} \cdot 0.938e^{j103^o} = 243.858e^{j125^o} \ V \ .$$

$$\underline{U}_{Aa} = \underline{Z}_l \cdot \underline{I}_a = 50e^{j53^o} \cdot 2.973e^{-j42^o} = 148.655e^{j11^o}\ V.$$

$$\underline{U}_{Bb} = \underline{Z}_l \cdot \underline{I}_b = 50e^{j53^o} \cdot 0.808e^{-j166^o} = 40.40e^{-j113^o}\ V.$$

$$\underline{U}_{Cc} = \underline{Z}_l \cdot \underline{I}_c = 50e^{j53^o} \cdot 0.938e^{j103^o} = 46.9e^{j156^o}\ V.$$

A vector diagram for this condition is shown in Fig. 2.8.

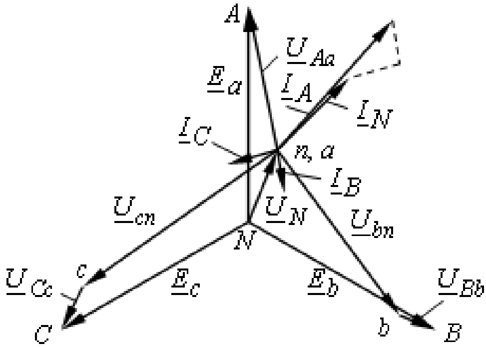


Fig. 2.8. A vector diagram for a short circuit in the phase *A*.

Short circuit without resistance in line wires (see 2.7 when all $Z_l = 0$) ***and without a neutral wire*** ($Z_N = \infty$).

Consider this figure when emf *E= 380* V, $\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = (240 + j100)\ \Omega$. For this case $\underline{Z}_A = 0$. Then admittance of phase *A* is equal to infinity, that is *YA = 0*. We can’t use formula (2.10), because it leads to an indefinite form $\frac{\infty}{\infty}$. The bias neutral voltage is determined from the contour NAanN, for which in accordance with Kirchhoff's second law we have $\underline{E}_A = \underline{U}_a + \underline{U}_N$. As at short circuit in phase *A* $\underline{U}_a = 0$, we’ve got

$$\underline{U}_N = \underline{E}_A - \underline{U}_a = 380 - 0 = 380 + j0 = 380e^{j0^o}\ B.$$

Calculate phase (line) currents in intact phases B and C by Ohm's law

$$\underline{I}_B = (\underline{U}_B - \underline{U}_N)\underline{Y}_B = (-190 - j329 - 380)(0{,}012 - j0{,}016) =$$

$$= -12{,}10 + j5{,}17 = 13{,}16e^{j157^o}\ A;$$

$$\underline{I}_C = (\underline{U}_C - \underline{U}_N)\underline{Y}_B = (-190 + j329 - 380)(0{,}012 - j0{,}016) =$$

$$= -1{,}58 + j13{,}07 = 13{,}16e^{j97^o}\ A.$$

Determine the phase (linear) current in the damaged phase A, using Kirchhoff's Current Law

$$\underline{I}_A = -(\underline{I}_B + \underline{I}_C) = -(-12{,}10 + j5{,}17 - 1{,}58 + j13{,}07) =$$

$$= 13{,}68 - j18{,}24 = 22{,}8e^{-j53^o}\ B.$$

Determine the phase load voltages in the intact phases B and C by Ohm's law

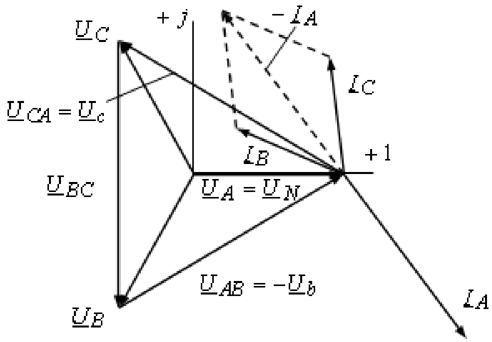
$$\underline{U}_b = \underline{I}_B \cdot \underline{Z}_ф = (-12{,}10 + j5{,}17)(30 + j40) =$$

$$= -569{,}8 - j328{,}9 = 658e^{-j150^o}\ B;$$

$$\underline{U}_c = \underline{I}_C \cdot \underline{Z}_ф = (-1{,}58 + j13{,}07)(30 + j40) =$$

$$= -570{,}2 + j328{,}9 = 658e^{j150^o}\ B.$$

The vector diagram for this case



The analysis of calculation and vector diagram shows that in the case of a short circuit in one phase of wye-wye connection, the values of phase voltages in intact phases are equal to the values of line voltages.

Short-circuit in some phases of a four-wire circuit, connected by wye-wye, is unacceptable, because it leads to a short circuit of the corresponding phase winding of a generator. **This is a dangerous emergency!**

An *open circuit* of the phase *A*, (Fig.2.9) the condition is also asymmetrical and a neutral voltage shift appears between “n” and “N”.

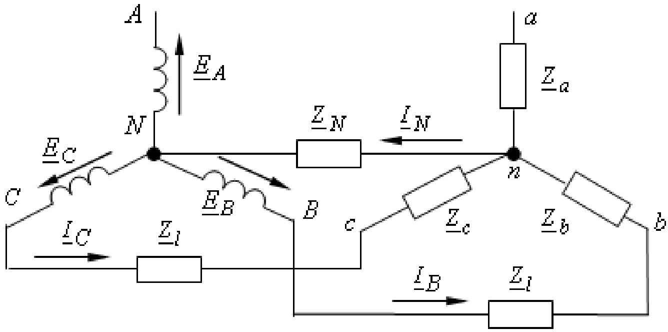


Fig. 2.9. An open circuit in the line wire *Aa*.

As $\underline{U}_A = 0$, Eq.(2.5) is used in the following condition:

$$\underline{U}_N = \frac{\underline{E}_B \underline{Y}_B + \underline{E}_C \underline{Y}_C}{\underline{Y}_B + \underline{Y}_C + \underline{Y}_N} \qquad (2.11)$$

$$\underline{U}_N = \frac{220e^{-j120^o} \cdot 0.0033e^{-j27^o} + 127e^{j120^o} \cdot 0.0033e^{-j27^o}}{0.003 - j0.0015 + 0.003 - j0.0015 + 0.02 - j0.01} =$$

$$= -24.997 - j0.284 = 24.998e^{-j179^o} \ V.$$

Calculate the phase currents and the current in the neutral wire:

$$\underline{I}_B = (\underline{E}_B - \underline{U}_N) \cdot \underline{Y}_b = \underline{U}_{bn} \cdot \underline{Y}_b = 0.0033e^{-j27^o} \cdot (127e^{-j120^O} -$$

$$- 24.998e^{j179^o}) = -0.537 - j0.43 = 0.69e^{-j141^o} \ A;$$

$$\underline{I}_C = (\underline{E}_C - \underline{U}_N) \cdot \underline{Y}_c = \underline{U}_{cn} \cdot \underline{Y}_c = 0.0033e^{-j27^o} \cdot (127e^{j120^O} -$$

$$- 24.998e^{j179^o}) = 0.04 + j0.684 = 0.69e^{j87^o} \ A$$

$$\underline{I}_N = \underline{U}_N \cdot \underline{Y}_N = 0.0022 e^{-j27^o} \cdot 24.998 e^{-j179^O} = 0.559 e^{-j206^o} \ A.$$

Find the phase voltages and the voltage drops in line wires:

$$\underline{U}_{Bb} = \underline{Z}_l \cdot \underline{I}_b = 50 e^{j53^o} \cdot 0.69 e^{-j141^o} = 34.5\, e^{-j88^o} \ V$$

$$\underline{U}_{Cc} = \underline{Z}_l \cdot \underline{I}_c = 50 e^{j53^o} \cdot 0.69 e^{j87^o} = 34.5 e^{j140^o} \ V$$

$$\underline{U}_{bn} = \underline{Z}_b \cdot \underline{I}_b = 0.69 e^{-j141^o} \cdot 260 e^{j23^o} = 179.4 e^{-j118^o} \ V$$

$$\underline{U}_{cn} = \underline{Z}_c \cdot \underline{I}_c = 0.69 e^{j87^o} \cdot 260 e^{j23^o} = 179.4 e^{j110^o} \ V$$

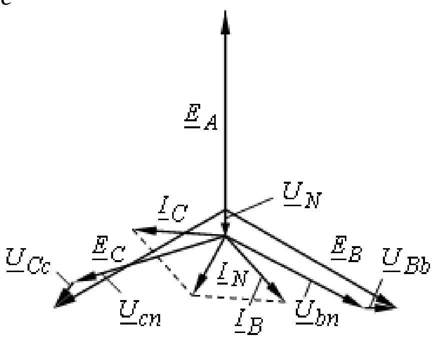


Fig. 2.10. A phasor diagram for the open circuit in the line wire *Aa*

2.7. Delta-Connected Load

In a delta-connected load, phase currents I_{ab}, I_{bc}, I_{ca} flowing through phase impedances Z_{ab}, Z_{bc}, Z_{ca}. The positive direction of a current flow is assumed to be clockwise.

U_{AB}, U_{BC}, U_{CA} are the line voltages in a three-phase delta-connected supply;

U_{ab}, U_{bc}, U_{ca} are the phase voltages in a delta-connected load;

I_A, I_B, I_C are the line currents in line wires connecting the supply and the load;

I_{ab}, I_{bc}, I_{ca} are the phase currents in a delta-connected load.

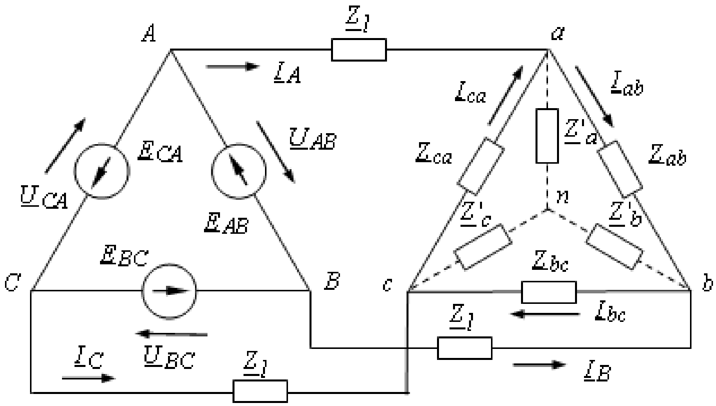


Fig. 2.11. The delta connection.

If there are no impedances in line wires, the line voltages in a delta-connected supply are equal to the phase voltages in a delta-connected load $U_l = U_p$. If the load is balanced, the line currents, however, $\sqrt{3}$ times more than the phase currents. If the load is unbalances, the line currents can be found in terms of the phase currents by Kirchhoff's current law:

$$\underline{I}_A = \underline{I}_{ab} - \underline{I}_{ca},$$
$$\underline{I}_B = \underline{I}_{bc} - \underline{I}_{ab}, \qquad (2.12)$$
$$\underline{I}_C = \underline{I}_{ca} - \underline{I}_{bc}.$$

It is often necessary to determine currents in delta-connected circuit when there are line impedances in line wires. For this case the line voltages across the load aren't equal to the generator line voltages.

That is why we must at first change delta-connected load into wye-connected one (see Fig. 2.11). by the following formulas:

$$\underline{Z}'_a = \frac{\underline{Z}_{ab} \cdot \underline{Z}_{ca}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}};$$
$$\underline{Z}'_b = \frac{\underline{Z}_{bc} \cdot \underline{Z}_{ab}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}; \qquad (2.13)$$
$$\underline{Z}'_c = \frac{\underline{Z}_{ca} \cdot \underline{Z}_{bc}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}.$$

If there is a symmetrical load and the phase impedances are equal ($\underline{Z}_{ab} = \underline{Z}_{ba} = \underline{Z}_{ca}$), the neutral voltage shift (or the bias neutral voltage) $\underline{U}_N$ is equal to *0*. Then we can calculate currents:

$$\underline{I}_A = \frac{\underline{E}_A}{\underline{Z}_l + \underline{Z}'_a}; \quad \underline{I}_B = \frac{\underline{E}_B}{\underline{Z}_l + \underline{Z}'_b}; \quad \underline{I}_C = \frac{\underline{E}_C}{\underline{Z}_l + \underline{Z}'_c}. \qquad (2.14)$$

The phase currents are $\sqrt{3}$ times less the line currents and angle shift between them is 30^{O} (that is a phase current leads the corresponding line current).

$$\underline{I}_{ab} = \frac{\underline{I}_A}{\sqrt{3}} e^{j30^o}; \ \underline{I}_{bc} = \frac{\underline{I}_B}{\sqrt{3}} e^{j30^o}; \ \underline{I}_{ca} = \frac{\underline{I}_C}{\sqrt{3}} e^{j30^o} \qquad (2.15)$$

As the result we draw a vector diagram (Fig. 2.12).

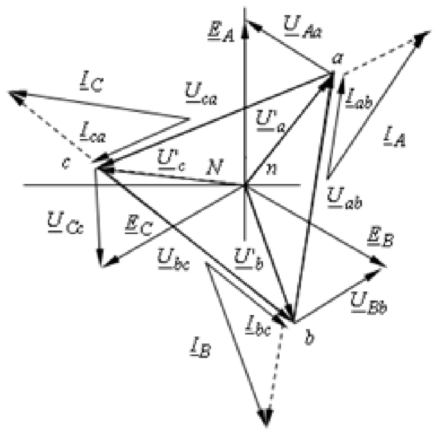


Fig. 2.12. A vector diagram for a symmetrical delta connection.

Example 2.4. The load $\underline{Z}_{ab} = \underline{Z}_{bc} = \underline{Z}_{ca} = (500 - j450)\,\Omega$ is connected up by way of line wires with line impedances $\underline{Z}_l = (35 + j45)\,\Omega$ to the symmetrical generator with the phase voltage equaled to $U_p = 380\,V$. Calculate phase and line currents and voltages.

Solution: Change a delta-connection into a wye-connection (see Fig. 3.11). As we have a balanced load, one can use the following equation:

$$\underline{Z}'_a = \underline{Z}'_b = \underline{Z}'_c = \frac{\underline{Z}_{ab}}{3} = 166{,}44 - j150 = 224.23e^{-j42^o}\,\Omega\ .$$

$$\underline{U}_A = 380\ V;\ \underline{U}_B = 380e^{-j120^o}\,V;\ \underline{U}_C = 380e^{j120^o}\,V\ .$$

Define phase impedances:

$$\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = \underline{Z}_l + \underline{Z}_a = 35 + j45 + 166.44 - j150 = 227.36e^{-j28^o}\,\Omega\ .$$

Calculate line currents by Ohm's law:

$$\underline{I}_A = \frac{\underline{U}_A}{\underline{Z}_a} = \frac{380}{227.36e^{-j28^o}} = 1.67e^{j28^o}\,A\ .$$

$$\underline{I}_B = \frac{\underline{U}_B}{\underline{Z}_b} = 1.67e^{-j92^o}\,A;\quad \underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_c} = 1.67e^{j148^o}\,A.$$

Determine phase currents across the load by Eq. (2.15) (we can use these formulas as there is a symmetrical load in this problem):

$$\underline{I}_{ab} = \frac{\underline{I}_A}{\sqrt{3}}e^{j30^o} = \frac{1.67e^{j28^o}}{\sqrt{3}}e^{j30^o} = 0.965e^{j58^o}\,A;$$

$$\underline{I}_{bc} = 0.965e^{-j62^o}\,A;\quad \underline{I}_{ca} = 0.965e^{j178^o}\ \text{A}.$$

Define line and phase voltages:

$$\underline{U}_{ab} = \underline{Z}_{ab}\cdot\underline{I}_{ab} = 672.68e^{-j42^o}\cdot 0.965e^{j57^o} = 649.14e^{j58^o}\,V\ ;$$

$$\underline{U}_{bc} = \underline{Z}_{bc}\cdot\underline{I}_{bc} = 649.14e^{-j104^o}\,V\ ;$$

$$\underline{U}_{ca} = \underline{Z}_{ca}\cdot\underline{I}_{ca} = 649.14e^{j136^o}\,V;$$

$$\underline{U}_{Aa} = \underline{Z}_l\cdot\underline{I}_A = 57e^{j52^o}\cdot 1.67e^{j27^o} = 92.27e^{j80^o}\,V;$$

$$\underline{U}_{Bb} = \underline{Z}_l\cdot\underline{I}_B = 92.27e^{-j40^o}\,V;$$

$$\underline{U}_{Cc} = \underline{Z}_l\cdot\underline{I}_C = 92.27e^{j200^o}\,V\ .$$

At last we draw a vector diagram for this case.

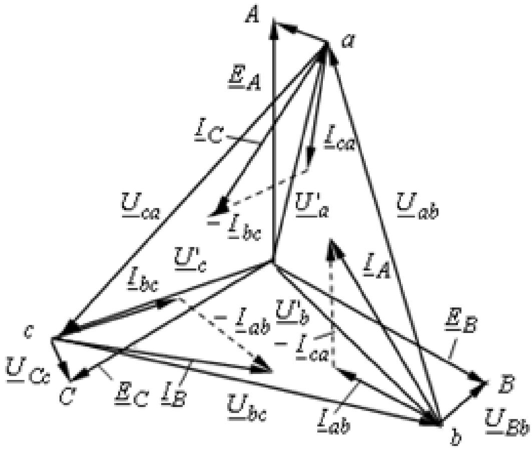


Fig. 2.13. A vector diagram for a symmetrical delta connection

2.8 Damage Conditions in a Delta-Connected Load

The first damage condition is ***an open circuit in any phase***, for example, when phase *bc* is broken (see Fig. 2.14).

As it is the asymmetrical wye connection without a neutral wire, a neutral voltage shift (bias voltage) appears between two neutral nodes:

$$\underline{U}_N = \frac{\underline{E}_A\underline{Y}_A + \underline{E}_B\underline{Y}_B + \underline{E}_C\underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C}$$

There are such admittances in this case:

$$\underline{Y}_A = \frac{1}{\underline{Z}_l} \qquad \underline{Y}_B = \frac{1}{\underline{Z}_{ab} + \underline{Z}_l} \qquad \underline{Y}_C = \frac{1}{\underline{Z}_{ca} + \underline{Z}_l}$$

The phase currents may be determined by Eq. (2.7 - 2.9) as

$\underline{I}_A = (\underline{E}_A - \underline{U}_N)\cdot\underline{Y}_A$, $\underline{I}_B = (\underline{E}_B - \underline{U}_N)\cdot\underline{Y}_B$, $\underline{I}_C = (\underline{E}_C - \underline{U}_N)\cdot\underline{Y}_C$, the voltages $\underline{U}_{Aa}, \underline{U}_{Bb}, \underline{U}_{Cc}$ in line wires and the phase voltages $\underline{U}_{ab}, \underline{U}_{ca}$ are determined in a common way.

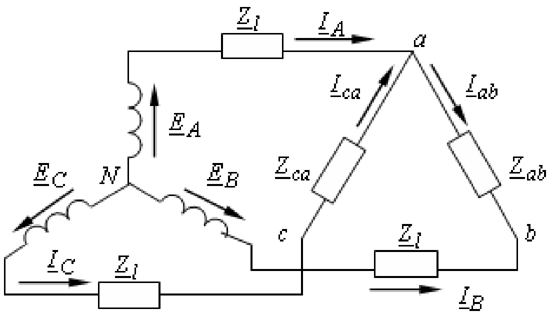


Fig. 2.14. An open circuit in phase *bc*

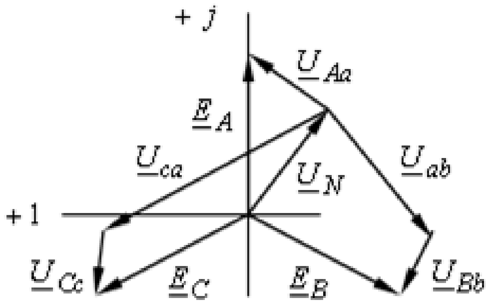


Fig. 2.15. A vector diagram for OC in phase *bc* ($\underline{Z}_{ab} \neq \underline{Z}_{ca}$).

The 2nd damage rate when there is ***a short circuit in phase bc***. A Δ-connected circuit changes into asymmetrical wye connection without a neutral wire (see Fig. 2.16). The neutral voltage shift:

$$\underline{U}_N = \frac{\underline{E}_A \underline{Y}_A + \underline{E}_B \underline{Y}_B + \underline{E}_C \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C}$$, where phase admittances

$$\underline{Y}_A = \frac{1}{\underline{Z}_l + \dfrac{\underline{Z}_{ab} \cdot \underline{Z}_{ca}}{\underline{Z}_{ab} + \underline{Z}_{ca}}} = \frac{1}{\underline{Z}_l + \underline{Z}_{abc}}$$; and $\underline{Y}_B = \underline{Y}_C = {}^1\!/\!_{\underline{Z}_l}$.

Knowing the neutral voltage shift, one can determine currents.

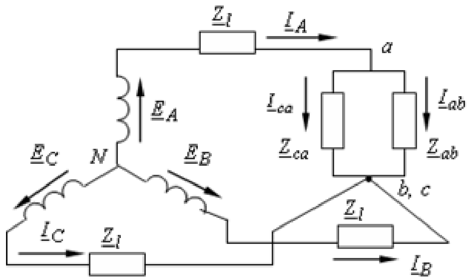


Fig. 2.16. The circuit with a short circuit in phase *bc*.

Phase currents of wye connection are found by Ohm's law:

$$\underline{I}_A = \frac{\underline{E}_A - \underline{U}_N}{\underline{Z}_l + \dfrac{\underline{Z}_{ab} \cdot \underline{Z}_{ca}}{\underline{Z}_{ab} + \underline{Z}_{ca}}} = (\underline{E}_A - \underline{U}_N) \cdot \underline{Y}_A$$

The currents in two parallel branches are determined by using so-called "resolving of the total current":

$$\underline{I}_{ab} = \underline{I}_A \frac{\underline{Z}_{ca}}{\underline{Z}_{ca} + \underline{Z}_{ab}}; \qquad \underline{I}_{ca} = \underline{I}_A \frac{\underline{Z}_{ab}}{\underline{Z}_{ca} + \underline{Z}_{ab}}$$

More detailed description of this damage mode can be found in the schoolbook "Theoretical Electrical Engineering", chapter 5 “Three-phase Circuits”or in any textbook of the recommended literature.

3 THE TECHNICAL TASK TO THE CGW

3.1 Part 1. **Wye-Wye connection**

3.1.1 The calculation of wye-wye connection with a neutral wire. Considering that the power supply is symmetric, one must consider the following modes of operation in a electric circuit:
- calculate nominal mode of wye-wye connection with a neutral wire.;
- calculate a short circuit in one of the phases of wye-wye connection **without** a neutral wire when $\mathbf{Z_L} = \mathbf{0}$.

3.1.2 Calculate phase currents and voltages for each mode, and construct a combined vector diagram of currents and voltages.

3.2 Part 2. **Wye-Delta connection**

3.2.1 Calculate of wye-delta connection for the following rates:
- nominal mode;
- open circuit in one of the phases.

3.2.2 Construct vector diagrams of currents and voltages for both modes.

3.2.3 Calculate the values of active, reactive and apparent powers in three-phase wye and delta connections (for nominal rates).

The figures of electric circuits are given in Fig. 4.1.

Data for calculation of wye-connection are given in Appendix 2.

Data for calculation of delta-connection are given in Appendix 3.

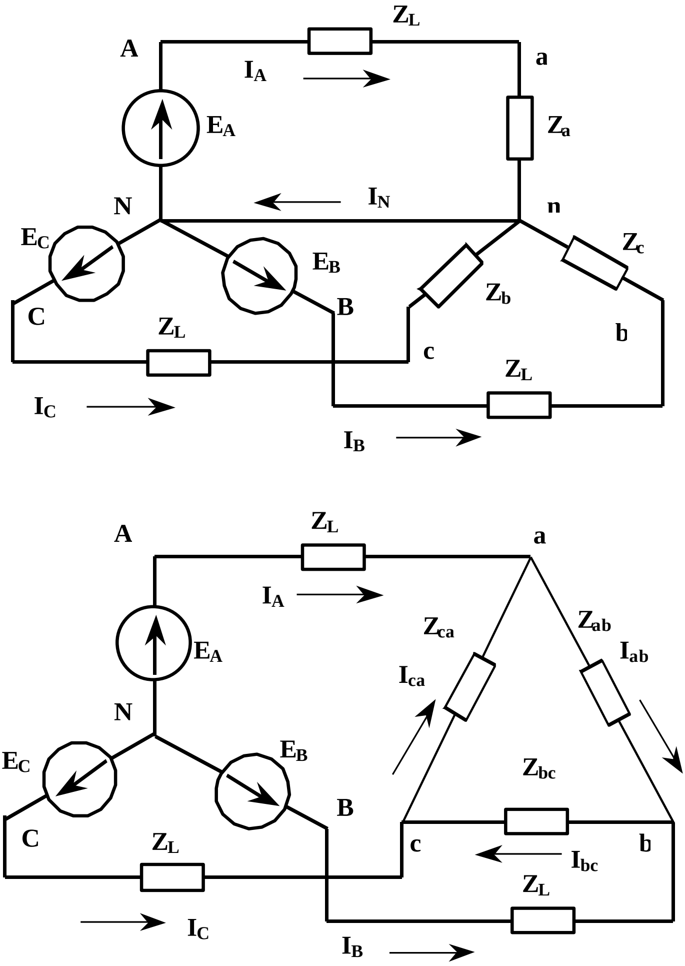


Fig. 3.1. The diagrams of three-phase electric circuits

Appendix 1 Data for calculation of Wye-connected load

№	$Z_A=Z_B=Z_C$, Ω	Z_L, Ω	O.C.	S.C.	U_L, V
1	10+j10	5-j5	A-n	b-n	220
2	-j20	20	B-n	c-n	380
3	14+j14	-j28	C-n	a-n	173
4	20+j20	-j40	A-n	b-n	220
5	15-j15	5+j5	B-n	c-n	380
6	-j20	10+j10	C-n	a-n	173
7	20+j20	4+j4	A-n	b-n	380
8	10-j10	j5	B-n	c-n	220
9	j30	15-j10	C-n	a-n	173
10	30+j40	-j10	A-n	b-n	380
11	j15	10-j5	B-n	c-n	173
12	10+j15	-j5	C-n	a-n	220
13	10-j15	j5	A-n	b-n	380
14	j12	10	B-n	c-n	173
15	20-j20	-j5	C-n	a-n	380

Appendix 2 Data for calculation of Delta-connected load

№	U_L,	$Z_{AB}=Z_{BC}=Z_{CA}$,	Z_L,	OC in ph.	SC in ph.	OS in l.w.
1	200	24+j24	4+j4	ca	-	Aa
2	100	12-j12	4-j4	-	ab	Cc
3	220	11+j11	7+j7	bc	-	Bb
4	150	-j30	j20	-	bc	Cc
5	120	9-j9	9-j9	ab	-	Aa
6	220	j24	-j16	-	bc	Bb
7	380	12+j12	6+j6	ca	-	Aa
8	150	15-j15	5-j5	-	ab	Cc
9	380	j60	j20	bc	-	Aa
10	220	12+j12	6+j6	-	ca	Bb
11	380	24-j24	6-j6	ab	-	Cc
12	220	15-j15	j10	-	bc	Bb
13	380	30+j30	-j20	ca	-	Cc
14	200	9+j9	5+j5	ab	-	Aa
15	380	24+j24	-j8	-	bc	Bb

RECOMMENDED LITERATURE

1. The schoolbook "Theoretical Electrical Engineering" / Nabokova O.V. - Zaporozhye: ZNTU, 2014. - 530 p.

2. Мілих В.І. Електротехніка та електромеханіка. К.: Каравела, 2010. – 376 с.

3. "Analysis and Synthesis of Elecnric Circuits" / G.Zeveke, P. Ionkin, A,Netushil, S.Strakhov. – Moscow: Mir Publishers, 1979. - 850 p.

4. Paul C.R., Nazzsar S.A. Introduction to electrical engineering. - New York: Electrical Engineering University of Kentucky, 1999. - 800 p.